HIGHER SECONDARY – SECOND YEAR

MATHEMATICS

QUESTION BANK

BASED ON NEW SYLLABUS

VOLUME – II

Contains

Classification of Text Book Problems

Creative Questions

Supportive Materials

Prepared under the Guidance of

Dr.A.Anitha

CHIEF EDUCATIONAL OFFICER

CHENNAI DISTRICT

Prepared By

Thiru. S.ANANTHA KRISHNAN, M.Sc.,B.Ed.,M.Phil Head Master, M.F.S.D.Hr.Sec.School, Sowcarpet, Chennai – 79	**Thiru. R.AVOODAIAPPAN** M.Sc.,B.Ed., P.G.Assistant (Mathematics), Govt.Girls.Hr.Sec.School, Ashok Nagar, Chennai – 83
Tmt.C.BAGAVATHI M.Sc.,B.Ed.,M.Phil., P.G.Assistant (Mathematics), Sri Akobila Mutt Oriental Hr.Sec.School, West Mambalam, Chennai - 33	**Thiru. M.MADHUSUDAN** M.Sc.,M.Ed.,M.Phil., P.G.Assistant (Mathematics), AB Parekh Hr.Sec.School, Park Town, Chennai – 3
Tmt.D.PHILOMINA RANI M.Sc.,M.Ed.,M.Phil., P.G.Assistant (Mathematics), Manilal. M.Mehta Girls Hr.Sec.School, Park Town, Chennai – 3	**Tmt.S.INDRA** M.Sc.,B.Ed.,M.Phil., P.G.Assistant (Mathematics), K.C.SankaralingaNadarHr.Sec.School, , Chennai – 21
Tmt.S.VISALATCHI M.Sc.,B.Ed., P.G.Assistant (Mathematics), MurugaDhanushkodi Girls Hr.Sec.School, Chennai – 81	**Thiru.S.SENTHILVEL** M.Sc.,B.Ed.,M.Phil., P.G.Assistant (Mathematics), T.T.V. Hr.Sec.School, Sowcarpet, Chennai – 79
Thiru.D.RAMAN M.Sc.,B.Ed.,M.Phil., P.G.Assistant (Mathematics), H.U.C. Hr.Sec.School, Choolai, Chennai – 112	**Thiru.RAJEDNDRA PRATAP YADAV** M.Sc.,B.Ed., P.G.Assistant (Mathematics), M.F.S.D.Hr.Sec.School, Sowcarpet, Chennai – 79
Tmt.V.BALAMEENAKSHI M.Sc.,M.Ed.,M.Phil., P.G.Assistant (Mathematics), Dr.AmbedkarGovt.Hr.Sec.School, Egmore, Chennai - 8	**Thiru.A.MARIAPPAN** M.Sc.,B.Ed.,(Type Set) P.G.Assistant (Mathematics) Municipal Hr.Sec.School, ZameenPallavaram Chennai - 117

Created Questions

Chapter 7 – Application of Differential Calculus

Two Marks Questions

Exercise	Exercise problems	Example Problems
7.1	4, 6	7.2, 7.3 and 7.16
7.2	1(i), (ii),	
7.3	1 each problem	7.33, 7.34, 7.35
7.5	1,2,3 and 4	7.42, 7.46, 7.47 and 7.52
7.9	I (i), (ii)	

Three Marks Questions

Exercise	Exercise problems	Example Problems
7.1	1, 5 and 7	7.1, 7;4l 7.5, 7.8, 7.11, 7.12
7.2	2, 3, 4 and 5(i), (ii) and (iii), 9 and 10	7.19, 7.20, 7.21, 7.22, 7.25, 7.26, 7.27, 7.28 and 7.29
7.4	1 (i), (ii), (iii) 2, 3 and 4	7.32,
7.5	5, 6, 7 and 12	7.36, 7.37, 7.38, 7.39, 7.40 and 7.41
7.6	1(i), (II), (III) and (iv)	7.48, 7.49, 7.51
7.8	1, 2, 3 and 11	7.58, 7.59
7.9	1 (iii), (iv) and (v)	7.66, 7.67 and 7.68

Five Marks Questions

Exercise	Exercise problems	Example Problems
7.1	2, 3, 8, 9 and 10	7.6, 7.7, 7.9, 7.10
7.2	5, 6 and 7	7.13, 7.14, 7.15, 7.17 and 7.18
7.4	1(iv), (v) and (vi)	7.24
7.6	2 (ii), (iii), (iv) and (v)	7.43, 7.44 and 7.45
7.7	1 (i), (ii) and (iii) 2 (i), (ii) and (iii) 3	7.50, 7.53, 7.55, 7.55, 7.56, 7.57, 7.60 and 7.61
7.8	4, 5, 6, 7, 8, 9, 10 and 12	7.62, 7.63, 7.64 and 7.65
7.9	2 (i), (ii), (iii), (iv) and (v)	7.69, 7.70, 7.71 and 7.72

Created Questions

Two marks Questions

1. Find x if the rate of decrease of $\frac{x^2}{2} - 2x + 5$ is twice the decrease of x

2. Find the point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa

3. Find the point at which the curve $y - e^{xy} + x = 0$ has a vertical tangent

4. Find the equation of the tangent to the curve $y^2 = 4x + 5$ and which is parallel to y = 2x + 7.

5. Using Rolle's theorem find the value of c for f(x) = sin x in$[0, 2\pi]$.

6. Verify Rolle 's Theorem for $f(x) = |x - 1|, 0 \le x \le 2$.

7. If $f(1) = 10$ and $f'(x) \ge 2$ for $1 \le x \le 4$ how small can f (4) possible be?

8. Verify Lagrange's Mean Value theorem for $f(x) = \sqrt{x-2}$ in the interval $[2, 6]$

9. Obtain Maclaurin's Series expansion for e^{3x}

10. Expand the polynomial $f(x) = x^2 - 3x + 2$ in powers of (x – 2)

11. Evaluate the following limits, if necessary using L'Hopitalrule

    i) $\lim_{x\to 2} \frac{\sin \pi x}{2-x}$ ii) $\lim_{x\to 2} \frac{x^n - a^n}{x-2}$ iii) $\lim_{x\to \infty} \frac{\sin\frac{2}{x}}{\frac{1}{x}}$ iv) $\lim_{x\to \infty} \frac{x^2}{e^x}$

12. Find the absolute extreme of the function $f(x) = x^2 - 2x + 2$ on the closed interval [0, 3]

13. Prove that the function $f(x) = 2x^2 + 3x$ is strictly increasing on $\left[-\frac{1}{2}, \frac{1}{2}\right]$

14. Prove that the function $f(x) = e^{-x}$ is strictly increasing on [0, 1]

15. Find the stationary points of each of the following functions

    i) $f(x) = 2x - 3x^2$ ii) $f(x) = x^3 - 3x + 1$

16. Find the critical number of $f(\theta) = \theta + \sin\theta$ in $[0, 2\pi]$

17. Determine the domain of concavity of the curve $y = 2 - x^2$

18. Determine the domain of convexity of the function $y = e^x$

Three Marks Questions

1. The side of a square is equal to the diameter of a circle. If the side and radius change at the same rate then find the ratio of the change of their areas.

2. The volume of a cube is increasing at the rate of $8cm^3/s$. How fast is the surface area increasing when the length of an edge is 12 cm?

3. Find the equation of normal to the curve $y^4 = ax^3$ at (a, a)

4. Find the angle between two curves $2x^2 + y^2 = 20$ and $x^2 - 4y^2 + 8 = 0$

5. Apply Rolle's theorem to find points on curve $y = -1 + \cos x$, where the tangent is parallel to x – axis in $[0, 2\pi]$

6. Verify Rolle's theorem for $f(x) = e^x \sin x,\ 0 \le x \le \pi$

7. A cylindrical hole 4 mm in diameter and 12 mm deep in a metal block is reboared to increase the diameter to 4.12mm. Estimate the amount of metal removed.

8. Verify Mean Value theorem for $f(x) = (x-1)(x-2)(x-3)$ in $[0, 4]$

9. Obtain Maclaurin's series for $\frac{1}{1+x}$

10. Write down the Taylor series expansion of the function cos x in ascending powers $x - \frac{\pi}{4}$ upto three non-zero terms.

11. Evaluate the following limits, if necessary use L'Hopitals rule

    i) $\lim_{x \to 0^+} x^{\sin x}$ ii) $\lim_{x \to 0} \frac{\cot x}{\cot 2x}$ iii) $\lim_{x \to \frac{\pi}{2}^-} (\tan x)^{\cos x}$

12. Find the absolute extrema of the following functions on the given closed interval.

    i) $f(x) = x^3 - 3x^2 + 1, -\frac{1}{2} \le x \le 4$

    ii) $f(x) = x - 2\sin x, 0 \le x \le 2\pi$

13. Prove that the function $f(x) = x(x-1)(x+1)$ is strictly increasing in $[-2, -1]$

14. Find the intervals of monotonicities of the function $f(x) = \sin x, x \in [0, 2\pi]$

15. Find the intervals of monotonicities and find the local extremum for the following functions

    i) $f(x) = 20 - x - x^2$ ii) $f(x) = x(x-1)(x+1)$ on [0, 2]

16. Find the intervals of concavity and the point of inflection of the function $f(x) = 2x^3 + 5x^2 - 4x$

17. Find the local extrema for the following functions using second derivate test.

    i) $x^3 - x$ ii) $x^3 - 3x^2 + 1, -\frac{1}{2} \le x \le 4$

18. Find two numbers whose sum is 100 and whose product is a maximum.

19. Find two positive numbers whose product is 100 and whose sum is minimum.

20. Find the slant asymptote for the function $f(x) = \frac{x^2-6x+11}{x+4}$

Five Marks Questions

1. Sand is pouring from a pipe at the rate of 12 cm^3/sec. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing, when the height is 4 cm?

2. Gas is escaping from a spherical balloon at the rate of 900 cm^3/sec. How fast is the surface area and radius of the balloon shrinking when the radius of the balloon is 30 cm?

3. The length x of a rectangle is decreasing at the rate of 3cm/min and the width is increasing at the rate of 2cm/min. When x = 10cm, y = 6cm, find the rate of change of (i) the perimeter (ii) the area of the rectangle.

4. A particle moves along the curve $y = \frac{4}{3}x^3 + 5$. Find the points on the curve at which y coordinate changes as fast as x-coordinates.

5. Find the points on the curve $y = x^3 - 2x^2$ at which the tangent lines are parallel to the line y = 3x – 2.

6. If the curves $4x = y^2$and 4xy = k cut at right angles show that k^2=512.

7. A missle fired from ground level rises x metres vertically upwards in t seconds and $x = 100t - \frac{25}{2}t^2$. Find the (i) initial velocity of the missile (ii) the time when the height of the missile is maximum (iii) the maximum height reached (iv) the velocity which the missile strikes the ground.

8. Show that the equation of the normal to the curve $x = acos^3\theta, y = asin^3\theta$ at $'\theta'$ is $xcos\,\theta - y\sin\theta = a\cos 2\theta$.

9. A water tank has a shape of an inverted cone with its axis vertical and vertex lower most. Its semi vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cm^3/hr. Find the rate at which the level of the water is rising at that instant when the depth of the water is 4 m.

10. Evaluate :$\lim_{x\to 1} \frac{1}{x^{x-1}}$

11. If $f(x) = a\log x + bx^2 + x$ has extreme values at x = 1, x = 2 then find a and b.

12. Find the intervals of concavity and points of inflexion for $f(x) = x^3 - 15x^2 + 75x - 50$.

13. Find the intervals for which the function $f(x) = 2x^3 - 9x^2 - 12x + 1$ is increasing or decreasing and find the local extremes.

14. Show that the surface area of a closed cuboid with a square base and given volume is minimum, when it is a cube.

15. Find the local maximum and local minimum values for $f(x) = (x-1)(x+2)^2$ using second derivative test.

16. If the curve $y^2 = x$and xy = k are orthogonal, then prove that $8k^2 = 1$

17. Find the intervals of concavity and the points of inflection of $f(x) = 12x^2 - 2x^3 - x^4$

18. Find the points of inflection and determine the intervals of concavity of $y = e^{-x^2}$

19. A manufacturer can sell x items at a price of rupees $\left(5 - \frac{x}{100}\right)$ each. The cost price of x items is $Rs.\left(\frac{x}{5} + 500\right)$. Find the numbers of items he should sell to earn maximum profit.

20. Find the local maximum and local minimum values of $f(x) = x^4 - 3x^3 + 3x^2 - x$.

21. Find the equation of the tangent and the normal to the curve $y = 2sin^2 3x, x = \frac{\pi}{6}$

Chapter 8 – Differentials and Partial Derivatives

Two Marks Questions

Exercise	Exercise problems	Example Problems
8.1		8.3
8.2	1 each, 2 each, 6, 7 and 8	8.5, 8.6 and 8.7
8.3	1, 2 and 4	-
8.4	1 (i), (ii)	-
8.5	-	8.16
8.6	-	-
8.7	1 each	8.21

Three Marks Questions

Exercise	Exercise problems	Example Problems
8.1	1, 2 each, 3 each, 4, 5, 6 and 7	8.1, 8.2 and 8.4
8.2	3 each, 4, 5, 9, 10 and 11	-
8.3	3, 5, 6 and 7	8.8, 8.9 and 8.10
8.4	1 (iii), (iv), 2(iii), 3, 4, 5 each, 6, 9 and 10	8.11, 8.12 and 8.15
8.5	1, 2, 3, 4 and 5	8.17
8.6	1, 2, 3, 4, 5 and 8	8.18, 8.19 and 8.20
8.7	4	-

Five Marks Questions

Exercise	Exercise problems	Example Problems
8.1	-	-
8.2	-	-
8.3	-	-
8.4	2 (i), (ii), 7 and 8	8.13 and 8.14
8.5	-	-
8.16	6, 7 and 9	-
8.7	2, 3, 5 and 6	8.22

Created Questions

Two Marks Questions

1. If $u = \log\sqrt{x^2+y^2}$ then prove that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \frac{1}{x^2+y^2}$

2. If $u = x^2y + y^2z + z^2x$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = (x+y+z)^2$.

3. If $u = x^3 + y^3 + z^3 - 3xyz$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 3u$

4. If $u = x^3 + 3xy^2 + y^3$, then prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

5. If $f(x,y) = 10x - x^2 + xy$find (i) $f_x(2,6)$ (ii) $f_y(2,6)$

6. If $u = e^{\frac{x}{y}} \sin\left(\frac{x}{y}\right) + e^{\frac{y}{x}} \cos\left(\frac{y}{x}\right)$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

7. If $w = e^{xy}, x = at^2, y = 2at$, find $\frac{dw}{dt}$

8. If $w = \log(x^2+y^2), x = \cos\theta, y = \sin\theta$, find $\frac{dw}{d\theta}$

9. If $w = e^{x^2+y^2}, x = \cos\theta, y = \sin\theta$, find $\frac{dw}{d\theta}$

10. If $w = xye^{xy}$ find $\frac{\partial^2 u}{\partial x \partial y}$

11. Using differentials, find the approximate value of $\sin\left(\frac{22}{14}\right)$

12. Show that the percentage error in computing the volume of a sphere, due to an error in measuring the radius, is approximately equal to three times the percentage error in the radius.

13. The pressure P and the volume V of a gas are connected by the relation $PV^{1.4} =$ constant. Find the % error in P corresponding to a decrease of $\frac{1}{2}\%$ in V.

14. If y = sin x and x changes from $\frac{\pi}{2}$to $\frac{22}{14}$, what is the approximate change in y.

15. Estimate $\cos\left(\frac{\pi}{4} + 0.01\right) - \cos\left(\frac{\pi}{4}\right)$

16. Find a linear approximation to $f(x) = 3xe^{2x-10}$at x = 5.

17. Find the linear approximation to $g(z) = \sqrt[4]{z}$at z = 2.

18. Without using any kind of computational aid use linear approximation to find the value of $e^{0.1}$

19. Show that $(1+x)^n = 1+nx$ approximately if x is close to zero.

20. Calculate df for $f = \sqrt{2x+5}$, when x = 22 and dx = 3.

Three Marks Questions

1. If $u = \log(x^2+y^2+z^2)$, then prove that $x\frac{\partial^2 u}{\partial y \partial x} = y\frac{\partial^2 u}{\partial z \partial x} = z\frac{\partial^2 u}{\partial x \partial y}$

2. If $f(x,y) = 3x^2+4y^3+6xy-x^2y^3+5$ then find (i) $f_x(1,-1)$ (ii) $f_{yy}(1,1)$

   (iii) $f_{xy}(2,1)$

3. If $u = \sin 3x cos\ 4y$ then prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

4. If $u = \frac{x}{y^2} - \frac{y}{x^2}$, then prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

5. If $u = (x-y)(y-z)(z-x)$, then prove that $u_x + u_y + u_z = 0$

6. If $v = \log(\tan x + \tan y + \tan z)$, then prove that $\sum \sin 2x \frac{\partial v}{\partial x} = 2$

7. If $z = e^{x^3+y^3}$ then prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2$

8. If $w = \log(x^2+y^2)$ and$x = rcos\ \theta$ and $y = rsin\ \theta$ then, find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial \theta}$

9. If $w = xy + z$ and$x = \cos t, y = \sin t, z = t$ then find $\frac{dw}{dt}$

10. If u is a homogeneous function in x and y of degree n then prove that $x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x \partial y} = (n-1)\frac{\partial u}{\partial x}$

11. Using linear approximation find$\sqrt{0.082}$

12. Find the approximate value of $\left(\frac{17}{81}\right)^{\frac{1}{4}}$ using linear approximation.

13. Find the limit for the following if it exists. $\lim_{(x,y)\to(1,1)} \frac{2x^2-xy-y^2}{x^2-y^2}$

14. Find the limit for the following if it exists. $\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^4+3y^4}$

15. If $w = x^2 + y^2$ and $x = u^2 - v^2, y = 2uv$ then find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$

16. Evaluate : $\lim_{(x,y)\to(2,0)} \frac{\sqrt{2x-y}-2}{2x-y-4}$

17. Evaluate : $\lim_{(x,y,z)\to(-1,0,4)} \frac{x^3-ze^{2y}}{6x+2y-3z}$

18. Evaluate : $\lim_{(x,y)\to(0,0)} \frac{x^2-xy}{\sqrt{x}-\sqrt{y}}$

19. z is a homogeneous function in x and y of degree n then prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = (ax + by + n)V$, where $V = ze^{ax+by}$

20. If $w = x + 2y + z^2$ and $x = \cos t, y = \sin t, z = t$ then find $\frac{dw}{dt}$.

Five Marks Quesitons

1. If $u = \tan^{-1}\left(\frac{x^2+y^2}{x-y}\right)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\sin 2u$

2. If $P = CL^\alpha k^\beta, c > 0, \alpha + \beta = 1$, then prove that $k\frac{\partial P}{\partial k} + L\frac{\partial P}{\partial L} = P$. (Without using Euler's theorem)

3. If $u = \log\sqrt{x^2 + y^2 + z^2}$, then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{x^2+y^2+z^2}$

4. If $w = u^2 e^v$ where $u = \frac{x}{y}$ and $v = y\log x$. Find $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$

5. Verify Euler's theorem for the function $f(x,y) = \frac{1}{\sqrt{x^2+y^2}}$

6. The height of a cone is increased by k%, its semi vertical angle remaining the same. What is the approximate percentage increases in (i) T.S.A (ii) Volume assuming k is small.

7. Find $\frac{\partial w}{\partial u}, \frac{\partial w}{\partial v}$ if $w = \sin^{-1}(xy)$ where x = u + v, y = u – v

8. If $w = x^2\sin\left(\frac{x}{y}\right) + y^2\cos\left(\frac{x}{y}\right) + xy\tan\left(\frac{x}{y}\right)$, then prove that $x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} = 2w$

9. If $u = \log(xy + yz + zx)$, then prove that $\sum x\frac{\partial u}{\partial x} = 2$

10. Find the approximate value of $\sqrt[3]{1.02} + \sqrt[4]{1.02}$

Chapter 9 – Application of Integration

Two Marks Questions

Exercise	Exercise problems	Example Problems
9.1	-	-
9.2	-	-
9.3	1(i). (ii), 2 (ii), (iii), (v) and (vi)	9.5, 9.7 and 9.24
9.4	-	-
9.5	-	9.35
9.6	1 (i), (ii) and (v)	9.37, 9.39 (i) and (ii) and 9.42
9.7	1 (i)	9.44
9.8	-	-
9.9	1, 2	-

Three Marks Questions

Exercise	Exercise problems	Example Problems
9.1	1, 2 and 3	-
9.2	-	9.2 and 9.3
9.3	1 (ii), (iii), (iv), (v) and (vi) 2 (i) to (vi) each	9.6, 9.8, 9.9, 9.10, 9.11, 9.19, 9.20, 9.22, 9.23, 9.25, 9.26 and 9.29
9.4	4	9.31, 9.32, 9.33 and 9.34
9.5	1 (i), (ii)	9.36
9.6	1 (iii), (iv) and (vi)`	9.38, 9.40, 9.42
9.7	1 (ii) and 2	9.43, 9.45 and 9.46
9.8	1 and 2	9.47, 9.48, 9.49, 9.50, 9.51, 9.52, 9.53
9.9	3, 4, 5 and 6	9.62, 9.63, 9.64, 9.65, 9.66, 9.67, 9.68 and 9.69

Five Marks Questions

Exercise	Exercise problems	Example Problems
9.1	-	9.1
9.2	1 (i), (ii) each	9.4
9.3	2 (vi), (vii), (viii), (ix), (x) and (xi)	9.13, 9.14, 9.15, 9.16, 9.17, 9.18, 9.21, 9.27, 9.28 and 9.30
9.4	1, 2 and 3	-
9.5	-	-
9.6	-	-
9.7	-	-
9.8	3, 4, 5, 6, 7, 8, 9, and 10	9.54, 9.55, 9.56, 9.57, 9.58, 9.59, 9.60 and 9.61
9.9	5 and 6	-

Created Questions

Two Marks Questions

1. Evaluate $\int_0^{\frac{\pi}{2}} \log(\tan x)\ dx = 0$

2. Evaluate $\int_0^{\frac{\pi}{2}} e^{2x} \cos x\, dx$

3. Evaluate $\int_0^{\frac{\pi}{2}} e^{3x} \cos x\, dx$

4. Evaluate $\int_0^{\frac{\pi}{2}} e^{-x} \sin x\, dx$

5. Evaluate $\int_{-1}^{1} \log\left(\frac{3-x}{3+x}\right)\ dx$

6. Evaluate $\int_0^1 \left(\frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}}\right) dx$

7. Evaluate $\int \sum_{r=0}^{\infty} \frac{x^r 2^r}{r!} dx$

8. Evaluate $\int e^{3x} 3^{2x} 5^x dx$

9. Evaluate $\int_0^{\infty} (a^{-x} - b^{-x}) dx$

10. Find the slope of the tangent to the curve $y = \int_0^x \frac{dt}{1+t^3}$at x = 1.

11. Find the area bounded by the curve $y = \sin 2x$ between the ordinates $x = 0, x = \pi$ and x-axis.

12. Find the volume of the solid generated when the region enclosed by $y = \sqrt{x}$, y = 2 and x = 0 is revolved about y-axis.

13. Find the volume of the solid $y = x^3$, x = 0,y = 1 is revolved about the y-axis.

14. If $\int_0^{\infty} \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)(x^2+c^2)} = \frac{\pi}{2(a+b)(b+c)(c+a)}$, then find $\int_0^{\infty} \frac{dx}{(x^2+4)(x^2+9)}$

15. Evaluate $\int_0^1 \frac{e^x}{1+e^{2x}} dx$

16. Evaluate $\int_1^2 \frac{3x}{9x^2-1} dx$

17. Find the area of the region enclosed by the curve $y = \sqrt{x} + 1$ the axis of x and the lines x = 0 and x = 4.

18. Find the volume of the solid obtained by revolving the area of the triangle whose sides are x = 4, y = 0 and 3x – 4y = 0 about x axis.

19. Find the area of the region bounded by the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$

(x, y > 0) and the co-ordinate axes.

20. Find the area of the region bounded by the curves $y = 2^x$,

$y = 2x - x^2$ and the lines x = 0 and x = 2

21. Find the area bounded by $y = x^2 + 2$, x-axis, x = 1 and x = 2.

22. Find the area of the region bounded by the curve y = sin x and the ordinate x = 0, $x = \frac{\pi}{3}$

23. Find the area enclosed between the parabola $y^2 = 4ax$ and the line x = a, x = 9a.

24. Find the area bounded by y = cos x, y = x + 1, y = 0.

25. Find the area bounded by the curve y = cosax in one arc of the curve.

Three Marks

1. If $f(x) = \begin{vmatrix} x+1 & 2x+1 & 3x+1 \\ 2x+1 & 3x+1 & x+1 \\ 3x+1 & x+1 & 2x+1 \end{vmatrix}$, then find $\int_0^1 f(x)dx$.

[Hint : $R_2 \to R_2 - R_1$; $R_3 \to R_3 - R_1$]

2. If $f(x) = \begin{vmatrix} 2cos^2x & \sin 2x & -\sin x \\ \sin 2x & 2sin^2x & \cos x \\ \sin x & -\cos x & 0 \end{vmatrix}$, then find $\int_0^{\frac{\pi}{2}} [f(x) + f'(x)]dx$

[Hint : $f(x) = 2, f'(x) = 0$]

3. If $f(x) = \begin{vmatrix} \cos x & 1 & 0 \\ 1 & 2\cos x & 1 \\ 0 & 1 & 2\cos x \end{vmatrix}$, then find $\int_0^{\frac{\pi}{2}} f(x)dx$

[Hint : f(x) = cos 3x]

4. Evaluate $\int_0^{\pi} \sqrt{1 + 4sin^2\frac{x}{2} - 4\sin\frac{x}{2}}\,dx$

5. Evaluate $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$

6. Evaluate $\int_2^4 \frac{\log x^2}{\log x^2+\log(36-12x+x^2)}dx$

7. Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{1+tan^3x}$

8. Evaluate $\int_0^1 x(1-x)^n dx$

9. Evaluate $\int_0^1 x(1-x)^{10} dx$

10. If $\int_0^\infty \frac{dx}{(x^2+4)(x^2+9)} = k\pi$, then find the value of k.

11. Find the area bounded by the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line y = 2.

12. The area of the curve $y = \sin ax$ bounded by y = 0, $x = \frac{\pi}{a}$ and $x = \frac{3\pi}{a}$ is 3a, then find a.

13. Find the area bounded by the curve $y = x|x|$, x-axis and the ordinates $x = -1, x = 1$

14. Find the area between the curve $y = 1 - |x|$ and x-axis.

15. Find the area of the region described by $A = \{(x, y), x^2 + y^2 \leq 1 \& y^2 \leq 1 - x\}$

16. Find the area bounded by the curves $y = x^3, y = x^2$ and the ordinates x = 1 and x = 2.

17. Find the area bounded by the curve $y = \sin x, y = \cos x$ and the y-axis.

18. Find the area bounded by $x = 0, x = 6 + 5y - y^2$

19. Find the area bounded by $y = 3x$ and $y = x^2$

20. Evaluate $\int_0^\pi cos^3x dx$

21. Evaluate $\int_0^{\frac{\pi}{2}} sin^9x\, dx$

22. Evaluate $\int_0^{\frac{\pi}{2}} cos^{10}x\, dx$

23. Evaluate $\int_0^{50} [x - |x|]\, dx$

24. Evaluate $\int_{-2}^{3} |1 - x^2|\, dx$

25. Evaluate $\int_0^{\frac{\pi}{2}} \frac{cos^{3/2}x}{cos^{3/2}x+sin^{3/2}x} dx.$

Five Marks Questions

1. Find the area of the curve $y^2 = (x-5)^2(x-6)$ between

   i) x = 5 and x = 6   ii) x = 6 and x = 7.

2. Find the area of the region bounded by the curves $x^2 + 2y^2 = 0$and $x + 3y^2 = 1$.

3. Find the ratio of the area between the curves $y = \cos x$ and $y = \cos 2x$ and x-axis from x = 0 to $x = \frac{\pi}{3}$

4. Find the area bounded by the curves $y = |x| - 1$ and $y = -|x| + 1$

5. Find the value of 'c' for which the area bounded by the curve

   $y = 8x^2 - x^5$, the lines x = 1, x = c and x-axis is $\frac{16}{3}$.

6. Find the area of the loop of the curve $3ay^2 = x(x-a)^2$

7. Find the area of the region common to the circle $x^2 + y^2 = 16$ and the parabola $y^2 = 6x$.

8. Prove that $\int_0^{\frac{\pi}{2}} \sin 2x \ \log(\tan x)\, dx$

9. Find the area bounded by $x = at^2, y = 2at$ between the ordinates corresponding to t = 1 and t = 2.

10. Using integration find the area of the triangle with sides y = 2x + 1,

    y = 3x + 1and x = 4.

11. Find the area bounded by the curve $y = xe^x$ and $y = xe^{-x}$ and the line x = 1.

12. Find the area of the region bounded by $y = e^x$ and $y = e^{-x}$ and the line x = 1.

13. Find the area of the region bounded by $a^2y^2 = a^2(a^2 - x^2)$

14. Find the area of the region enclosed by the two circles $x^2 + y^2 = 1$and $(x-1)^2 + y^2 = 1$.

15. AOB is the positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where OA = a and OB = b. Find the area between the arc AB and chord AB of the ellipse.

16. Show that the ratio of the area under the curve y = sin x and

y = sin 2x between x = 0 and $x = \frac{\pi}{3}$ and x- axis are as 2 : 3.

17. Find volume of the solid generated by the revolution of the loop of the curve $x = t^2, y = t - \frac{t^3}{3}$ about the x-axis.

18. Find the area bounded by the curve $y^2(2a - x) = x^3$ and the line

x = 2a.

19. Find the area bounded by the curve $xy^2 = a^2(a - x)$ and the y-axis.

20. Find the area enclosed by the parabolas $5x^2 - y = 0$ and $2x^2 - y + 9 = 0$.

Chapter 10 – Ordinary Differential Equations

Two Marks Questions

Exercise	Exercise problems	Example Problems
10.1	1 (i) to (x)	10.1 each
10.2	1 each 2 (i) and (v) each	-
10.3	1	10.5
104	1 (i) and (ii) each, 2 (i)	10.7 and 10.8
10.5	4 (i)	10.16

Three Marks Questions

Exercise	Exercise problems	Example Problems
10.1	-	-
10.2	-	-
10.3	2, 3, 5, 7 and 8	10.4 and 10.6
10.4	2 (ii), 3, 4, 5 and 8	10.9 and 10.10
10.5	1, 4 (ii), (iii), (iv), (vii), (viii) and (ix)	10.11, 10.12, 10.13 and 10.14
10.6	-	10.20
10.7	1, 5, 6 and 10	10.22, 10.24 and 10.26
10.8	4	10.28 and 10.30

Five Marks Questions

Exercise	Exercise problems	Example Problems
10.1	-	-
10.2	-	-
10.3	4 and 6	-
10.4	6 and 7	-
10.5	2, 3, 4 (v), (vi) and x	10.15
10.6	1,2,3,4,5,6,7 and 8	10.17, 10.18, 10.19 and 10.21
10.7	2, 3, 4, 7, 8, 9, 11, 12, 13, 14 and 15	10.23 and 10.25
10.8	1, 2, 3, 5, 6, 7, 8, 9 and 10	10.27, 10.29 and 10.30

Created Questions

Two Marks Questions

1. Determine the order and degree of $\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = k$

2. Find the order and degree of $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$

3. Find the order and degree of $y + \frac{dy}{dx} = \frac{1}{4}\int y\, dx$

4. Form the Differential Equation representing the family of curves

   $y = Acos\ (x + B)$where A and B are parameters.

5. Form the D.E of family of parabolas having vertex at the origin and axis along positive y-axis.

6. Form the D.E corresponding to $y = e^{mx}$ by eliminating 'm'

7. Solve: $\frac{dy}{dx} = \frac{2x}{x^2+1}$

8. Solve : $\frac{dy}{dx} = \frac{e^x-e^{-x}}{e^x+e^{-x}}$

9. Solve: $\frac{dy}{dx} + y = 1$

10. solve: $xdy + ydx = xydx$

Three Marks

1. Form the D.E of the family of curves $c(y + c)^2 = x^3$, where c is the parameter.
2. Form the D.E of family of curves represented by $y = c(x - c)^2$, where c is the parameter.
3. Form the D.E to $y^2 = a(b - x)(b + x)$ by eliminating a and b as its parameters.
4. Find the D.E of all circles touching x-axis at the origin.
5. Find the D.E of all circles touching y-axis at the origin.
6. Obtain the D.E of all circles of radius 'r'

7. Find the D.E of all circles in the first quadrant which touch the co-ordinate axes.
8. Form the D.E corresponding to $y^2 - 2ay + x^2 = a^2$ by eliminating 'a'
9. Show that the function $y = Acos\ 2x - B \sin 2x$ is a solution of the D.E $y_2 + 4y = 0$
10. Verify that $y = -x - 1$ is a solution of the D.E$(y - x)dy - (y^2 - x^2)dx = 0$
11. Solve: $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$
12. Solve : $\frac{dy}{dx} + 2y^2 = 0, y(1) = 1$ and find the corresponding solution of the curve.
13. Solve : $\cos x(1 + \cos y)dx - \sin y(1 + \sin x)dy = 0$
14. Solve : $(1 + e^{2x})dy + (1 + y^2)e^x dx = 0$
15. Solve : $(x^2 - yx^2)dy + (y^2 + x^2y^2)dx = 0$
16. At any point (x, y) of a circle the slope of the tangent line segment is the slope of the line segment joining the point of contact to the point $(-4, -3)$. Find the equation of the curve given that it passes through $(-2,1)$
17. Solve : $\frac{dy}{dx} - \frac{y}{x} = 2x^2, x > 0$
18. Solve : $(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{1}{x^2-1}$
19. Solve : $\frac{dy}{dx} - y = e^x$
20. Solve : $ydx + (x - y^3)dy = 0$

Five Marks Questions

1. Sovle$(x + 2)\frac{dy}{dx} = x^2 + 4x - 9$. Also find the domain of the function.
2. Solve: $\frac{dy}{dx} = (sin^3xcos^2x + xe^x)dx$
3. Solve : $e^{\frac{dy}{dx}} = x + 1,\ y(0) = 5$

4. Solve : $(1+x^2)\frac{dy}{dx} - x = 2\tan^{-1}x$

5. Solve : $(x^3+x^2+x+1)\frac{dy}{dx} = 2x^2+x$

6. Solve : $(1+y^2)(1+\log x)dx + xdy = 0$, given that x = 1, y = 1.

7. In a bank principal increases at the rate of 5% per year. In how many years Rs.1000 doubled itself.

8. Sovle : $(x+y+1)^2dy = dx, y(-1) = 0$

9. Solve : $x^2dy + y(x+y)dx = 0$ given that y = 1 when x = 1.

10. Solve : $(x^2+xy)dy = (x^2+y^2)dx$

11. Solve : $x\frac{dy}{dx}\sin\left(\frac{y}{x}\right) + x - y\sin\left(\frac{y}{x}\right) = 0, y(1) = \frac{\pi}{2}$

12. Solve : $2x^2\left(\frac{dy}{dx}\right) - 2xy + y^2 = 0, y(e) = e$

13. Solve : $cos^2x\frac{dy}{dx} + y = \tan x \quad \left(0 \le x \le \frac{\pi}{2}\right)$

14. Solve : $\frac{dy}{dx} = -\frac{x+y\cos x}{1+\sin x}$. Also find the domain of the function

15. Solve : $(x-\sin y)dy + \tan y dx = 0\,, y(0) = 0$

16. It is given that the rate at which some bacteria multiply is proportional to the instantaneous number present. If the original number of bacteria doubles in two hours, in how many hours will it be five times.

17. A thermometer reading $80°F$ is taken outside. Five minutes later the thermometer reads $60°\mathrm{F}$. After another five minutes the thermometer reads $50°\mathrm{F}$. What is the temperature outside?

18. Water at temperature $100℃$ cools in 5 minutes to $80℃$ in a room of temperature $30℃$. Find (i) the temperature of water after 10 minutes. (ii) the time when the temperature is $40℃$.

Chapter 11 – Probability Distributions

Two Marks Questions

Exercise	Exercise problems	Example Problems
11.1	1	11.3 and 11.4
11.2	1	11.5
11.3	1	-
11.4	5 and 6	-
11.5	1 each , 3 each and 4	11.19 each

Three Marks Questions

Exercise	Exercise problems	Example Problems
11.1	2, 3, 4 and 5	11.1 and 11.2
11.2	2, 3, 4 and 5	11.6, 11.7 and 11.9
111.3	2, 3, 5 and 6	11.13
11.4	1 each, 2, 3, 4, 7 and 8	11.18
11.5	2, 5, 8 and 9	-

Five Marks Questions

Exercise	Exercise problems	Example Problems
11.2	6 and 7	11.8, 11.10
11.3	4	11.11, 11.12, 11.14 and 11.15
11.4	-	11.16, 11.17
11.5	6 and 7	11.20, 11.21 and 11.22

Created Questions

Two Marks Quesitons

1. Define a random variable.
2. How many types of random variables are there? What are they?
3. Define discrete random variable.
4. Define Probability Mass Function.
5. Define Distribution function.
6. Define Probability Density function.
7. Define Continuous random variable.
8. What is meant by the expected value of a random variable X?
9. Define Variance of a random variable X?
10. Prove that $E(aX+b) = aE(X) + b$
11. Prove that $Var(X) = E(X^2) - [E(X)]^2$
12. Prove that $Var(aX+b) = a^2 Var(X)$
13. When do we say that a discrete random variable X is a binomial random variable.
14. What is the Probability Mass function of a binomial random variable.?
15. The probability distribution of a random variable X is given below.

X	0	1	2	3
P(X)	k	$\frac{k}{2}$	$\frac{k}{4}$	$\frac{k}{8}$

i) Find k ii) P(X > 2)

16. A coin is tossed twice. If X is a random variable defined as the number of heads minus the number of tails, then obtain its probability distribution.
17. A coin is tossed until a head appears or the tail appears 4 times in succession. Find the probability distribution of the number of tosses.
18. The probability distribution of a random variable X is given under :

$$P(X=x)=\begin{cases} kx^2, for\ x=1,2,3 \\ 2kx, for\ x=4,5,6 \\ o, otherwise \end{cases}$$

Find (i) k (ii) $E(X)$

19. Is it possible that the mean of a binomial distribution is 15 and its standard deviation is 5?

20. Find the variance of the binomial distribution with parameters 8 and $\frac{1}{4}$

21. Let X be a continuous random variable with p.d.f

$f(x)=\begin{cases} 2x, 0\le x\le 1 \\ o, otherwise \end{cases}$. Find E(X)

22. In an investment a man can make a profit of Rs.5000 with a probability of 0.62 or a loss of Rs.8000 with a probability of 0.38. Find the expected gain or loss %

23. The p.d.f of a continuous random variable X is

$f(x)=\begin{cases} k, 0\le x\le 4 \\ o, otherwise \end{cases}$. Find k.

24. Prove that $Var(X)=E(X^2)$ if $E(X)=0$

25. Prove that $Var(X+b)=Var(X)$

Three Marks Questions

1. Give any three properties of distribution function.

2. Give any three properties on expectation and variance.

3. A random variable X has the following probability distribution

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Evaluate (i) k (ii) $P(X\ge 6)$ (iii) $P(0<X<3)$

4. Let X denote the number of hours you study on a Sunday. Also it is known that

$$P(X = x) = \begin{cases} 0.1\ if\ x = 0 \\ kx\ if\ x = 1\ or 2 \\ k(5 - x) if x = 3 or 4 \\ 0, otherise \end{cases}$$ where k is a constant.

i) Find k
ii) Find the probability that you study for atleast 2 hours
iii) Find the probability that you study for atmost 2 hours.

5. Two cards are drawn successively without replacement from a well shuffled pack of 52 cards. Find the probability distribution of number of spades.

6. From a lot of 10 bulbs, which includes 3 defective bulbs, a sample of 2 bulbs is drawn at random. Find the probability distribution of defective bulbs.

7. Out of a group of 60 architects 40 are qualified and co-operative while the remaining are qualified but remain reserved. Two architects are selected from the group at random. Find the probability distribution of the number of architects who are qualified and co-operative. Which of the two values, namely co-cooperativeness or reservedness, mentioned above, do you prefer and why?

8. Four defective oranges are accidentally mixed with sixteen good ones. Three oranges are drawn from the mixed lot. Find the probability distribution of X, the number of defective oranges.

9. Two cards are drawn simultaneously from a well shuffled pack of 52 cards. Find the probability distribution of number of jacks.

10. A box contains 12 bulbs of which 3 are defective. A sample of 3 bulbs is selected from the box. Let X denote the number of defective bulbs in the sample. Find the probability distribution of X.

11. Find the variance of the probability distribution

X	0	1	2	3	4	5
P(X)	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$

12. In a meeting, 70% of the members favour a certain proposal while remaining 30% oppose it. A member is selected at random and we let X = 0 if he opposes, and X = 1 if he is in favour. Find E(X) and Var(X)

13. Find the mean, variance and standard deviation of the number of heads in two tosses of a coin.

14. In a game, a man wins Rs.5 for getting a number greater than 4 and loses Re.1 otherwise when a fair dice is thrown. The man decided to throw a die thrice but to quit as and when he gets a number greater than 4. Find the expected value of the amount he wins/loses.

15. A random variable X has the following probability distribution.

x_i	-2	-1	0	1	2	3
p_i	0.1	k	0.2	2k	0.3	k

i) find k ii) find the mean of the distribution

16. In 3 trials of a binomial distribution, the probability of 2 success is 9 times the probability of 3 success. Find the parameter of p of the distribution.

17. For 6 trials of an experiment, let X be a binomial variate which satisfies the relation $9P(X=4)=P(X=2)$. Find p.

18. How many times must a man toss a coin so that the probability of having atleast one head is more than 80%?

19. The probability of a shooter hitting a target is $\frac{3}{4}$. How many minimum number of times must he fire so that the probability of hitting the target atleast once is more than 0.99?

20. If the mean and variance of a binomial distribution are respectively 9 and 6, find the distribution.

21. The distribution of a continuous random variable X in range $(-3,3)$ is given by p.d.f

$$f(x)=\begin{cases}\frac{1}{16}(3+x)^2, -3\le x\le -1\\ \frac{1}{16}(6-2x^2)^2, -1\le x\le 1\\ \frac{1}{16}(3-x)^2, 1\le x\le 3\end{cases}$$

Verify that the area under the curve is unity.

22. Consider a random variable X with p.d.f $f(x)=\begin{cases}3x^2, 0<x<1\\ 0, otherwise.\end{cases}$.

Find Var(3X – 2).

23. Let X be a random variable with distribution function

$$F(x) = \begin{cases} 0\,, & x < 0 \\ \frac{x}{8}, & 0 \le x < 1 \\ \frac{1}{4} + \frac{x}{8}, & 1 \le x < 2 \\ \frac{3}{4} + \frac{x}{12}, & 2 \le x < 3 \end{cases}$$

Find (a) $P(1 \le X \le 2)$ (b) $P(X = 3)$ (c) $P(1 \le X \le 3)$.

24. A person tosses a coin and is to receive Rs.4 for a head and has to pay Rs.2 for a tail. Find the variance of the game.

25. Let X be a continuous random variable with p.d.f $f(x) = \begin{cases} \frac{3}{x^4}, x \ge 1 \\ o, otherwsie \end{cases}$.

    Find the mean and the variance of X.

Five Marks Questions

1. Four bad oranges are accidentally mixed with sixteen good ones. Find the probability distribution of bad oranges in a draw of two oranges. Also find the mean, variance and standard deviation of the distribution.

2. Two cards are drawn simultaneously from a well shuffled pack of 52 cards. Find the mean and variance of the number of red cards.

3. From a lot of 10 items containing 3 defective items, 4 items are drawn at random. Find the mean and variance of the number of defective items drawn.

4. Suppose that a pair of fair dice are tossed and let random variable X denote the sum of outcomes. Find the mean and variance of the probability distribution of X.

5. A box contains 4 red and 5 black marbles. Find the probability distribution of the red marbles in a random draw of three marbles. Also find the mean and standard deviation of the distribution.

6. The mean and variance of a binomial distribution are 4 and 2 respectively. Find the probability of at least 6 success.

7. If X is a binomial random variable with mean 4 and variance 2, find $P(|X - 2| \le 2)$.

8. If the sum and the product of the mean and variance of a binomial distribution are 1.8 and 0.8 respectively, find the probability distribution and the probability of at least one success.

9. The difference between the mean and variance of a binomial distribution is 1 and the difference of their squares is 11. Find the distribution.

10. If the sum of the mean and variance of a binomial distribution is 15 and the sum of their squares is 117, then find the distribution.

11. In a business venture a man can make a profit of Rs.2,000 with a probability of 0.4 or have a loss of Rs.1,000 with a probability of 0.6. What is his expectation, variance and S.D of profit?

12. The p.d.f of a continuous random variable X is $f(x) = \begin{cases} a + bx^2, 0 \leq x \leq 1 \\ 0, otherwise. \end{cases}$ where a and b are some constants. Find

(a) a and b if $E(X) = \frac{3}{5}$ (b) Var (X)

13. Ten coins are tossed simultaneously. What is the probability of getting (a) exactly 6 heads (b) at least 6 heads (c) at most 6 heads?

14. The probability that an engineering college student will graduate is 0.3. Find the probability that out of 6 students (i) none (ii) one (iii) at least one will graduate.

15. The probability distribution of a random variable X is given by

X	0	1	2	3
P(X)	0.1	0.3	0.5	0.1

If $Y = X^2 + 3X$, find the mean and the variance of Y.

Chapter 12 – Discrete Mathematics

Two Marks Questions

Exercise	Exercise problems	Example Problems
12.1	1 each, 2, 3, 4, 6 and 7	12.1 each, 12.5 and 12.8
12.2	1 each, 2, 3 (each) and 4	-

Three Marks Questions

Exercise	Exercise problems	Example Problems
12.1	8	12.2, 12.3, 12.4 and 12.6
12.2	5 each, 6 each, 7 each 8, 9, 10, 11 and 13	12.12, 12.13 each, 12.14 12.15, 12.16, 12.17 and 12.18

Five Marks Questions

Exercise	Exercise problems	Example Problems
12.1	5, 9 and 10	12.7, 12.9 and 12.10
12.2	12, 14, 15, 7 (iv)	12.9

Created Questions

Two Marks Questions

1. In a non empty set S on which a binary operation * is defined and for an element $a \in S$, $a * a * a = e$, where e is the identity element. Find the inverses of a and $a * a$.

2. Check whether dot product is defined on the set of vectors. Explain?

3. Is cross product commutative on the set of vectors? Justify your answer.

4. In Z, the set of integers, an operation * is defined as $a * b = 2(a + b)$. Check whether * is associative.

5. Is it possible to define a binary operation * on any non empty set S such that $a * b = \frac{a+b}{a-b}$, given that a and b are integers.

6. If $a \equiv b(mod\ n)$ and $b \equiv c(mod\ n)$, check whether $a \equiv c(mod\ n)$.

7. If $a \equiv b(mod\ n)$ and $c \equiv d(mod\ n)$, check whether $a + c \equiv (b + d)(mod\ n)$.

8. Give any four Boolean Matrices of order 2×2

9. How many Boolean matrices of order 2×2 are there?

10. A and B are Boolean matrices of order 2×2.If $A \vee B = A$, is it necessary that $B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

11. If A and B are two Boolean matrices of order $2 \times 2$and $A \cap B = A$, is it necessary that $B = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

12. An operation * is defined as $m \times n = m^n - n^m$. Is it binary on N?

13. Form the truth table of $(\sim q) \wedge p$.

14. Form the truth table of $(\sim p) \rightarrow (\sim q)$

15. Write the equivalent forms of $p \rightarrow q$ and $(\sim p) \rightarrow q$.

16. Are $\sim p \vee (p \vee q)$ and $p \vee (\sim p \vee q)$ tautology statements. Justify your answer

17. Are $\sim p \vee (p \wedge q)$ and $p \wedge (\sim p \wedge q)$contradictions

18. p : N is divisible by 4 and q : N is an even number. Whether $p \rightarrow q$ is true.

19. Give the truth table of $\sim p \rightarrow \sim q$

20. Give the truth value of $(\sim p \vee q) \vee (\sim q)$

Three Marks Questions

1. Give an example for (S, *) in which a^3= e where e is the identity element.

2. If in a pair (S,*) where S is a nonempty set and * is a binary operation defined on S as a^2=e for all $a \in S$, then prove that * is commutative given that e is the identity element.

3. Give an example for (S, *) where a^2=e for all $a \in S$. Given that e is identity element.

4. The pair (S, *) satisfied closure, associative, identity and inverse axioms, prove that * is commutative if and only if $(a * b)^2 = a^2 * b^2$

5. On the set Q of rational numbers, an operation * is defined as $a * b = k(a + b)$ where k is a given non zero number. Is it associative

6. If on the set Q of rational numbers, a binary operation * is defined as $a * b = \lambda(a + b)$were λ is a nonzero fixed rational number and it is given that * is associative, then find the value of λ and what can we say about the operation *?

7. If $a \equiv b(mod\ n)$ and $c \equiv d(mod\ n)$, prove that $ac \equiv bd(mod\ n)$.

8. If $a \equiv b(mod\ n)$ show that $a^m \equiv b^m(mod\ n)$, where m is a natural number.

9. Show that $(z_3 - [0],\ X_3)$ satisfies closure, identity and inverse properties.

10. Show that $(Z_4, +_4)$ satisfies closure, associative and commutative properties.

11. On the set of real numbers * is defined as $a * b = k(a + b + ab)$ where k is a non zero real number. What are the conditions on R and k so that * is associative.

12. If in (S, *) satisfying closure, associative, identity and inverse axiom, $a * b^2 = a^2 * b$ for some $a, b \in S$, then prove that a = b.

13. In in (S, *) satisfying closure, associative, identity and inverse axioms and $(a * b)^{-1} = a^{-1} * b^{-1} \forall a, b \in S$, then prove that * is commutative.

14. If $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ are Boolean matrices, find i) $A \vee B$ ii) $A \vee C$ iii) $A \vee B \vee C$.

15. If $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$and $C = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, find $(A \vee B) \vee C$

Five Marks Questions

1. M be the set of all 2×2 matrices each of whose determinant value is 1. Show that M satisfies the closure, associative, identity and inverse axioms under multiplication.

2. $M = \left\{ \begin{pmatrix} \alpha & 0 \\ 0 & \alpha \end{pmatrix} / \alpha \neq 0 \; and \;\; \alpha \in R \right\}$. Show that M satisfies the closure, associative, inverse, identity and commutative axioms under multiplication.

3. Show that $Z_7 - \{[0]\}$ satisfies the closure, associative, identity, inverse and commutative axioms under multiplication modulo 5.

4. Let Q, be the set of all nonzero rational numbers and k is a nonzero fixed rational number and * be a binary operation defined as a*b=kab. Show that (Q, *) satisfies closure, associative, inverse and commutative properties.

5. Show that $(2018)^{2017} + (2020)^{2017} \equiv 0(mod\ 2019)$.

6. Show that $(Z_7 - [0], X_7)$ satisfies closure, identity, inverse and commutative properties.

7. Prove by using truth table $\sim \big(p \vee (q \vee r)\big) \equiv (\sim p) \wedge (\sim q \wedge \sim r)$

8. Prove without using the truth table

$$\sim \big(p \wedge (q \wedge r)\big) \equiv (\sim p \vee \sim q) \vee (\sim r)$$

9. Examine whether there is a nonempty subset S of the set of real numbers such that it satisfies closure, associative, identity and inverse properties under a binary operation * defined as $a * b = k$ where k is a fixed real number.

10. Prove that $(2019)^{10} + (2020)^{10} \equiv 1025(mod\ 2018)$

ANSWERS

CHAPTER 7.

APPLICATIONS OF DIFFERENTIATION

Two Marks

1. x = 4
2. $\left(\frac{9}{8}, \frac{9}{2}\right)$
3. (1, 0)
4. 2x – y + 3 = 0.
5. $e = \frac{\pi}{2}, \frac{3\pi}{2} \in (0,2\pi)$
6. Rolle's theorem is not valid.
7. smallest value of f(x) possibly be 16.
8. c = 3
9. $e^{3x} = 1 + \frac{3x}{1!} + \frac{(3x)^2}{2!} + \frac{(3x)^3}{3!} + \cdots$
10. $(x-2) + (x-2)^2$
11. (i) $-\pi$ (ii) $n \times 2^{n-1}$ (iii) 2 (iv) 0
12. Absolute minimum value = 1, Absolute maximum value = 6
13. f(x) is strictly increasing $\left[-\frac{1}{2}, \frac{1}{2}\right]$
14. f(x) is strictly decreasing on [0, 1]
15. (i) $\left(\frac{1}{3}, \frac{1}{3}\right)$ (ii) $(1,-1), (-1,3)$
16. critical number is π
17. concave downward everywhere
18. concave upward everywhere

Three Marks

1. $2:\pi$
2. $\frac{8}{3}$cm^2 / sec
3. 4x + 3y = 7a
4. $\theta = \frac{\pi}{2}$
5. $(\pi, -2)$
6. $C = \frac{3\pi}{4}$
7. 2.89πcu.mm
8. $2 \pm \frac{2}{\sqrt{3}}$
9. $1 - x + x^2 - x^3 + x^4 + \cdots$
10. $\frac{1}{\sqrt{2}}\left[1 - \frac{\left(x-\frac{\pi}{4}\right)}{1!} - \frac{\left(x-\frac{\pi}{4}\right)^2}{2!} + \cdots\right]$
11. (i) 1 (ii) 2(iii) 1
12. (i) Absolute minimum value = −3, Absolute maximum value = 17

    (ii) Absolute minimum value = $\frac{\pi}{3} - \sqrt{3}$. Absolute max value = $\frac{5\pi}{3} + \sqrt{3}$
14. f(x) is increasing on $\left[0, \frac{\pi}{2}\right]$ and $\left[\frac{3\pi}{2}, 2\pi\right]$

    f(x) is decreasing on $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

15. (i) f(x) is strictly increasing on $\left(-\infty,-\frac{1}{2}\right)$ and f(x) is strictly decreasing on $\left[-\frac{1}{2},\infty\right]$

Local maximum value $=\frac{81}{4}$

(ii) f(x) is strictly decreasing on $\left(0,\frac{1}{\sqrt{3}}\right)$ and strictly increasing on $\left(\frac{1}{\sqrt{3}},2\right)$

Local minimum value $=-\frac{2}{3\sqrt{3}}$

16. $\left(-\infty,-\frac{5}{6}\right)$concave downward

$\left(-\frac{5}{6},\infty\right)$concave upward

Points of inflection is$\left(-\frac{5}{6},\frac{305}{54}\right)$

17. (i) Local min value $=-\frac{2}{3\sqrt{3}}$ and local max value = $\frac{2}{3\sqrt{3}}$

(ii) local min value = -3 and local max.value = 1

18. 50, 50

19. 10, 10

20. y = x – 10

Five Marks

1. $\frac{1}{48\pi}$ cm / sec

2. $\frac{ds}{dt}$ = 60cm^2 / sec$\frac{dr}{dt}=\frac{1}{4\pi}$ cm/sec

3. 2 cm / min and 2 cm^2 / min

4. $\left(\frac{1}{2},\frac{31}{6}\right)and\left(-\frac{1}{2},\frac{29}{6}\right)$

5. $\left(-\frac{2}{3},-\frac{14}{7}\right)and(2,-2)$

7. 100 m / s, t = 4 sec, 200 m / s, -100 m / s

9 $\frac{35}{88}$ m / h

10. e

11. a = 2 , $b=-\frac{1}{2}$

12. $(-\infty,-2)$concave downward

$(5,\infty)$concave upward

Point of inflection is (5, 75)

13. $(-\infty,-2)$decreasing

$(-2,-1)$increasing

$(-1,\infty)$decreasing

Local maximum value = 6

Local minimum value = 5

15. Local min value $=-4$

Local max.value = 0

17. $(-\infty,-2)$concave downward

$(-2,1)$concave upward

$(1,\infty)$concave downward

Points of inflections are $(-2,48),(1,9)$

18. $\left(-\frac{1}{\sqrt{2}},e^{-\frac{1}{2}}\right),\left(\frac{1}{\sqrt{2}},e^{-\frac{1}{2}}\right)$

19. 240

20. local min value $=-\frac{27}{256}$

No local maximum

21. Equation of tangent is y – 2 = 0

Equation of normal is $x = \frac{\pi}{6}$

8.Differentials and Partial Derivatives

Two marks

5. $f_x = 12, f_y = 2$

7. $\frac{dw}{dt} = 6a^2t^2e^{2a^2t^3}$

8. $\frac{dw}{d\theta} = 0$

9. $\frac{dw}{d\theta} = 0$

10. $\frac{\partial^2 u}{\partial x \partial y} = e^{xy}[3xy + 1 + x^2y^2]$

11. $\sin\left(\frac{22}{14}\right) = 1$

13. % error in P = 0.7%

16. 33x – 150

17. $L(z) = 2^{1/4} + \frac{1}{4}\left(2^{-3/4}\right)(z-2)$

18. 1.1

20. $\frac{3}{14}$

Three Marks

2. (i) 2 (ii) 24 (iii) -12

8. $\frac{\partial w}{\partial r} = \frac{2}{r}, \frac{\partial w}{\partial \theta} = 0$

9. $2\,cos^2 t$

11. 0.2867

12. 0.677

13. $\frac{1}{4}$

14. $\frac{3}{2}$

15. $\frac{\partial W}{\partial U} = 4u(u^2 + v^2)$; $\frac{\partial w}{\partial v} = 4v(u^2 + v^2)$

16. $\frac{1}{4}$

17. $\frac{5}{18}$

18. 0

20. $-\sin t + 2\cos t + 2t$

Five Marks

4. $\frac{\partial w}{\partial x} = x^y \cdot \frac{x}{y^2}(2 + y)$;$\frac{\partial w}{\partial y} = \frac{x^2}{y^2}x^y[y\log x - 2]$

6. percentage increase in volume is 3k%

7. $\frac{\partial w}{\partial u} = \frac{2u}{\sqrt{1-(u^2-v^2)^2}}$; $\frac{\partial w}{\partial v} = \frac{-2v}{\sqrt{1-(u^2-v^2)^2}}$

10. 2.0116 approx.

9. Applications of Integration

Two Marks

1. 0

2. $\frac{1}{5}(e^5 - 2)$

3. $\frac{1}{10}\left[e^{\frac{3\pi}{2}} - 1\right]$

4. $\frac{1}{2}\left[1 - e^{-\frac{\pi}{2}}\right]$

5. 0

6. $\frac{x^3}{3} + c$

7. $\frac{e^{2x}}{2} + c$

8. $\frac{2^{3x}3^{2x}5^x}{\log 360} + c$

9. $\frac{1}{loga} - \frac{1}{logb}$

10. $\frac{1}{2}$

11 2

12. $\frac{32\pi}{3}$

13. $\frac{3\pi}{5}$

14. $\frac{\pi}{60}$

15. $\tan^{-1} e - \frac{\pi}{4}$

16. $\frac{1}{6}\left[\log\left(\frac{35}{8}\right)\right]$

17. $\frac{28}{3}$

18. 12π

19. $\frac{a^2}{6}$

20. $\frac{3}{\log 2} - \frac{20}{3}$

21. $\frac{13}{3}$

22. $\frac{1}{2}$

23. $\frac{208a^2}{3}$

24. $\frac{3}{2}$

25. $\frac{2}{a}$

Three Marks

1. $-\frac{15}{2}$

2. π

3. $-\frac{1}{3}$

4. $4\sqrt{3} - 4 - \frac{\pi}{3}$

5. $\frac{3}{2}$

6. 1

7. $\frac{\pi}{4}$

8. $\frac{1}{(n+1)(n+2)}$

9. $\frac{1}{132}$

10. $\frac{1}{60}$

11. $\frac{20\sqrt{2}}{3}$

12. $\frac{1}{2}$

13. $\frac{2}{3}$

14. 1

15. $\frac{\pi}{2} + \frac{4}{3}$

16. $\frac{17}{12}$

17. $\sqrt{2} - 1$

18. $\frac{343}{6}$

19. $\frac{9}{2}$

20. 0

21. $\frac{128}{315}$

22. $\frac{63\pi}{512}$

23. 25

24. $\frac{28}{3}$

25. $\frac{\pi}{4}$

Five Marks

1. (i) not exist (ii) $\frac{32}{15}$ sq.units.

2. $\frac{4}{3}$

3. 2 : 1

4. 2 sq.units

5. $c = -1$

6. $\frac{8\sqrt{3}a^2}{45}$

7. $\frac{4}{3}\left(4\pi + \sqrt{3}\right)$

8. prove

9. $\frac{56a^2}{3}$

10. 8 sq.units.

11. $\frac{2}{e}$

12. $e + e^{-1} - 2$

13. $\frac{4a^2}{3}$sq.units.

14. $\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$

15. $\frac{ab(\pi-2)}{4}$

16. prove

17. $\frac{3\pi}{4}$

18. $3\pi a^2$

19. πa^2

20. $12\sqrt{3}$

Chaoter 10. Differential Equations

Two Marks Questions

1. order – 2 : degree 2

2. order 2 ; degree not defined

3. order 2 ; degree 1

4. $\frac{d^2y}{dx^2} + y = 0$

5. $x\frac{dy}{dx} = 2y$

6. $x\frac{dy}{dx} = y\log y$

7. $y = \log(x^2 + 1) + c$

8. $y = \log|e^x + e^{-x}| + c$

9. $x + \log(1 - y) = c$

10. $|xy| = ce^x$

Three Marks

1. $8x\left(\frac{dy}{dx}\right)^3 - 12y\left(\frac{dy}{dx}\right)^2 = 27x$

2. $\left(\frac{dy}{dx}\right)^3 = 4y\left(x\frac{dy}{dx} - 2y\right)$

3. $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \frac{y}{x}\frac{dy}{dx}$

4. $(x^2 - y^2)\frac{dy}{dx} = 2xy$

5. $y^2 - x^2 = 2xy\frac{dy}{dx}$

6. $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = y^2\left(\frac{d^2y}{dx^2}\right)^2$

7. $(x-y)^2\left[1 + \left(\frac{dy}{dx}\right)^2\right] = \left(x + y\frac{dy}{dx}\right)^2$

8. $\left(\frac{dy}{dx}\right)^2(x^2 - 2y^2) - 4\left(\frac{dy}{dx}\right)xy - x^2 = 0$

9. prove

10. prove

11. $y = 2\tan\frac{x}{2} - x + c$

12. $y = \frac{1}{2x-1}, x \neq \frac{1}{2}$

13. $(1 + \sin x)(1 + \cos y) = c$

14. $\tan^{-1} y + \tan^{-1}(e^x) = c$

15. $\log|y| + \frac{1}{y} + \frac{1}{x} = x + c$

16. y + 3 = 2(x + 4)

17. $y = x^3 + cx$

18. $y(x^2 - 1) = \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + c$

19. $ye^{-x} = x + c$

20. $xy = \frac{y^4}{4} + c$

Five Marks

1. $y = \frac{x^2}{2} + 2x - 13\log|x+2| + c, x \in R - \{2\}$

2. $y = -\frac{1}{3}cos^3x + \frac{cos^5x}{5} + xe^x - e^x + c$

3. $y = x\log(x+1) + \log(x+1) - x + 5$

4. $y = \frac{1}{2}\log(x+1) + (\tan^{-1} x)^2 + c$

5. $y = \frac{1}{2}\log(x+1) + \frac{3}{4}\log(x^2+1) - \frac{1}{2}\tan^{-1} x + c$

6. $\tan^{-1} y = \frac{\pi}{4} + \frac{1}{2} - \frac{1}{2}(1 + \log x)^2$

7. $20\log_e 2$years

8. $y = \tan^{-1}(x + y + 1)$

9. $y = \frac{2x}{3x^2-1}, x \neq \pm\frac{1}{\sqrt{3}}$

10. $c(x-y)^2 = |x|e^{-y/x}, x \neq 0$

11. $\log|x| = \cos\left(\frac{y}{x}\right), x \neq 0$

12. $y = \frac{2x}{1+\log|x|}, x \neq 0, \pm\frac{1}{e}$

13. $y\, e^{\tan x} = e^{\tan x}(\tan x - 1) + c$

14. $y = \frac{2c - x^2}{2(1+\sin x)}, x \neq n\pi + (-1)^n\frac{\pi}{2}, \forall n \in Z$

15. $y = \sin^{-1}(2x), x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

16. $\frac{2\ \log 5}{\log 2}$hours

17. $S = 40°\text{F}$

18. (i) $65.33°\text{C}$ (ii) 53.46

11. Probability Distribution

Two Marks

15. (i) $k = \frac{8}{15}$ (ii) $\frac{1}{15}$

16.

Z	-2	0	2
P(z)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

17.

X	1	2	3	4
P(X)	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

18. (i) $\frac{1}{44}$ (ii) $\frac{95}{22}$

19.not possible

20. $\frac{3}{2}$

21. $\frac{2}{3}$

22. Rs.60

23. $k = 1.4$

Three Marks

3. (i) $k = \frac{1}{10}$ (ii) $\frac{19}{100}$ (iii) $\frac{3}{10}$

4. (i) k = 0.15 (ii) 0.75 (iii) 0.55

5.

X	0	1	2
P(X)	$\frac{19}{34}$	$\frac{13}{34}$	$\frac{1}{17}$

6.

X	0	1	2
P(X)	$\frac{7}{15}$	$\frac{7}{15}$	$\frac{1}{15}$

7.

X	0	1	2
P(X)	$\frac{19}{177}$	$\frac{80}{177}$	$\frac{78}{177}$

8.

X	0	1	2	3
P(X)	$\frac{28}{57}$	$\frac{24}{57}$	$\frac{24}{285}$	$\frac{1}{15}$

9.

X	0	1	2
P(X)	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

11. $\frac{35}{18}, \frac{665}{324}$

12. 0.3, 0.21

13. $1, \frac{1}{2}, \frac{1}{\sqrt{2}}$

14. $Rs. \frac{19}{9}$

15. 0.8

16. $p = \frac{1}{4}$

17. $p = \frac{1}{4}$

18. 3

19. 4

20. $27C_x \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)$

21. 1

22. $\frac{27}{80}$

23. $\frac{13}{24}, 0, \frac{5}{8}$

24. 1, 9

25. $\frac{3}{2}, \frac{3}{4}$

Five Marks

1.

X	0	1	2
P(X)	$\frac{60}{95}$	$\frac{32}{95}$	$\frac{3}{95}$

$$\frac{38}{95}, \frac{2}{5}, 0.6614$$

2. $1, \frac{25}{57}$ (or) 0.481

3. 1.2, 2.56

4. 7, 5.83

5. 1.33, 0.5611

6. $\frac{37}{256}$

7. $\frac{163}{256}$

8. 0.6723

9. $P(X = x): 36C_x \left(\frac{1}{6}\right)^x \left(\frac{5}{6}\right)^{36-x}$ x = 0 t0 36.

10. $P(X = x): 27C_x \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{27-x}$ x = 0 to 27

11. 200, 2160000, 1469.69

12. $\frac{3}{5}, \frac{6}{5}, \frac{2}{25}$

13. (a) $\frac{105}{512}$ (a) $\frac{99}{256}$ (c) $\frac{53}{64}$

14. (i) 0.1176 (ii) 0.3025 (iii) 0.8824

15. 8, 23.2