SCHOOL EDUCATION DEPARTMENT

CHENNAI DISTRICT

LEARNING MATERIAL
2022 – 2023

X – STD
MATHEMATICS

Preface

We convey our sincere gratitude to our respected Chief Educational Officer, who has given this opportunity to bring out a unique material for the students (X Standard Maths) in the name of Learning Material.

The learning material is prepared based on the selected chapters. This includes classification for selected chapters, solved textbook exercise problems (2 marks, 5 marks and 8 marks).

Students can prepare the example problems based on the classification. All the text book MCQ problems have to be practiced regularly. Students must practice all the problems in the classification. This material mainly focus on the late bloomers to achieve their goals.

Good effort always lead to success
All the best ! ! !

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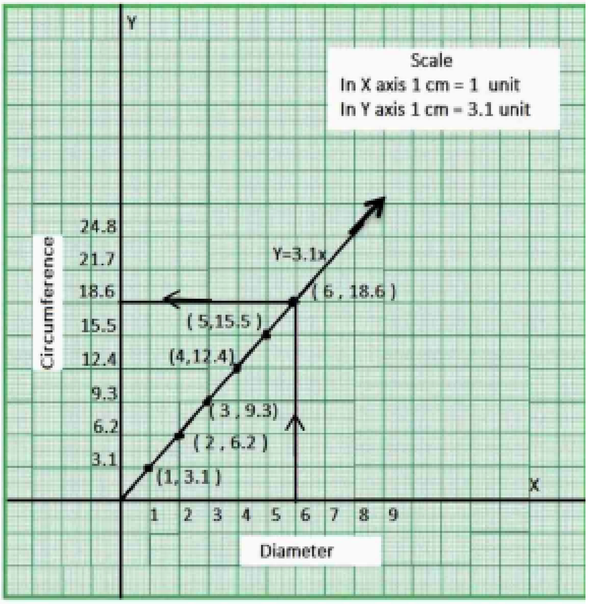
ALGEBRA – VARIATION OF GRAPHS – 8 MARKS

1. Varshika drew 6 circles with different sizes. Draw a graph for the relationship between the diameter and Circumference (approximately related) of each circle as shown in the table and use it to find the circumference of a circle when its diameter is 6 cm.

Diameter (x) cm	**1**	**2**	**3**	**4**	**5**
Circumference (y)cm	**3.1**	**6.2**	**9.3**	**12.4**	**15.5**



Solution :

1.Table :

Diameter (x) cm	1	2	3	4	5	6
Circumference (y)cm	3.1	6.2	9.3	12.4	15.5	18.6

2.Variation :

Direct Variation

3.Equation :

$$y = kx$$

$$k = \frac{y}{x} = \frac{3.1}{1} = 3.1$$

$$y = 3.1x$$

4. Points :

$(1,\ 3.1)$, $(2, 6.2)$, $(3, 9.3)$, $(4, 12.4)$, $(5, 15.5)$, $(6, 18.6)$

5.Solution :

From the graph , When diameter is 6 cm , its circumference **= 18.6 cm**

2. A bus is travelling at a uniform speed of 50 km/hr. Draw the distance – time graph and hence find

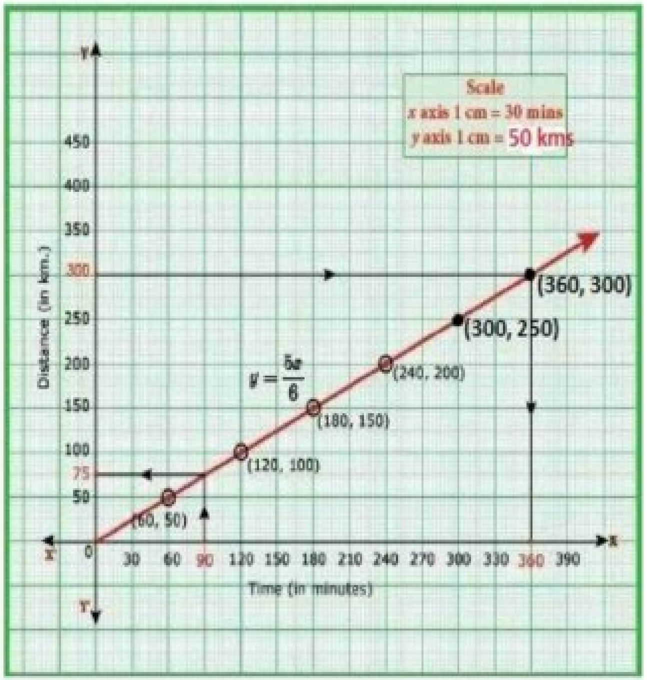
(i). the constant of variation **(ii). how far will it travel in 90 minutes ?**

(iii). the time required to cover a distance of 300 km from the graph

Solution :

1. Table

Time taken x (in mins)	60	120	180	240	300	360
Distance y (in km)	50	100	150	200	250	300



2. Variation :

Direct Variation

3. Equation :

$$y = kx$$

$$k = \frac{y}{x} = \frac{50}{60} = \frac{5}{6}$$

$$y = \frac{5}{6}x$$

4. Points :

$(60, 50)$, $(120, 100)$, $(180, 150)$, $(240, 200)$, $(300, 250)$, $(360, 300)$

5. Solutin :

From the graph ,

(i).. Constant of variation $\boldsymbol{k} = \frac{5}{6}$

(ii). The distance travelled in 90 minutes $= \mathbf{75}\ \boldsymbol{km}$

(iii). The time taken to cover 300 km $= \mathbf{360\ minutes} = \mathbf{6\ hours}$

3. A company initially started with 40 workers to complete the work by 150 days . Later , it decided to fasten u the work increasing the number of workers as shown below.

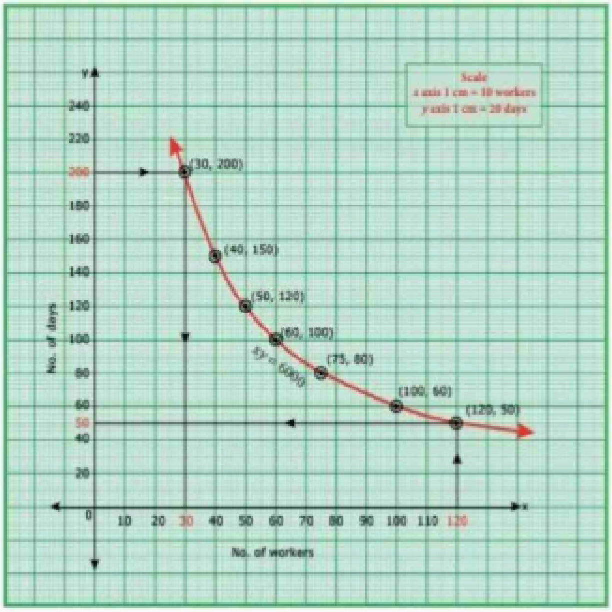
Number of workers (x)	**40**	**50**	**60**	**75**
Number of days (y)	**150**	**120**	**100**	**80**

(i).. Graph the above data and identify the type of variation.

(ii). From the graph , find the number of days required to complete the work if the company decides to opt for 120 workers ?

(iii). If the work has to be completed by 200 days , how many workers are required ?

Solution :



1..Table :

Number of workers (x)	30	40	50	60	75	100	120
Number of days (y)	200	150	120	100	80	60	50

2. Variation :

Indirect Variation

3.Equation :

$$xy = k$$
$$xy = 40 \times 150 = 6000$$
$$xy = 6000$$

4.Points :

$(30\,,200), (40\,,150), (50, 120), (60, 100), (75\,, 80),$
$(100\,,\ 60), (120\,, 50)$

5. Solution :

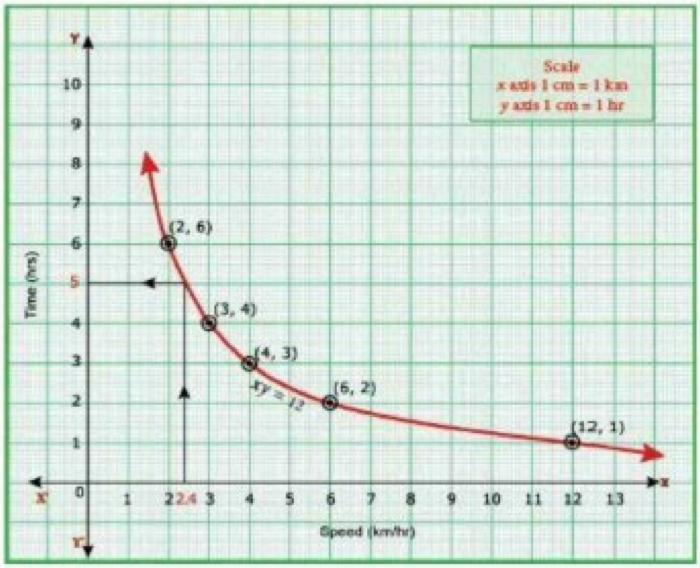
From the graph ,

(i). Type of variation = **Indirect variation**

(ii). The required number of days to complete the work when the company decides to work with 120 workers = **50 days**

(iii). If the work has to be completed by 200 days , the number of workers required = **30 workers**

4. Nishanth is the winner in a Marathon race of 12 km distance. He ran at the uniform speed of 12 km / hr and reached the destination in 1 hour . He was followed by Aradhana , Jeyanth , Sathya and Swetha with their respective speed of 6 km/hr , 4 km/hr , 3 km/hr and 2 km/hr . And , they covered the distance in 2 hrs , 3 hrs , 4 hrs and 6 hours respectively. Draw the speed – time graph and use it to find the time taken to kaushik with his speed of 2.4 km/hr.



Solution :

1. Table :

Speed x (km / hr)	12	6	4	3	2
Time y (hours)	1	2	3	4	6

2..Variation :

Indirect Variation

3. Equation :

$$xy = k$$
$$xy = 12 \times 1 = 12$$
$$xy = 12$$

4.Points :

$(\,12\,,1\,), (\,6\,,2\,), (\,4\,,3\,), (\,3\,,4\,), (\,2\,,6\,)$

5. Solution :

From the graph , The time taken by Kaushik to go at a speed of 2.4 km/hr = **5 hours**

5. A garment shop announces a flat 50% discount on every purchase of items for their customers. Draw the graph for the relation between the Marked Price and the Discount. Hence find

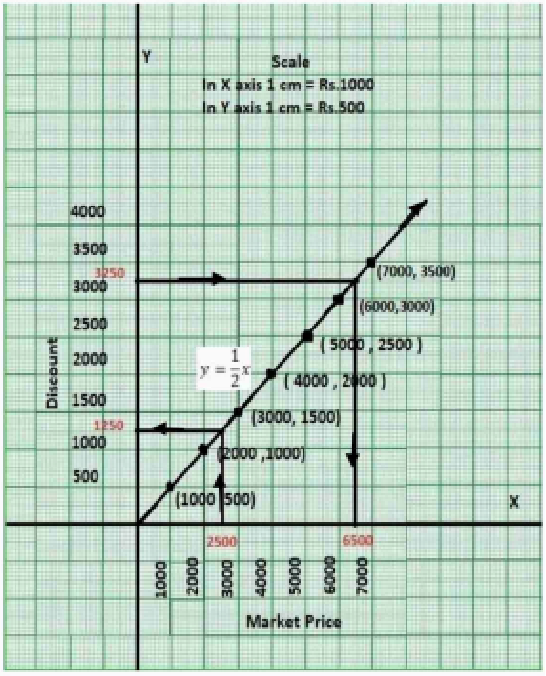
(i). the Marked Price when a customer gets a discount of Rs.3250 (from graph)

(ii). the discount when the Marked Price is Rs.2500

Solution:

1.Table :

Marked Price (x)	1000	2000	3000	4000	5000	6000	7000
Discount (y)	500	1000	1500	2000	2500	3000	3500



2. Variation :

Direct Variation

3. Equation :

$$y = kx$$

$$k = \frac{y}{x} = \frac{500}{1000} = \frac{1}{2}$$

$$y = \frac{1}{2}x$$

4. Points :

(1000 , 500) , (2000 , 1000) , (3000 , 1500), (4000 , 2000)
(5000 , 2500) , (6000 , 3000) , (7000 , 3500) ,

5. Solution :

From the graph ,

(i). If the customer gets a discount of Rs.3250 , then the Marked Price = **Rs. 6500 /-**

(ii). If the Marked Price is Rs.2500 , then the discount = **Rs. 1250 /-** .

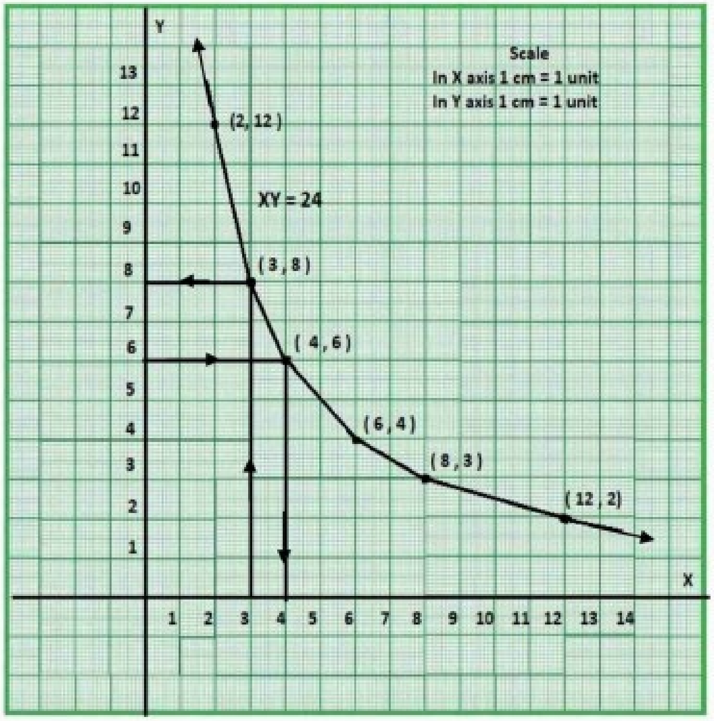
6. Draw the graph of $xy = 24$, x , $y > 0$. Using the graph find ,

(i) . y when $x = 3$ and (ii). x when $y = 6$

Solution :

1.Table :

x	12	8	6	4	3	2
y	2	3	4	6	8	12



2. Variation :

Indirect Variation

3. Equation :

$$xy = k$$

$$xy = 12 \times 2 = 24$$

$$xy = 24$$

4. Points :

(12 , 2), (8 , 3), (6 , 4),
(4, 6), (3, 8), (2 , 12),

5. Solution :

From the graph ,

(i) .. If $x = 3$ then , $\mathbf{y = 8}$

(ii). If $y = 6$ then , $\mathbf{x = 4}$.

7. Graph the following linear function $y = \frac{1}{2}x$. Identify the constant of variation and verify it with the graph. Also , (i). find y when $x = 9$ (ii). find x when $y = 7.5$.

Solution :

1.Table :

x	2	4	6	8	10	12	14	16
y	1	2	3	4	5	6	7	8

2.Variation :

Direct Variation

3. Equation :

$$y = kx$$

$$k = \frac{y}{x} = \frac{1}{2}$$

$$y = \frac{1}{2}x$$

4. Points :

$(2,1),(4,2),(6,3),(8,4),(10,5)$

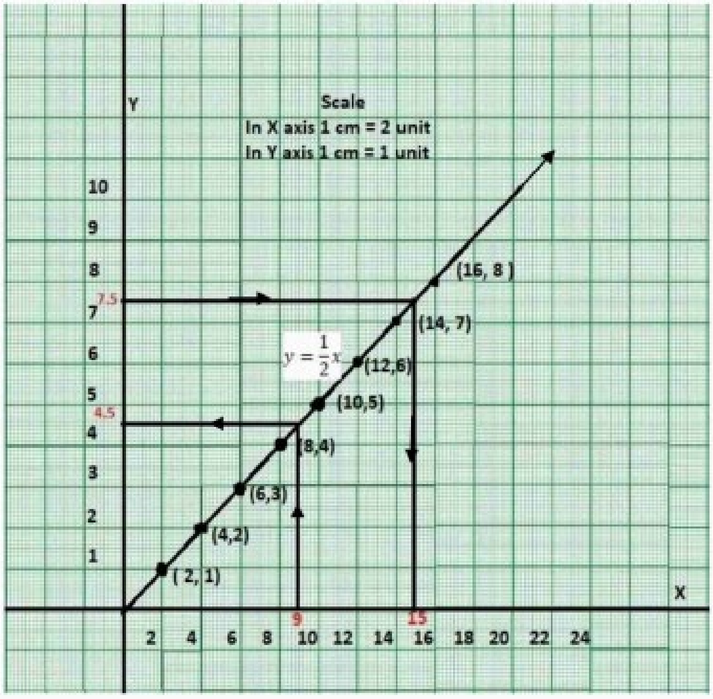
$(12,6)\ ,(14,7),(16,8)$

5. Solution :

From the graph ,

(i).. If $x = 9$ then , $\mathbf{y = 4.5}$

(ii). If $y = 7.5$ then , $\mathbf{x = 15}$



8. The following table shows the data about the number of pipes and the time taken to till the same tank.

No. of pipes (x)	**2**	**3**	**6**	**9**
Time taken (in minutes) (y)	**45**	**30**	**15**	**10**

Draw the graph for the above data and hence

(i). Find the time taken to fill the tank when five pipes are used

(ii). Find the number of pipes when the time is 9 minutes.

Solution :

1.Table :

No. of pipes (x)	2	3	5	6	9	10
Time taken (y) (mins)	45	30	18	15	10	9

2. Variation :

Indirect Variation

3. Equation :

$$xy = k$$

$$xy = 2 \times 45 = 90$$

$$xy = 90$$

4. Points :

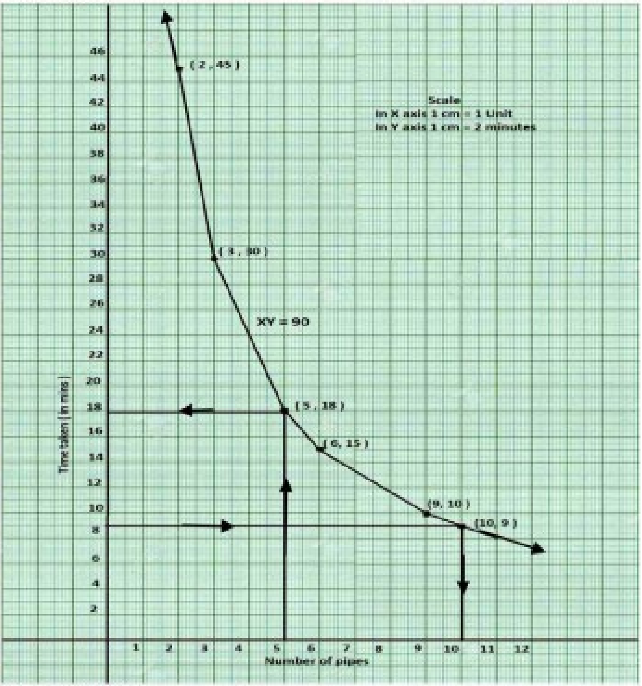
$(2,45),(3,30)\ (5,18),(6,15),(9,10),(10,9)$

5. Solution :

From the graph

(i). Time taken to fill the tank if using 5 pipes = **18 minutes**

(ii). Number of pipes used if the tank fills up in 9 minutes = **10 pipes**



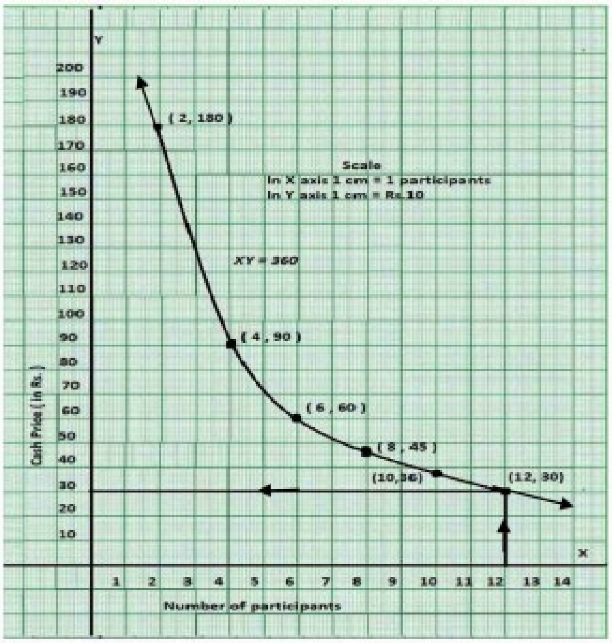
9. A school announces that for a certain competitions , the cash price will be distributed for all the participants equally as show below.

No. of participants (x)	**2**	**4**	**6**	**8**	**10**
Amount for each participants in Rs. (y)	**180**	**90**	**60**	**45**	**36**

(i). Find the constant of variation.

(ii). Graph the above data and hence , find how much will each participant get if the number of participants are 12.

Solution :



1. Table :

No. of participants (x)	2	4	6	8	10	12
Amount for each participants in Rs. (y)	180	90	60	45	36	30

2. Variation :

Indirect Variation

3. Equation :

$$xy = k$$

$$xy = 2 \times 180 = 360$$

$$xy = 360$$

4.Points :

(2, 180), (4 , 90), (6 , 60) , (8 , 45), (10 , 36), (12, 30)

5. Solution :

(i). Constant of Variation $\boldsymbol{k = 360}$

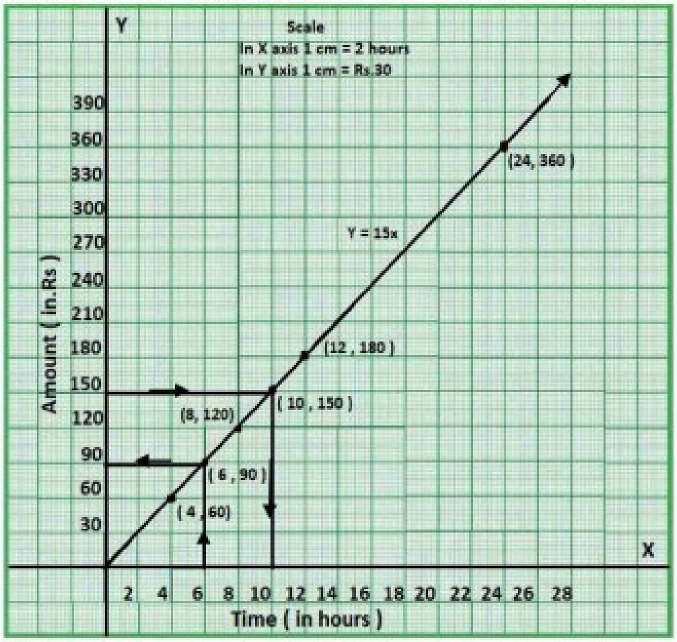
(ii). Cash Price each participant will get if 12 participants participate $= \boldsymbol{Rs.30}$

10. A two wheeler parking zone near bus stand charges as below.

Time (in hours) (x)	**4**	**8**	**12**	**24**
Amount in Rs (y)	**60**	**120**	**180**	**360**

Check if the amount charged are in direct variation or in inverse variation to the parking time. Graph the data. Also (i). Find the amount to be paid when parking time is 6 hr (ii). Find the parking duration when the amount paid is Rs.150

Solution :



1.Table :

Time (in hours) (x)	4	6	8	10	12	24
Amount in Rs (y)	60	90	120	150	180	360

2.Variation :

Direct Variation

3. Equation :

$$y = kx$$

$$k = \frac{y}{x} = \frac{60}{4} = 15$$

$$y = 15x$$

4.Points :

(4 , 60) , (6 , 90), (8 , 120) , (10 , 150) , (12 , 180) , (24 , 320)

5.Solution :

From the graph

(i). If the parking time is 6 hours , then the parking charge **= Rs. 90**

(ii). If the amount Rs.150 is paid , then the Parking time **= 10 hours**

Quadratic Graph

11. Discuss the nature of solutions of the following quadratic equations.

(i) $x^2 + x - 12 = 0$ (ii) $x^2 - 8x + 16 = 0$ (iii) $x^2 + 2x + 5 = 0$

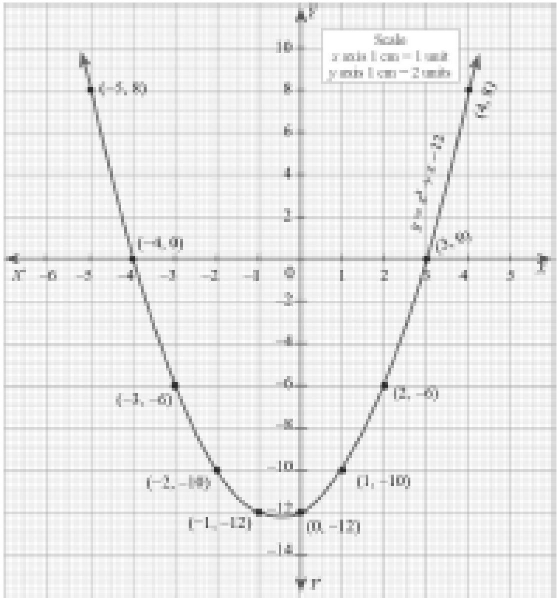
Solution:

i) Table:

x	–5	–4	–3	–2	–1	0	1	2	3	4
x^2	25	16	9	4	1	0	1	4	9	16
x	–5	–4	–3	–2	–1	0	1	2	3	4
–12	–12	–12	–12	–12	–12	–12	–12	–12	–12	–12
y	8	0	–6	–10	–12	–12	–10	-6	0	8

Points: (–4, 72), (–3, 56), (–2, 42), (–1, 30), (0, 20), (1, 12), (2, 6), (3, 2), (4, 0)

Points of Parabola intersect *x* axis: (–4, 0) and (3, 0). *x*– coordinates –4 and 3



Nature of Solution:

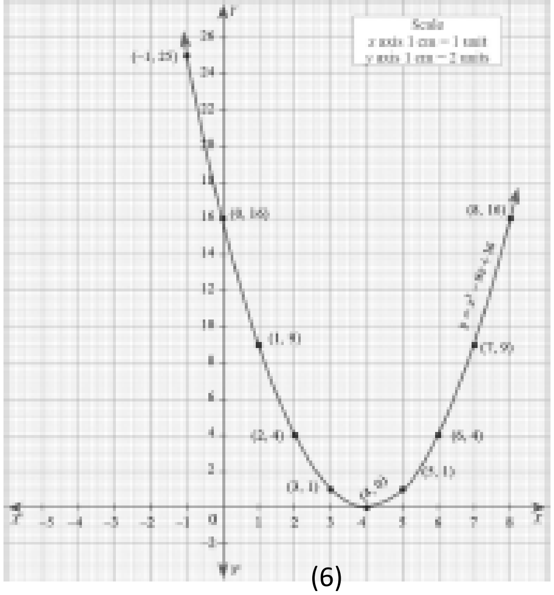
There are two points of intersection with the X axis, the quadratic equation has real and unequal roots.

ii) Table:

x	–1	0	1	2	3	4	5	6	7	8
x^2	1	0	1	4	9	16	25	36	49	64
–8x	8	0	–8	–16	–24	–32	–40	–48	–56	–64
+7	7	7	7	7	7	7	7	7	7	7
y	25	16	9	4	1	0	1	4	9	16

Points: (–1, 25), (0, 16), (1, 9), (2, 4), (3, 1), (4, 0), (5, 1), (6, 4), (7, 9), (8, 16)

Points of Parabola intersect *x* axis: (4, 0) *x*- coordinates 4



Nature of Solution:

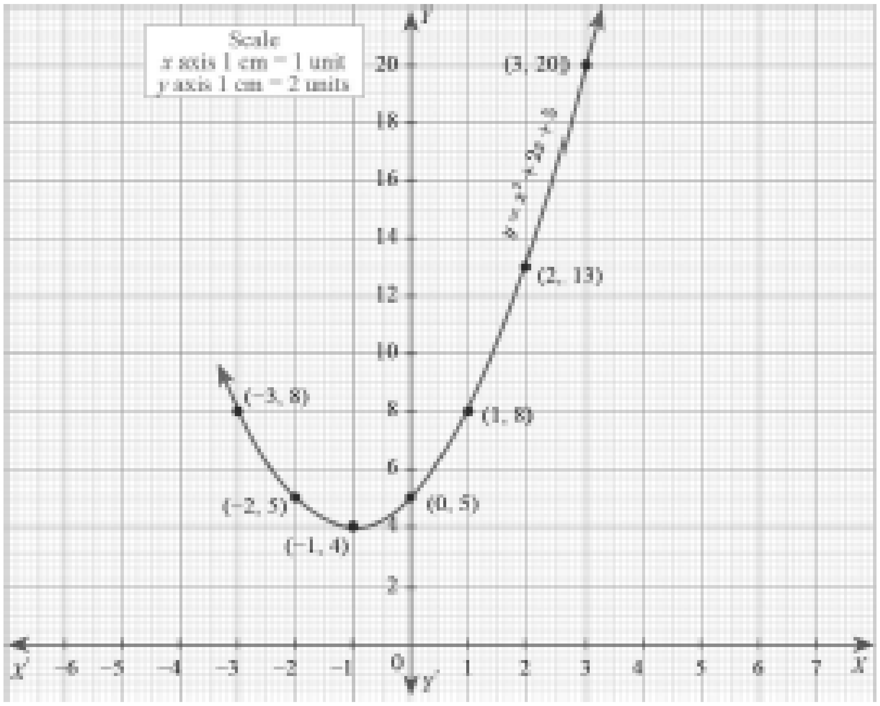
The quadratic equation has real and equal roots.

iii) **Table:**

x	–3	–2	–1	0	1	2	3	4
x^2	9	4	1	0	1	4	9	16
2x	–6	–4	–2	0	2	4	6	8
5	5	5	5	5	5	5	5	5
y	8	5	4	5	8	13	20	29

Points: (–3, 8), (–2, 5), (–1, 4), (0, 5), (1, 8), (2, 13), (3, 20), (4, 29)

Points of Parabola intersect *x* axis: the parabola doesn`t intersect or touch the X axis.



Nature of Solution:

There is no real root for the given quadratic equation.

12. **Graph the following quadratic equations and state their nature of solutions.**

(i) $x^2 - 9x + 20 = 0$ **(ii) $x^2 - 4x + 4 = 0$** **(iii) $x^2 + x + 7 = 0$**

(iv) $x^2 - 9 = 0$ **(v) $x^2 - 6x + 9 = 0$** **(vi) $(2x - 3)(x + 2) = 0$**

Solution:

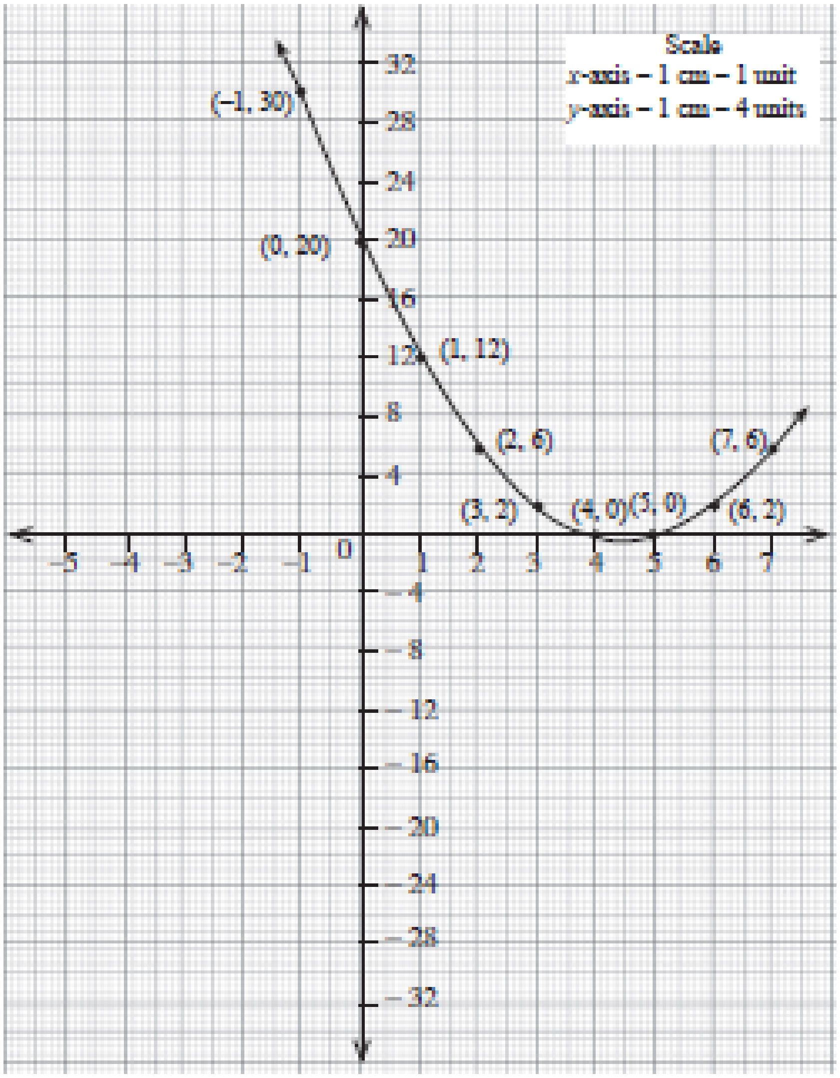
i. $x^2 - 9x + 20 = 0$

Table:

x	–4	–3	–2	–1	0	1	2	3	4	5
x^2	16	9	4	1	0	1	4	9	16	25
–9x	36	27	18	9	0	–9	–18	–27	–36	–45
+20	20	20	20	20	20	20	20	20	20	20
y	72	56	42	30	20	12	6	2	0	0

Points: (–4, 72), (–3, 56), (–2, 42), (–1, 30), (0, 20), (1,12), (2, 6), (3,2), (4, 0)

Point of intersection of Parabola at *x* axis: (4, 0) and (5, 0). X- Coordinates are 4 and 5



Solution:

Since there are two points of intersection with the x axis, the quadratic equation $x^2 - 9x + 20 = 0$ has real and unequal roots

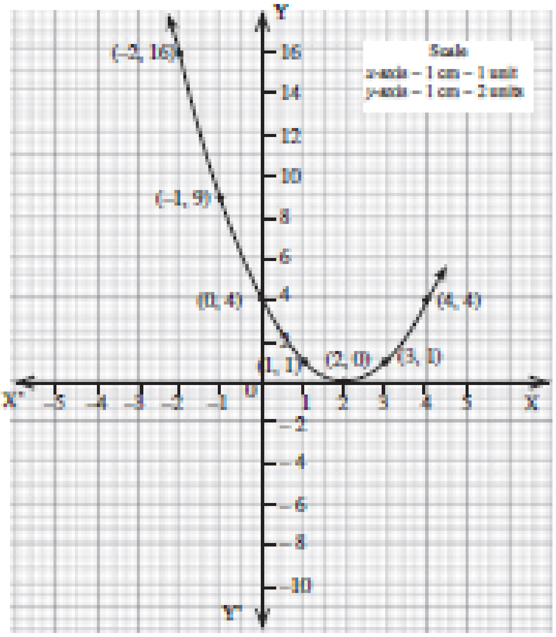
(ii) $x^2 - 4x + 4 = 0$

Table:

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
–4x	16	12	8	4	0	–4	–8	–12	–16
+4	4	4	4	4	4	4	4	4	4
y	36	25	16	9	4	1	0	1	4

Points:

(–4, 36), (–3, 25), (–2, 16), (–1, 9), (0, 4), (1, 1), (2, 0), (3, 1), (4, 4)



Solution:

Here, the curve meets the x - axis at (2, 0).

∴ The equation 2 equal roots. ∴ The x - co ordinates of the points is $x = 2$.

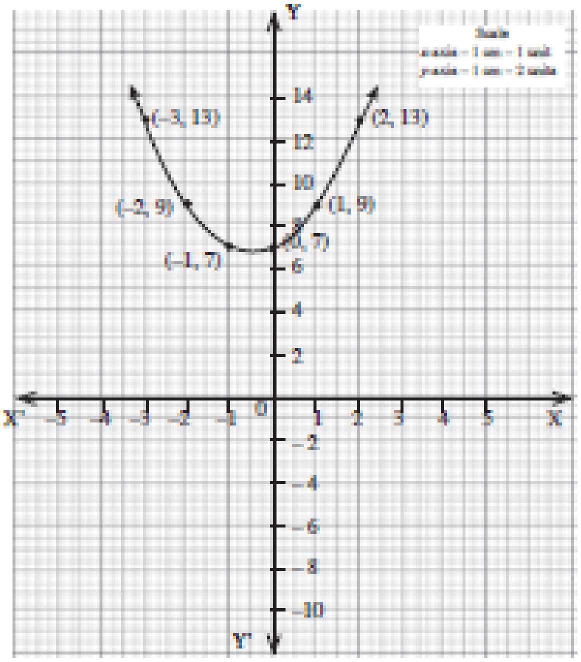
∴ Solution = {2, 2}

(iii) $x^2 + x + 7 = 0$.

Table

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
+x	–4	–3	–2	–1	0	1	2	3	4
+7	7	7	7	7	7	7	7	7	7
y	19	13	9	7	7	9	13	19	27

Points (–4, 19), (–3, 13), (–2, 9), (–1, 7), (0, 7), (1, 9), (2, 13), (3, 9), (4, 27)



Solution:

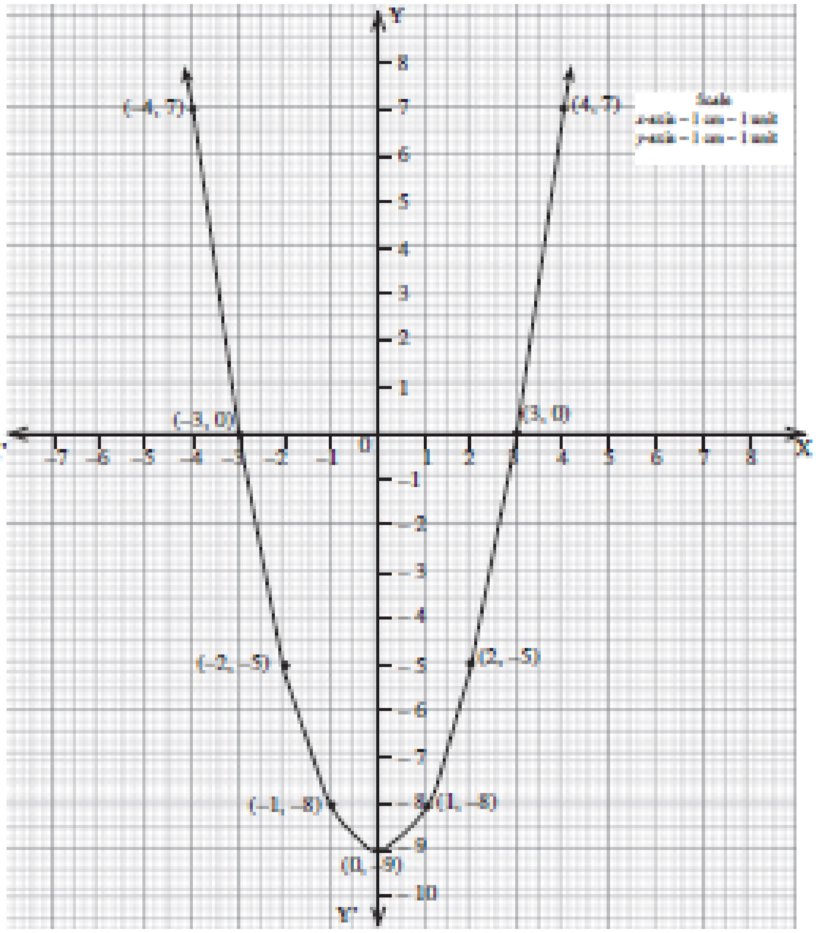
Here the curve does not meet the x – axis and the curve has no real roots

(iv) $x^2 - 9 = 0$.

Table

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
–9	–9	–9	–9	–9	–9	–9	–9	–9	–9
y	7	0	–5	–8	–9	–8	–5	0	7

Points: (–4, 7), (–3, 0), (–2, –5), (–1, –8), (0, –9), (1, –8), (2, –5), (3, 0), (4, 7)



Solution:

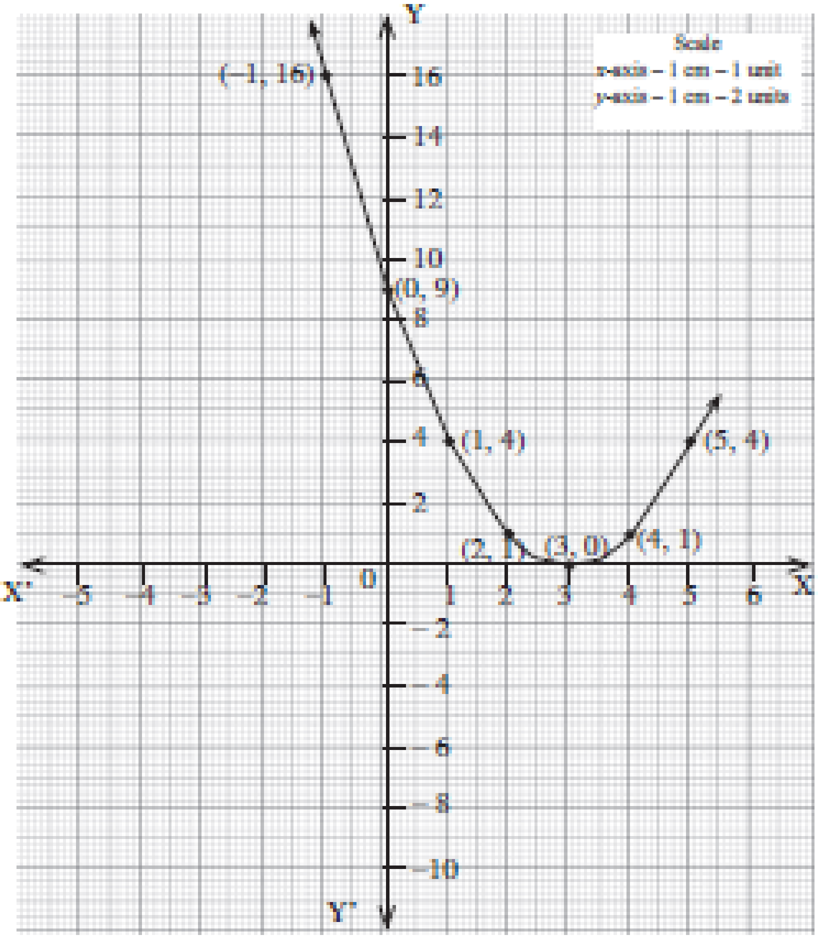
Here the curve meets the x – axis at 2 points (–3, 0), (3, 0). ∴ The equation has real and unequal roots. ∴ The x-coordinates are 3 , –3 will be the solution. ∴ Solution = {–3, 3}and the curve has no real roots

(v) $x^2 - 6x + 9 = 0$.

Table

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
-6x	24	18	12	6	0	–6	–12	–18	–24
+9	9	9	9	9	9	9	9	9	9
y	49	36	25	16	9	4	1	0	1

Points (–4, 49), (–3, 36), (–2, 25), (–1, 16), (0, 9), (1, 4), (2, 1), (3, 0), (4, 1)



Solution

Here the curve meets the x-axis at only one point (3, 0). and the equation has real and equal roots.

∴ The x-coordinates are 3 will be the solution. ∴ Solution = {3, 3}

(vi) $(2x - 3)(x + 2) = 0$.

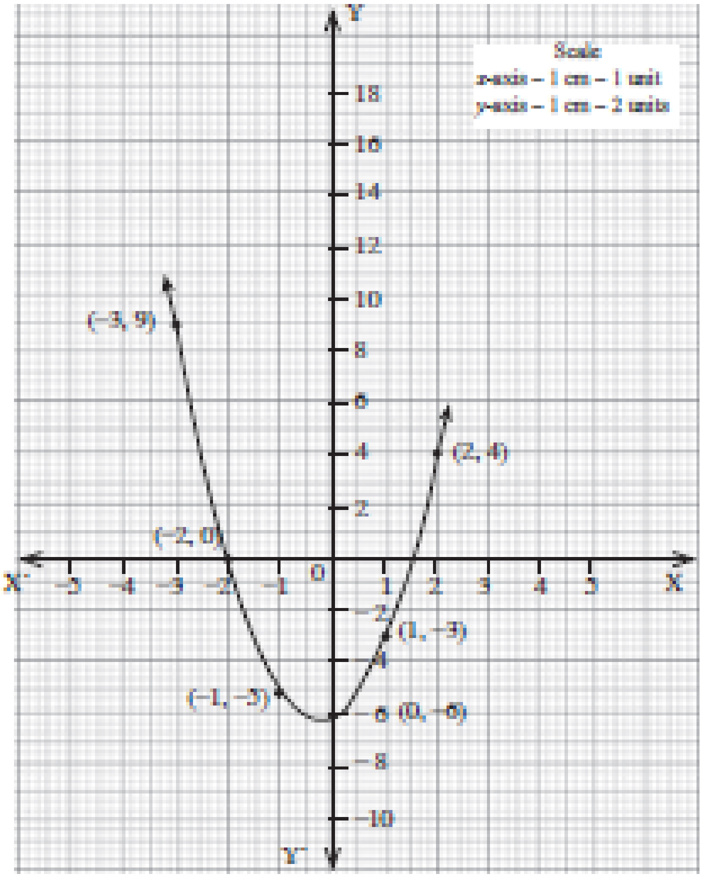
$$y = (2x - 3)(x + 2)$$
$$= 2x^2 + 4x - 3x - 6$$
$$= 2x^2 + x - 6$$

Table:

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$2x^2$	32	18	8	2	0	2	8	18	32
$+x$	–4	–3	–2	–1	0	1	2	3	4
–6	–6	–6	–6	–6	–6	–6	–6	–6	–6
y	22	9	0	–5	–6	–3	4	15	30

Points:

(–4, 22), (–3, 9), (–2, 0), (–1,–5), (0,–6), (1, –3), (2, 4), (3, 15), (4, 30)



Solution:

Here the curve meets the x-axis at 2 points (–2, 0), (1.5, 0).

∴ The equation has real and unequal roots.

∴ The x-coordinates are –2, 1.5 will be the solution.

∴ Solution = {–2, 1.5}

13. Draw the graph of $y = x^2 + x - 2$ and hence solve $x^2 + x - 2 = 0$.

Solution:

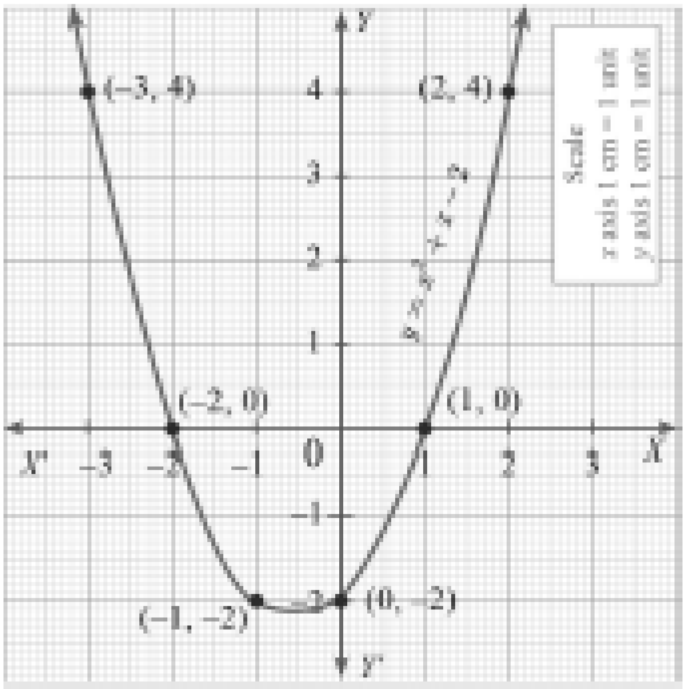
Table:

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
+x	–4	–3	–2	–1	0	1	2	3	4
–2	–2	–2	–2	–2	–2	–2	–2	–2	–2
y	10	4	0	–2	–2	0	4	10	18

Points: (–4, 10), (–3, 4), (–2, 0), (–1, –2), (0, –2), (1, 0), (2, 4), (3, 10), (4, 18)

Subtraction

$$\begin{array}{lcl} y & = & x^2 + x - 2 \\ 0 & = & x^2 + x - 2 \\ (-) & & (-)\ (-)\ (-) \\ \hline y & = & 0 \\ \hline \end{array}$$



Solution: –2 and 1

14. Draw the graph of $y = x^2 + 3x - 4$ and hence use it to solve $x^2 + 3x - 4 = 0$

Solution:

Table:

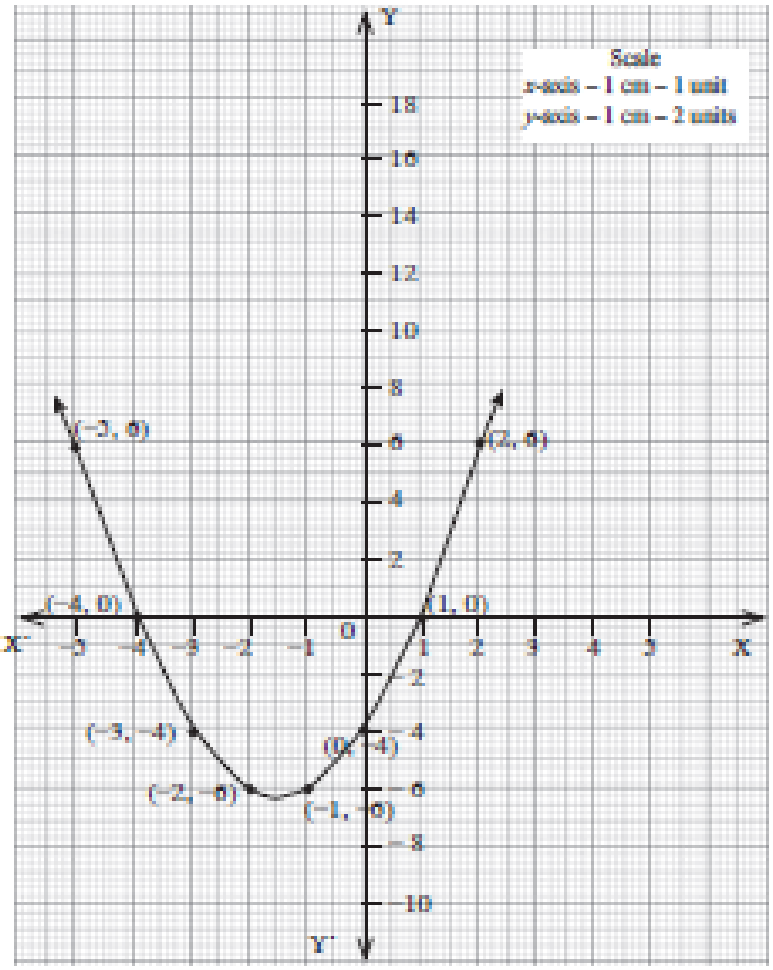
x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
3x	–12	–9	–6	-3	0	3	6	9	12
-4	-4	-4	-4	-4	-4	-4	-4	-4	-4
y	0	-4	-6	-6	-4	0	6	14	24

Points:

(–4, 0), (–3, –4), (–2, –6), (–1, –6), (0, –4), (1, 0), (2, 6), (3, 14), (4, 24)

Subtraction

$$\begin{array}{lcl} y & = & x^2 + 3x - 4 \\ 0 & = & x^2 + 3x - 4 \\ (-) & & (-)\ (-)\ (+) \\ \hline y & = & 0 \\ \hline \end{array}$$



Solution:

The curve meets x–axis at $(-4, 0)$, $(1, 0)$ and

the co-ordinates of the points $x = -4$, $x = 1$ will be the solution of $x^2 + 3x - 4 = 0$

$\therefore$ Solution = $\{-4, 1\}$

15. Draw the graph of $y = x^2 - 5x - 6$ and hence solve $x^2 - 5x - 14 = 0$

Solution:

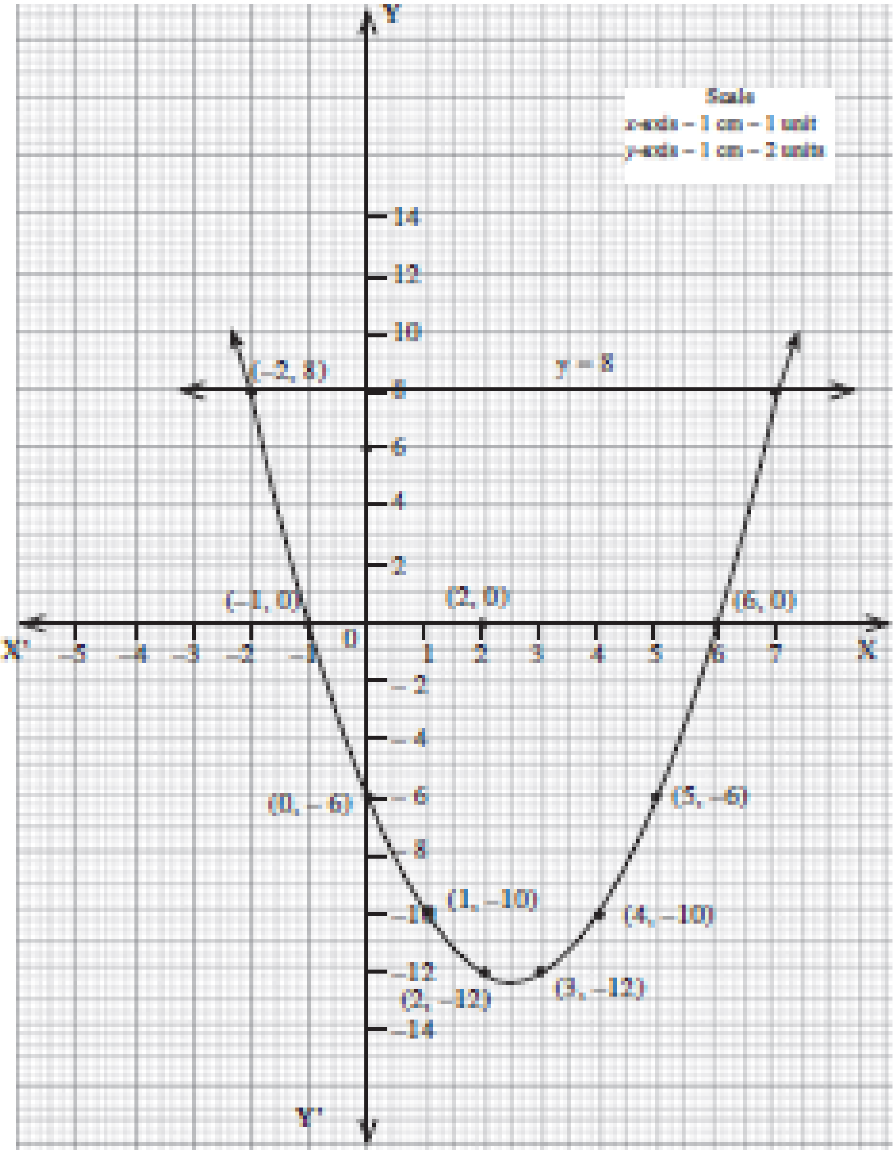
Table

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
–5x	20	15	10	5	0	–5	–10	–15	–20
–6	–6	–6	–6	–6	–6	–6	–6	–6	–6
y	30	18	8	0	–6	–10	–12	–12	–10

Points $(-4, 30)$, $(-3, 4)$, $(-2, 8)$, $(-1, 0)$, $(0, -6)$, $(1, -10)$, $(2, -12)$, $(3, -12)$, $(4, 10)$

Subtraction

$$\begin{array}{rcl} y & = & x^2 - 5x - 6 \\ 0 & = & x^2 - 5x - 14 \\ (-) & & (-)\ (+)\ \ (+) \\ \hline y & = & 8 \\ \hline \end{array}$$



Solution

The x co-ordinates of the points $x = -2$, $x = 7$ will be the solution $x^2 - 5x - 14 = 0$

$\therefore$ Solution = $\{-2, 7\}$

For Practice

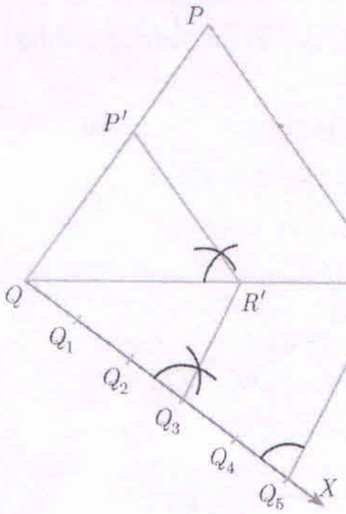
1. Draw the graph of $y = 2x^2$ and hence solve $2x^2 - x - 6 = 0$
2. Draw the graph of $y = x^2 + 4x + 3$ and hence find the roots of $x^2 + x + 1 = 0$
3. Draw the graph of $y = x^2 - 4x + 3$ and use it to solve $x^2 - 6x + 9 = 0$
4. Draw the graph of $y = x^2 - 4$ and hence solve $x^2 - x - 12 = 0$
5. Draw the graph of $y = x^2 + x$ and hence solve $x^2 + 1 = 0$
6. Draw the graph of $y = x^2 + 3x + 2$ and use it to solve $x^2 + 2x + 1 = 0$
7. Draw the graph of $y = 2x^2 - 3x - 5$ and hence solve $2x^2 - 4x - 6 = 0$
8. Draw the graph of $y = (x - 1)(x + 3)$ and hence solve $x^2 - x - 6 = 0$

***

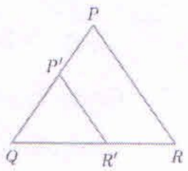
4 GEOMETRY

1. **Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{3}{5}$ of the corresponding sides of the triangle PQR (scale factor $\frac{3}{5} < 1$)**

Solution:



Rough diagram



Given a triangle PQR we are required to construct another triangle whose sides are 3/5 of the corresponding sides of the triangle PQR

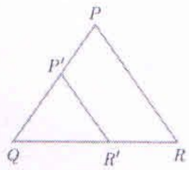
Steps of Construction:

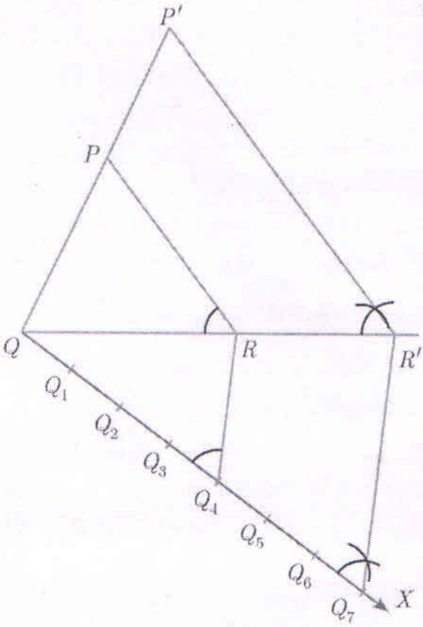
1. Construct a Δ PQR with any measurement
2. Draw a ray QX making an acute angle with QR on the side opposite to vertex P
3. Locate 5(the greater of 3 and 5 in $\frac{3}{5}$) points. Q_1, Q_2, Q_3, Q_4 and Q_5 on QX so that $QQ_1 = Q_1Q_2 = Q_2Q_3 = Q_3Q_4 = Q_4\,Q_5$.
4. Join Q_5R and draw a line through Q_3(the third point, 3 being smaller of 3 and 5 in $\frac{3}{5}$) parallel to Q5R to intersect QR at R.
5. Draw a line through R' parallel to the line RP to intersect QP at P'. Then, Δ P'QR' is the required triangle each of whose sides is three-fifths of the corresponding sides of ΔPQR.

2. **Cons Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{4}$ of the corresponding sides of the triangle PQR (scale factor $\frac{7}{4} > 1$)**

Solution:

Rough diagram



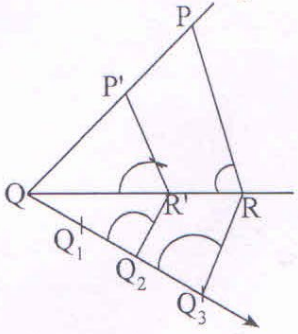


Given a triangle PQR, we are required to construct another triangle whose sides are $\frac{7}{4}$ of the corresponding sides of the triangle PQR

Steps of Construction:

1. Construct a Δ PQR with any measurement
2. Draw a ray QX making an acute angle with QR on the side opposite to vertex P
3. Locate 7 (the greater of 7 and 4 in $\frac{7}{4}$). Q_1, Q_2, Q_3, Q_4 and Q_5 on QX so that $QQ_1 = Q_1Q_2 = Q_2Q_3 = Q_3Q_4 = Q_4Q_5$.
4. Join Q_4 (the 4th point, 4 being smaller of 4 and 7 in $\frac{7}{4}$) to R and draw a line through Q7 parallel to Q_4R, intersecting the extended line segment QR at R'.
5. Draw a line through R' parallel to the line RP to intersect QP at P'. Then, Δ P'QR' is the required triangle each of whose sides is seven - fourths of the corresponding sides of ΔPQR.

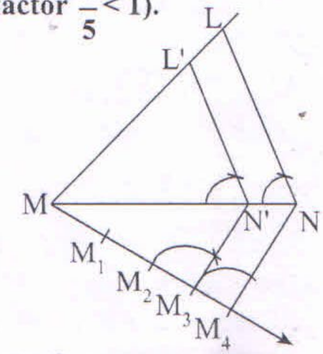
3. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{2}{3}$ of the corresponding sides of the triangle PQR (scale factor $\frac{2}{3} < 1$).



Steps of Construction

1. Draw a Triangle PQR with any measurement
2. Draw any ray QX making an acute angle with QR on the side opposite to the vertex P.
3. Locate 3 (the greater of 2 and 3 in $\frac{2}{3}$) points. Q_1, Q_2, Q_3 on QX so that $QQ_1 = Q_1Q_2 = Q_2Q_3$.
4. Join Q_3, R and draw a line through Q_2 (the second point, 2 being smaller of 2 and 3 in $\frac{2}{3}$) parallel to Q_3 R to intersect QR at R'.
5. Draw line through R^' parallel to the line RP to intersect QP at P'.
6. The ΔP'QR' is the required triangle each of the whose sides is $\frac{2}{3}$ of the corresponding sides of ΔPQR.

4. Construct a triangle similar to a given triangle LMN with its sides equal to $\frac{4}{5}$ of the corresponding sides of the triangle LMN (scale factor $\frac{4}{5} < 1$).

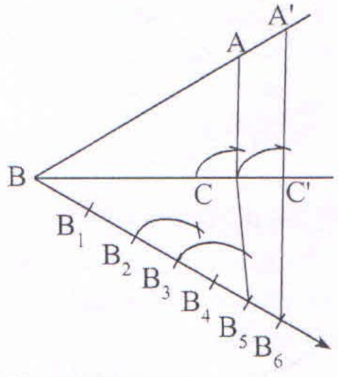


Steps of Construction

1. Draw a Triangle LMN with any measurement
2. Draw any ray making an acute angle to the vertex L.
3. Locate 5 points (the greater of 4 and 5 in –). M_1, M_2, M_3, M_4, M_5 and MX so that $MM_1 = M_1M_2 = M_2M_3 = M_3M_4 = M_4M_5$.
4. Join M_5N and draw a line parallel to M_5N through M_4 (the fourth point, 4 being smaller of 4 and 5 in $\frac{4}{5}$) to intersect in MN at N'.
5. Draw line through N' paralel to the line NL to intersect ML at L'

   The ΔL'MN' is the required triangle each of the whose sides is $\frac{4}{5}$ of the corresponding sides of ΔLMN.

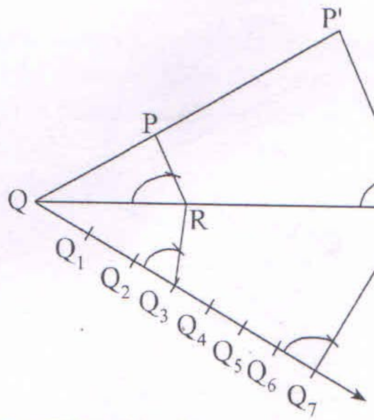
5. Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{6}{5}$ of the corresponding sides of the triangle ABC (scale factor $\frac{6}{5} > 1$). SEP-20



Steps of Construction

1. Draw a Triangle ABC with any measurement
2. Draw any ray BX making an acute angle with BC on the opposite side to the vertex A.
3. Locate 6(the greater of 6 and 5 in 6/5) points in BX. B_1, B_2, B_3, B_4, B_5, B_6 so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6$.
4. Join B_3, R(the fifth points, 5 being smaller of 5 and 6 in $\frac{6}{5}$) to C and draw a line through B6 parallel to B5C intersecting the extended line segment BC at C'.
5. Draw a line through C' parallel to CA intersecting the extended line segment BA at A'
6. The ΔA'BC' is the required triangle each of the whose sides is $\frac{6}{5}$ of the corresponding sides of ΔABC.

6. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{3}$ of the corresponding sides of the triangle PQR (scale factor $\frac{7}{3} > 1$).



Steps of Construction

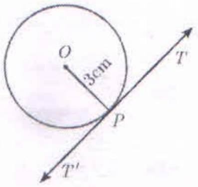
1. Draw a Triangle PQR with any measurement
2. Draw any ray QX making an acute angle with QR on the opposite side to the vertex P.
3. Locate 7 points(the greater of 7 and 3 in $\frac{7}{3}$) points. Q_1, Q_2, Q_3, Q_4, Q_5, Q_6 and Q_7 so that $QQ_1 = Q_1Q_2 = Q_2Q_3 = Q_3Q_4 = Q_4Q_5 = Q_5Q_6 = Q_6Q_7$
4. Join Q_3R and draw a line segment through Q_7 parallel to Q_3R to intersecting the extended line segment QR at R'.
5. Draw line segment through R' parallel to the PR to intersecting the extended line segment QP at P'
6. The ΔP'QR' is the required triangle each of the whose sides is $\frac{7}{3}$ of the corresponding sides of ΔPQR.

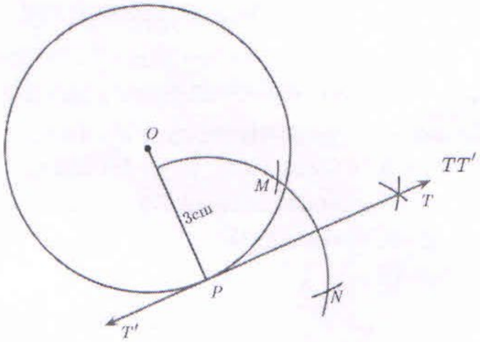
7. Draw a circle of radius 3 cm. Take a point P on this circle and draw a tangent at P.

Solution:

Given, radius r = 3 cm

Rough diagram





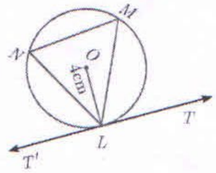
Construction

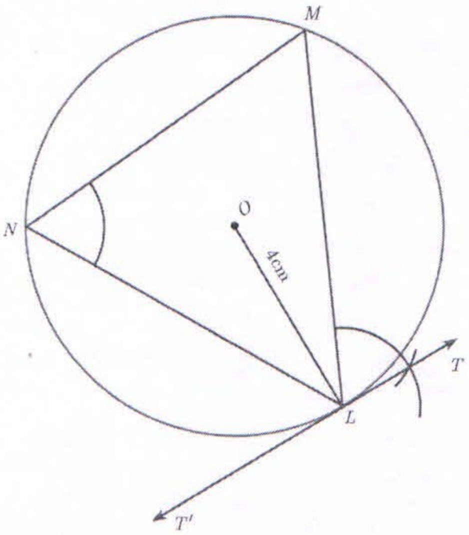
1. Draw a circle with centre at O of radius 3 cm.
2. Take a point P on the circle. Join OP.
3. Draw perpendicular time TT' to OP which passes through P.
4. TT' is the required tangent.

8. Draw a circle of radius 4 cm. At a point L on it draw a tangent to the circle using the alternate segment.

Solution:

Rough diagram





Construction

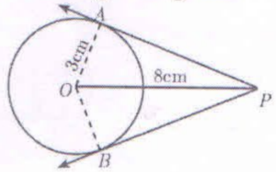
1. With O as the centre, draw a circle of radius 4cm.
2. Take a point L on the circle. Through L draw any chord LM.
3. Take a point N distinct from L and M on the circle, so that L, M and N are in anti-clockwise direction. Join LN and NM.

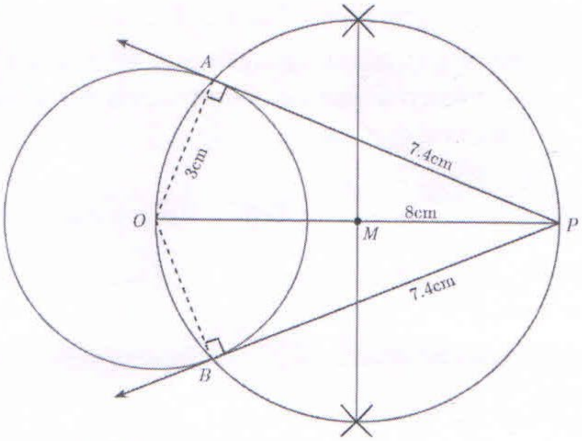
4. Through L draw a tangent TT' such that $\angle TLM = \angle MNL$.
5. TT' is the required tangent .

9. Draw a circle of diameter 6 cm from a point P, which is 8 cm away from its centre. Draw the two tangents PA and PB to the circle and measure their lengths.

Solution:

Rough diagram





Construction:

1. With centre at O, draw a circle of radius 3 cm.
2. Draw a line OP of length 8 cm.
3. Draw a perpendicular bisector of OP, which curs OP at M.
4. With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.
5. Join AP and BP. AP and BP are the required tangents. Thus length of the tangents are PA = PB = 7.4 cm,

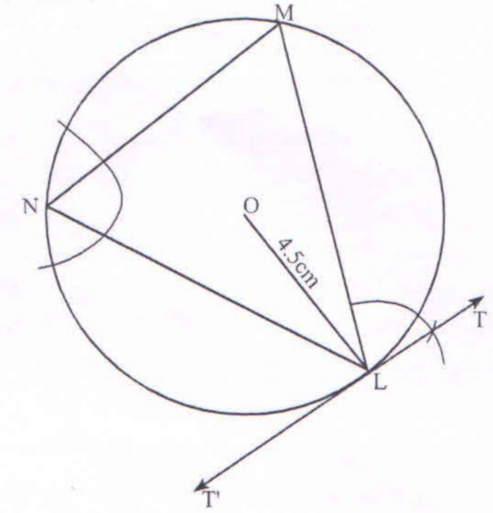
   Verification: In the right angle triangle OAP.

   $$PA^2 - OA^2 = 64 - 9 = 55$$

   $$PA = \sqrt{55} = 7.4 \text{ cm}$$

10. Draw a circle of radius 4.5 cm. Take a point on the circle. Draw the tangent at that point using the alternate segment theorem.

Solution:

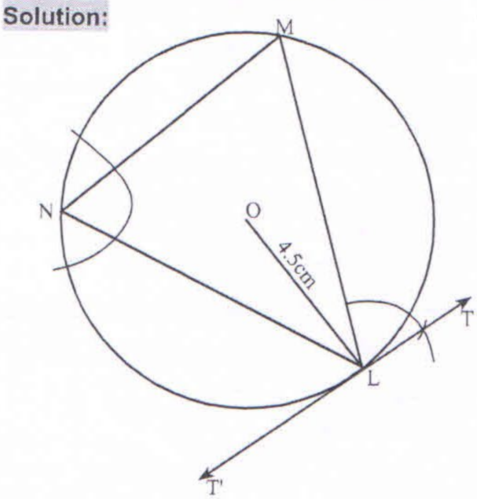


Construction:

1. Draw a circle with centre at P of radius 4.5 cm.
2. Take a point L on the circle. Though L draw any chord LM.
3. Take a point M distinct from L and N on the circle, So thatL, M and N are in anticlockwise direction. Join LN and NM.
4. Through L draw a tangent TT' such that $\angle TLM = \angle MNL$.
5. TT' is the required tangent.

11. Draw a circle of radius 4.5 cm. Take a point on the circle. Draw the tangent at that point using the alternate segment theorem.

Solution:



Construction:

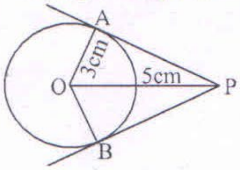
1. Draw a circle with centre at P of radius 4.5 cm.
2. Take a point L on the circle. Though L draw any chord LM.

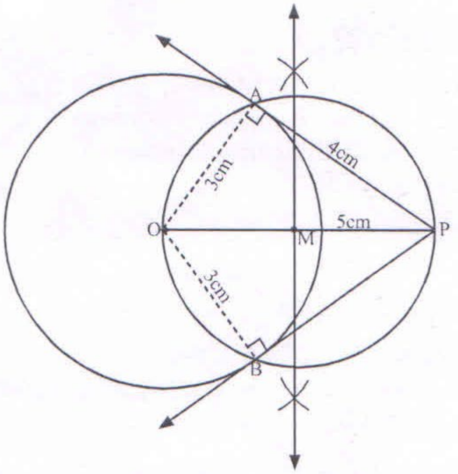
3. Take a point M distinct from L and N on the circle, So thatL, M and N are in anticlockwise direction. Join LN and NM.
4. Through L draw a tangent TT' such that $\angle TLM = \angle MNL$.
5. TT' is the required tangent.

12. Draw the two tangents from a point which is 10 cm away from the centre of a circle of radius 5 cm. Also, measure the lengths of the tangents. SEP-20

Solution:

Rough Diagram





Construction:

1. With centre at O, draw a circle of radius 5cm.
2. Draw a line OP = 10 cm
3. Draw a perpendicular bisector of OP, which cuts OP at M.
4. With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.
5. Join AP and BP. AP and BO are the required tangents. Thus length of the tangents are PA = PB = 8.7 cm.

Proof:

In ΔOPA

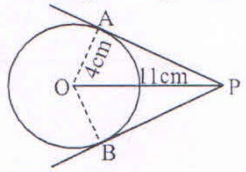
$PA^2 = OP^2 - OA^2$

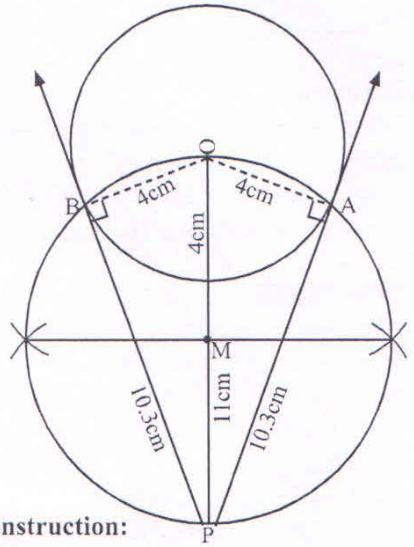
$= 10^2 - 5^2 = 100 - 25 = 75$

$PA = \sqrt{75} = 8.6$ cm (approx)

13. Take a point which is 11 cm away from the centre of a circle of radius 4 cm and draw the two tangents to the circle from that point.

Rough Diagram





Construction:

1. With centre at O, draw a circle of radius 4cm.
2. Draw a line OP = 11 cm
3. Draw a perpendicular bisector of OP, which cuts OP at M.
4. With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.
5. Join AP and BP. AP and BO are the required tangents. Thus length of the tangents are PA = PB = 10.2 cm.

Verification:

In ΔOPA $AP^2 = OP^2 - OA^2$

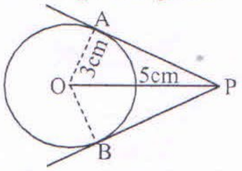
$= 11^2 - 4^2 = 121 - 16 = 105$

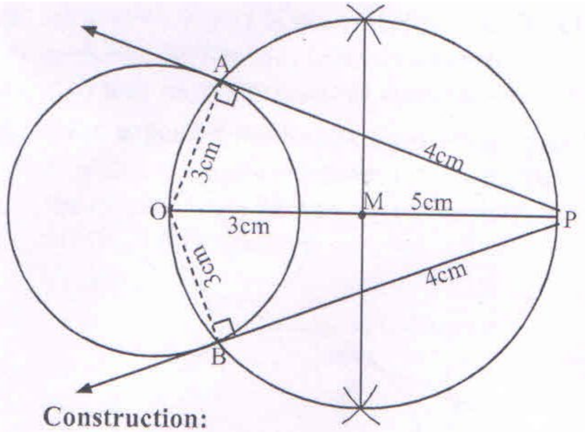
$AP = \sqrt{105} = 10.2$ cm

14. Draw the two tangents from a point which is 5 cm away from the centre of a circle of diameter 6 cm. Also, measure the lengths of the tangents. SEP-21

Solution:

Rough Diagram





Construction:

1. With centre at O, draw a circle of radius 3cm.
2. Draw a line OP = 5cm
3. Draw a perpendicular bisector of OP, which cuts OP at M.
4. With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.
5. Join AP and BP. AP and BO are the required tangents. Thus length of the tangents are PA = PB = 4 cm.

Verification

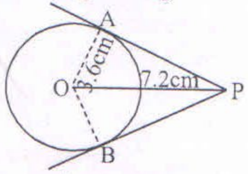
In $\Delta OPA\ AP^2 = OP^2 - OA^2$

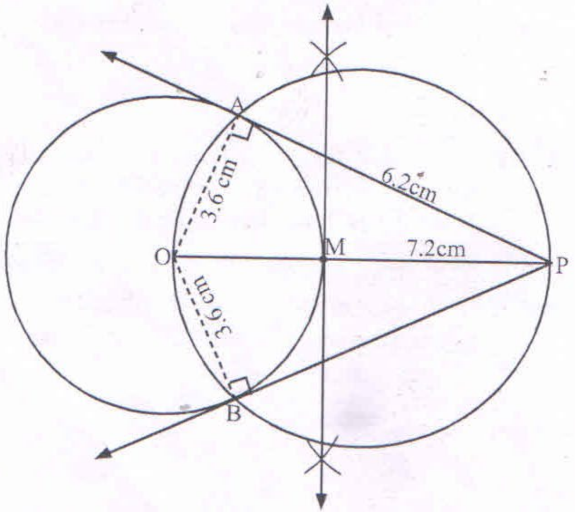
$= 5^2 - 3^2 = 25 - 9 = 16$

$AP = \sqrt{16} = 4$ cm

15. Draw a tangent to the circle from the point P having radius 3.6cm, and centre at O. Point P is at a distance 7.2 cm from the centre.

Solution: **Rough Diagram**





Construction:

1. With centre at O, draw a circle of radius 3.6 cm.
2. Draw a line OP = 7.2 cm
3. Draw a perpendicular bisector of OP, which cuts OP at M.
4. With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.
5. Join AP and BP. AP and BO are the required tangents. Thus length of the tangents are PA = PB = 6.2 cm.

Verification:

In $\Delta OPA,\ PA^2 = OP^2 - OA^2$

$= 7.2^2 - 3.6^2$

$= 51.84 - 12.96$

$= 38.88$

$PA = \sqrt{38.88} = 6.2$ cm (approx)

For Practice

1. Construct a ΔPQR in which PQ = 8 cm, R = 60° and the median RG from R to PQ is 5.8 cm. Find the length of the altitude from R to PQ.
2. Construct a triangle ΔPQR such that QR = 5 cm, ∠P = 30° and the altitude from P to QR is of length 4.2 cm.
3. Draw a triangle ABC of base BC = 8 cm, A = 60° and the bisector of ∠A meets BC at D such that BD = 6 cm.
4. Construct a ΔPQR in which QR = 5 cm, P = 40° and the median PG from P to QR is 4.4 cm. Find the length of the altitude from P to QR.
5. Construct a ΔPQR such that QR = 6.5 cm, P = 60° and the altitude from P to QR is of length 4.5 cm.
6. Construct a ΔABC such that AB = 5.5 cm, C = 25° and the altitude from C to AB is 4 cm.
7. Draw a triangle ABC of base BC = 5.6 cm, A = 40° and the bisector of ∠A meets BC at D such that CD = 4 cm.
8. Draw ΔPQR such that PQ = 6.8 cm, vertical angle is 50° and the bisector of the vertical angle meets the base at D where PD = 5.2 cm.

***

CLASSIFICATION OF TEXT BOOK PROBLEMS

CHAPTER: 1

RELATIONS AND FUNCTIONS

2 MARKS	5 MARKS
EX 1.1 (1-i, ii, iii) (2,3) Eg: 1.1 (i, ii, iii),1.2	Ex 1.1(4,5,6,7 each) Eg 1.3 (Each)
Ex 1.2 (2,3)	Ex 1.2(1,4 –i,ii) (5) Eg: 1.4,1.5
Ex 1.3 (2,3,5,7,9)	Ex 1.3(1,4,6,8,10) Eg : 1.8
Ex1.4 (1,4,5,6,11) Eg:1.10,1.12,1.13,1.14,1.17	Ex1.4(2,3,7,9,10,12) Eg: 1.15,1.16,1.18,1.11
Ex 1.5 (1-i, ii, iii)(2-i, ii)(4) Eg :1.19,1.20,1.21,1.22	Ex 1.5(3,5,6,7,8-i, ii, iii) (9,10) Eg:1.23, 1.24.

Chapter: 3

ALGEBRA

Exercise	2 Marks	5 Marks
3.4	Eg. 3.14 (i) (ii) (iii) Ex: 2 (i) (ii) (iii) (iv)	
3.5	Eg. 3.15 (i), 3.16 (i) Ex: 1 (i), (iii)	-
3.7	Eg: 3.19 (i) (ii) Ex: 1 (i) (iii)	-
3.8	-	Eg: 3.21, 3.22 Ex: 1, 2, 3
3.9	Eg: 3.24, 3.25 Ex: 1 (i), (ii), (iii) 2 (i) (iii) (iv)	-
3.11	-	Eg: 3.33, 3.34 Ex; 2 (i)
3.13	Eg: 3.40 Ex: 1 (i) (ii) (iii) (iv)	-
3.17	Eg: 3.56, 3.57, 3.58 Ex: 1, 2, 3, 4, 5, 6, 7 (i) (ii)	Eg: 3.59 Ex: 7 (iii)
3.18	Eg: 3.60, 3.61, 3.62, 3.63, 3.64 Ex: 1, 4	Eg: 3.65, 3.66 Ex: 2, 3, 5, 6, 7, 8
3.19	Eg: 3.67, 3.68 Ex: 1, 2, 3, 4, 6, 9, 10	Eg: 3.70, 3.71, 3.729, 3.73 Ex: 5, 6, 7, 8, 11, 12, 13

CHAPTER: 5
CO-ORDINATE GEOMETRY

Exercise	2 marks	5 marks
5.1	1(i),(ii), 2 (i) (ii) 3 (i), (ii) 4(i) ,(ii) Eg 5.1, 5.2, 5.3	5(i), (ii), 6, 7, 8, 9, 10, 11 Eg: 5.4, 5.5, 5.6, 5.7
5.2	1(i), (ii) 2(i), (ii) 3(i), (ii), 4, 5, 6, 7, 8, Eg: 5.8(i), (ii) Eg: 5.9(i), (ii), (iii) Eg: 5.10, 5.11, 5.12	9, 10, 11, 12, 13. 5.13, 5.14, 5.15, 5.16
5.3	1(i), (ii), 2, 3, 4, 5, 6, 7(i),7(ii), 8, 10, 12(i) ,(ii) 13(i) (ii) E.g: 5.17 (i), (ii) 5.18 (i), (ii) 5.19, 5.21, 5.22, 5.24, 5.25, 5.26	9, 11, 14 (i) (ii), 5.20, 5.27, 5.28, 5.29
5.4	1(i), (ii), 2 (i)(ii), 3 (i), (ii), 4 Eg 5.30, 5.31(i),(ii), 5.32, 5.33	5, 6, 7, 8, 9, 10, 11, 12. 5.35, 5.34, 5.36, 5.37
Unit exercise		1, 2, 3, 4, 5, 6, 7, 8, 9, 10

CHAPTER: 8
STATISTICS AND PROBABILITY

Exercise	2 marks	5 marks
8.1	1(i) (ii), 2, 3, 7, 8, 9 Eg: 8.1, 8.2, 8.3	4, 5, 6, 10, 11, 12, 13, 14, 15 Eg: 8.4, 8.5, 8.6, 8.7, 8.8, 8.9, 8.10, 8.11, 8.12, 8.13, 8.14
8.2	1, 2, 3, 4 Eg: 8.15	5, 6, 7, 8 Eg: 8.16
8.3	1, 2, 3, 4 Eg: 8.17, 8.18, 8.20, 8.22, 8.23	5, 6, 7, 8, 9, 10, 11, 12, 13, 14, Eg: 8.19, 8.21, 8.24, 8.25
8.4	1, 2, 3, 4, 5 Eg: 8.26, 8.27, 8.29	6, 7, 8, 9, 10, 11, 12, 13, 14 Eg: 8.28, 8.30, 8.31, 8.32
	Unit Exercise – 8 9	Unit Exercise – 8 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12

CHAPTER :1

SET, RELATIONS AND FUNCTIONS

Two marks:-

Exercise 1.1

1. Find AxB , AxA, BxA

(i) A={2,-2,3}   B={1,-4}

Sol:

AxB={2,-2,3} x (1,-4}

AxB={(2,1) (2,-4) (-2,1) (-2,-4) (3,1) (3,-4)}

AxA={2,-2,3} x {2,-2,3}

$$AxA=\left\{\begin{matrix}(2,2)(2,-2)(2,3)\\(-2,2)(-2,-2)(-2,3)\\(3,2)\ \ (3,-2)\ \ \ (3,3)\end{matrix}\right\}$$

BxA= {1,-4} x {2,-2,3}

$$=\left\{\begin{matrix}(1,2)(1,-2)(1,3)\\(-4,2)\ )(-4,-2)\ (-4,3)\end{matrix}\right\}$$

(ii) A=B = {p,q}

Sol:

$$AxB=\{p,q\}\ x\ \{p,q\}=\left\{\begin{matrix}(p,p)(p,q)\\(q,p)(q,q)\end{matrix}\right\}$$

$$AxA=\{p,q\}\ x\ \{p,q\}=\left\{\begin{matrix}(p,p)(p,q)\\(q,p)(q,q)\end{matrix}\right\}$$

$$BxA=\{p,q\}\ x\ \{p,q\}=\left\{\begin{matrix}(p,p)(p,q)\\(q,p)(q,q)\end{matrix}\right\}$$

(iii) A={m,n} B=ϕ

sol:

AxB= {m,n} x ϕ = ϕ

$$AxA=\{m,n\}x\{m,n\}=\left\{\begin{matrix}(m,m)(m,n)\\(n,m)(n,n)\end{matrix}\right\}$$

BxA= ϕ x {m,n} = ϕ.

2. A={1,2,3}

B={x/x is a prime number less than 10}

Find AxB and BxA.

Sol: A={1,2,3}

B={x/x is a prime number less than 10}

B={2,3,5,7}

AxB= {1,2,3} x {2,3,5,7}

$$AxB=\left\{\begin{matrix}(1,2)(1,3)(1,5)\ (1,7)\\(2,2)(2,3)(2,5)\ (2,7)\\(3,2)(3,3)(3,5)(3,7)\end{matrix}\right\}$$

BxA= {2,3,5,7} x{1,2,3}

$$BxA=\left\{\begin{matrix}(2,1)(2,2)(2,3)\\(3,1)\ (3,2)\ (3,3)\\(5,1)(5,2)(5,3)\\(7,1)(7,2)\ (7,3)\end{matrix}\right\}$$

---

3. If BxA={(-2,3) (-2,4) (0,3) (0,4)(3,3)(3,4)}

Find A and B.

Sol:

A= {3,4}   B={-2,0,3}

$\downarrow$ $\downarrow$

(2nd element)   (1st element)

EXERCISE 1.2

2. Let A={1,2,3,4,…,45} and R be the relation define as "is square of" on A. Write R as subset of AxA. Also find Domain and range R.

Sol: A= {1,2,3,4,…,45}

R={x,y) | y=x^2 and $x \in A \mid y \in A$}

x	1	2	3	4	5	6
y=x^2	1	4	9	16	25	36

R={(1,1) (2,4) (3,9) (4,16) (5,25) (6,36)

Domain = {1,2,3,4,5,6}

Range = {1,4,9,16,25,36}

---

3. A relation R is given by the set {(x, y)/ y = x +3, x ∈{0,1,2,3,4,5}

Determine Domain and Range:

x	0	1	2	3	4	5
y=x+3	3	4	5	6	7	8

Domain= {0,1,2,3,4,5}

Range = {3, 4, 5, 6, 7, 8}

Exercise -1.3

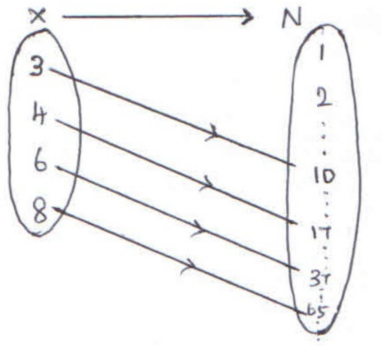
2. Let X= {3,4,6,8}. Determine whether the relation

R={x,f(x)/ $x \in X$, f(x)=x^2+1} is a relation from X to N?

Sol:

x	3	4	6	8
f(x)=x^2+1	10	17	37	65

R={(3,10) (4,17) (6,37) (8,65)



Yes, it is function (All elements from domain have image. in co-domain)

---

3. Given: the function f:$x \rightarrow x^2$-5x+6

Evaluate (i) f(-1)   (ii)f(2a)   (iii)f(2) (iv) f(x-1)

Sol: f(x)=x^2-5x+6

(i) f(-1)=$(-1)^2$-5(-1)+6

= 1+5+6 = 12

(ii) f(2a)=$(2a)^2 - 5(2a) + 6$

=$4a^2 - 10a + 6$

(iii) f(2)= 2^2-5(2)+6

= 4-10+6 = 0

(iv) f(x-1)=$(x-1)^2 - 5(x-1) + 6$

$= x^2 + 1 - 2x - 5x + 5 + 6$

$= x^2 - 7x + 12$

5. Let f(x)=2x+5. If x≠0 then find $\frac{f(x+2)-f(2)}{x}$

Sol: f(x+2) =2(x+2)+5

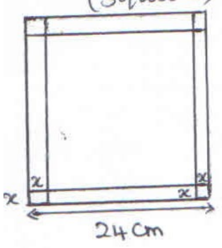
= 2x+4+5 = 2x+9

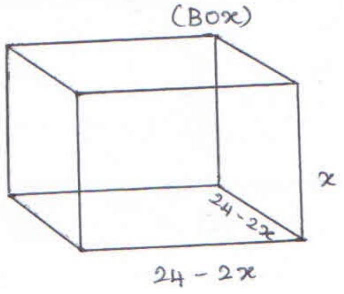
f(2)=4+5 =9

$\frac{f(x+2)-f(2)}{x} = \frac{2x+9-9}{x}$ =2

7. An open box is to be made from a square piece of material, 24cm on a side, by cutting equal squares from the corners and turning up the sides as shown in figure. Express the volume V of the box as a function of x.

Sol:





Side of square =24cm

Open box(cuboid)

$l = 24 - 2x$ b=24-2x h=x

Volume of box=lbh

= (24-2x)(24-2x)(x)

= $(24-2x)^2x$

=(576-96x+4x^2)x

=4x^3-96x^2+576x

9. A plane is flying at a speed of 500km/hr. Express the distance 'd' travelled by the plane as function of time 't' in hours.

Sol: Speed= 500km/hr.

Distance = 'd'

Time = 't'

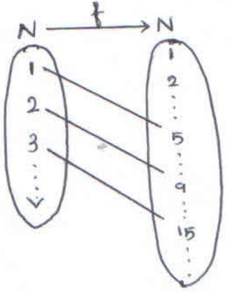
Distance=Speed x time

d= 500t

Exercise-1.3

5. Show that f:N→N defined by f(m)=m^2+m+3 is one-one function?

Domain = N= {1,2,3,…}



m	1	2	3	…
f(m)=m^2+m+3	5	9	15	…

Yes it is one-one, (Since all elements from Domain have unique image in codomain)

Exercise-1.5

1. Find fog and gof. Check whether fog = gof.

(i) f(x)=x-6 g(x)=x^2

fog =(fog)(x) =f[g(x)]=f(x^2)

=$x^2 - 6$

gof=g[f(x)]=g[x-6]=$(x-6)^2$

fog≠gof.

(ii) f(x)=$\frac{2}{x}$ g(x)=2x^2-1

fog= f(g(x))= f(2x^2-1)

fog=$\frac{2}{x^2-1}$

gof = g[f(x)]= g$\left[\frac{2}{x}\right]$

=2$(\frac{2}{x})^2$-1 = 2$(\frac{4}{x^2})$-1

gof=$\frac{8}{x^2}$-1

fog≠gof

(iii) f(x)=$\frac{x+6}{3}$,g(x)=3-x

fog=f[g(x)]=f[3-x]= $\frac{3-x+6}{3}$

=$\frac{9-x}{3}$

gof= g[f(x)] = g$\left[\frac{x+6}{3}\right]$

gof= 3-$\left(\frac{x+6}{3}\right)$ = $\frac{9-x+6}{3}$

gof= $\frac{3-x}{3}$

fog≠gof

(iv) f(x)=3+x g(x)=x-4

fog= f[g(x)]=f(x-4)

=3+(x-4) =x-1

gof= g[f(x)] =g[3+x] =3+x-4 = x-1

fog = gof

(v) f(x)=4x^2+1 g(x)=1+x

fog= f[g(x)] = f[1+x] = $4(1+x)^2 - 1$

= 4(1+2x+x^2) -1

fog= $4x^2 + 8x + 3$

gof= g[f(x)] = g[4x^2+1] =1+4x^2+1

gof=4x^2+2

fog≠gof

2. Find "k" such that fog=gof.

(i) f(x) =3x+2 g(x)=6x-k

Sol:

fog=gof

f[6x-k]=g[3x+2]

3(6x-k)+2=6(3x+2)-k

18x-3k+2=18x+12-k

-2k=10

k=-5

(ii) f(x)=2x-k g(x)4x+5

Sol: fog=gof

f[4x+5]=g[2x-k]

2(4x+5)-k=4(2x-k)+5

8x+10-k=8x-4k+5

3k=-5 $\Longrightarrow$ k=$\frac{-5}{3}$

4. f(x)=x^2-1 ; g(x)=x-2 find 'a' if gof(a)=1.

Sol: gof(a)=1

g[f(a)]=1

g[a^2-1]=1

a^2-1-2=1

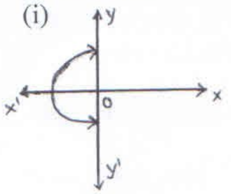
a^2-3=1$\longrightarrow a^2$=4

a=$\sqrt{4}$ =±2

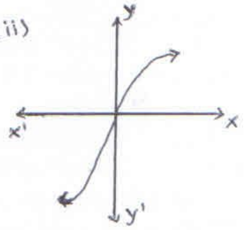
Exercise 1.4

1. Determine whether the graph given below represent functions. Give reason for your answers concerning each graph.

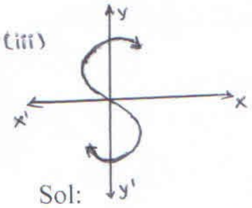
(i)



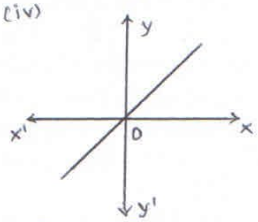
(ii)



(iii)

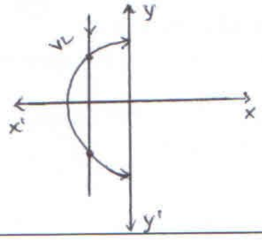


(iv)



Sol:

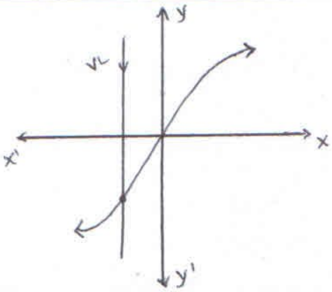
(i)



It is not function.

Reason:
Draw a vertical line it meets graph more than one points.

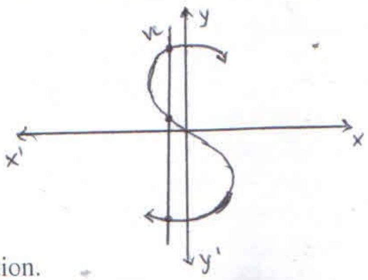
(ii)



It is function.

Reason: Draw vertical line it meets graph at one point.

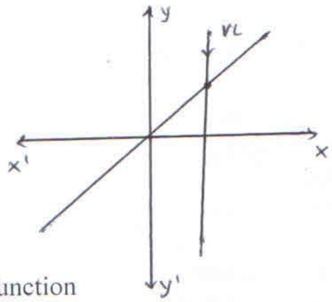
(iii)



It is not function.

Reason: Draw vertical line across the graph it meets more than one points.

(iv)

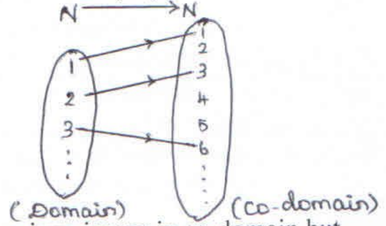


It is function

Reason: Draw a vertical line it meet graph at one point.

Show that function f:N→ N is defined by f(x)=2x-1 is one-one but not onto.

x	f(x)=2x-1
1	2-1=1
2	4-1=3
3	6-1=5



All elements in domain have unique image in co-domain but some elements from co-domain does not have pre image.

∴f is one-one but not onto.

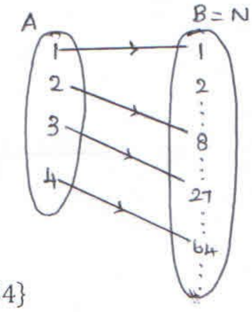
5. Let A={1,2,3,4} and B=N. Let f:A→B be defined by f(x)=x^3 then,

(i) Find the range of f.

(ii) Identify the type of function.

Sol:

x	f(x)=x^3
1	1^3=1
2	$(2)^3$=8
3	$(3)^3$=27
4	$(4)^3$=64



(i)Range of f ={1,8,27,64}

(ii)one –one function.

11. The distance S an object travels under the influence of gravity in time 't' seconds is given by S(t)=$\frac{1}{2}gt^2$+at+b where (g isthe acceleration due to gravity) a,b are constants. Verify whether the function s(t) is one-one or not.

Solution:

S(t)=$\frac{1}{2}gt^2$+at+b

S(t_1)=S(t_2)

$\frac{1}{2}$g$(t_1)^2$+at_1+b =$\frac{1}{2}$g$(t_2)^2$+at_2+b

$\frac{1}{2}$g$(t_1)^2$-$\frac{1}{2}$g$(t_2)^2$ = at_2+b-at_1-b

$\frac{1}{2}$g$[(t_1)^2 - (t_2)^2]$ = a(t_2-t_1)

$\frac{1}{2}$g(t_1+t_2)(t_1-t_2)-a(t_2-t_1) = 0

$\frac{1}{2}$g(t_1+t_2)(t_1-t_2)+a(t_1-t_2) = 0

$(t_1$-$t_2)\left[\frac{1}{2}g(t_1+t_2)+a\right]$= 0

t_1-t_2=0

t_1=t_2

∴ S(t) is 1-1.

Expected 2 marks from Examples

Eg.1.1: If A={1,3,5} and B={2,3} then
(i) Find AxB and BxA [Each]
(ii) Is AxB=BxA? If not why?
(iii) Show that n(AxB)=n(A) x n(B)

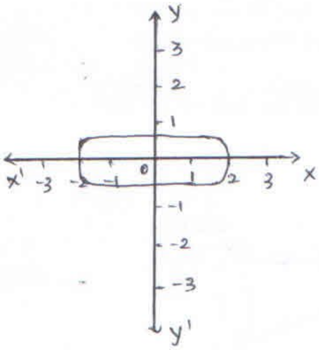
Eg 1.2: If AxB={(3,2) (3,4) (5,2) (5,4)} find A and B.

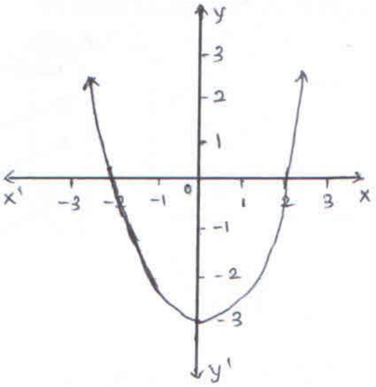
Eg 1.6: Let X={1,2,3,4} and Y={2,4,6,8,10} and R={(1,2) (2,4) (3,6) (4,8)}. Show that R is a function and find its domain, co-domain, Range.

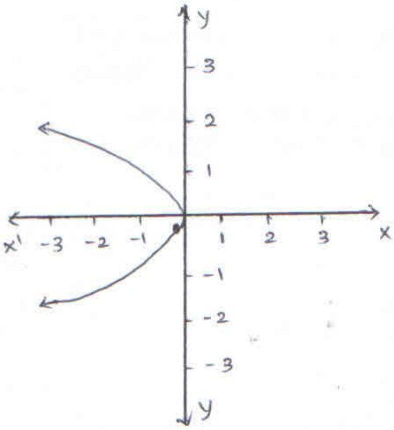
Eg.1.7 : A relation f:X→Y is defined by f(x)=x^2-2 where X={-2,-1,0,3} and Y=R.
(i) List the elements of f. (ii) Is f a function?

Eg.1.9 : Given f(x)=2x-x^2 find (i) f(1) (ii) f(x+1) (iii) f(x)+f(1).

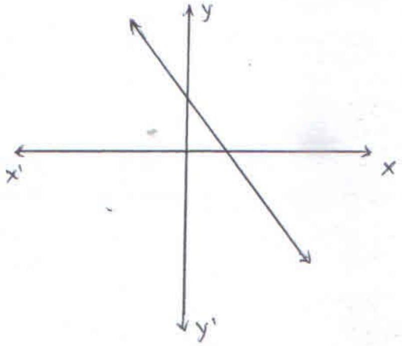
Eg.1.10: Using vertical line, determine which of the following curves represent a function?

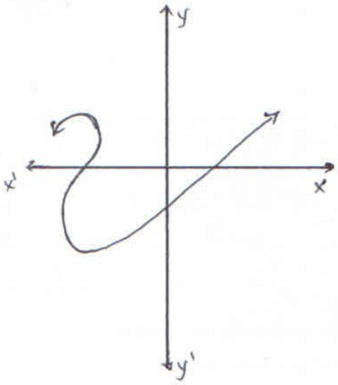


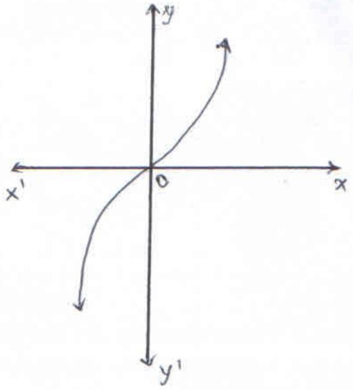




Eg.1.12 : Using horizontal test determine which of the following functions are one-one.







Eg.1.13 : Let A={1,2,3} , B={4,5,6,7} and f={(1,4) (2,5) (3,6)} be a function from A to B. Show that f is one-one but not onto function.

Eg.1.14 :If A={-2,-1,0,1,2} and f:A→B is an onto function defined by f(x)=x^2+x+1 then find B.

Eg.1.17 :Let f be a function from R to R defined by f(x)=3x-5. Find the values of a and b given that (a,4) and (1,b) belong to f.

Eg.1.19 :Find fog and gof when f(x)=2x+1 and g(x)=x^2-2.

Eg.1.20 : Represent the function f(x)=$\sqrt{2x^2-5x+3}$ as a composition of two functions.

Eg.1.21 :If f(x)=3x-2 , g(x)=2x+k and if fog=gof, then find the value of 'k'.

Eg.1.22 :Find k if fof(k)=5 where f(k)=2k-1.

FIVE MARKS:

Exercise-1.1

4. If A={5,6} B={4,5,6} C={5,6,7}.
Show that AxA=(BxB)∩(CxC).

Sol: L.H.S= AxA $=\left\{\begin{matrix}(5,5)(5,6)\\(6,5)\ (6,6)\end{matrix}\right\}\rightarrow(1)$

R.H.S = (BxB)∩(CxC)

$(BxB)=\left\{\begin{matrix}(4,4)(4,5)(4,6)\\(5,4)(5,5)(5,6)\\(6,4)(6,5)(6,6)\end{matrix}\right\}$

$(CxC)=\left\{\begin{matrix}(5,5)(5,6),(5,7)\\(6,5)(6,6)(6,7)\\(7,5)(7,6)(7,7)\end{matrix}\right\}$

R.H.S=(BxB)∩(CxC)$=\left\{\begin{matrix}(5,5)(5,6)\\(6,5)(6,6)\end{matrix}\right\}\rightarrow(2)$

From (1) and (2) L.H.S=R.H.S
Hence proved.

5. Given: A={1,2,3} B={2,3,5} C={3,4} and D={1,3,5}. Check if (A∩C)x(B∩D)=(AxB)∩(CxD) is true?

Sol: L.H.S = (A∩C)x(B∩D)
A∩C= {3} B∩D = {3,5}
(A∩C) x (B∩D) ={3 }x {3,5}
={(3, 3), (3, 5)} → (1)
R.H.S =(AxB)∩(CxD)

$AxB=\left\{\begin{matrix}(1,2)(1,3)(1,5)\\(2,2)(2,3)(2,5)\\(3,2)(3,3)(3,5)\end{matrix}\right\}$

CxD= {3,4} x{1,3,5}

= {(3,1) (3,3) (3, 5) (4, 1) (4, 3) (4, 5)}

$(\text{AxB}) \cap (C \times D) = \{(3,3)(3,5)\}$

From (1) and (2) L.H.S =R.H.S

Hence proved.

---

6. A=$\{x \in$ W/ x<2} B = { $x \in$ N/1< $x \leq$4} C={3,5}.

Verify (i) Ax(B∪C)=(AxB)∪(AxC)

(ii) Ax(B∩C)=(AxB)∩(AxC)

(iii)(A∪B)xC = (AxC)∪ (BxC).

Sol: (i) A={0,1} B={2,3,4} C={3,5}

L.H.S = Ax(B∪C)

B∪C = {2,3,4}∪{3,5} ={2,3,4,5}

$$\text{Ax(B}\cup\text{C)}=\{0,1\}\text{x}\{2,3,4,5\} = \left\{\begin{matrix}(0,2)(0,3)(0,4)(0,5)\\(1,2)(1,3)(1,4)(1,5)\end{matrix}\right\} \rightarrow (1)$$

R.H.S = (AxB)∪(AxC)

$$\text{AxB}=\left\{\begin{matrix}(0,2)(0,3)(0,4)\\(1,2)(1,3)(1,4)\end{matrix}\right\}$$

AxC= {(0,3) (0,5) (1,3) (1,5)}

$$(\text{AxB})\cup(\text{AxC})=\left\{\begin{matrix}(0,2)(0,3)(0,4)(0,5)\\(1,2)(1,3)(1,4)(1,5)\end{matrix}\right\} \rightarrow (2)$$

From (1) and (2) L.H.S = R.H.S Hence proved.

(ii)Ax(B∩C)=(AxB)∩(AxC)

Sol: L.H.S =Ax(B∩C)

B∩C={2,3,4}∩{3,5} ={3}

Ax(B∩C)={0,1}x{3}={(0,3) (1,3)} → (1)

R.H.S=(AxB)∩(AxC)

AxB={(0,2) (0,3) (0,4) (1,2) (1,3)(1,4)}

AxC={(0,3) (0,5) (1,3) (1,5)}

(AxB)∩(AxC)={(0,3) (1,3)} → (2)

From (1) and (2) L.H.S=R.H.S

Hence proved.

(iii) (A∪B)xC = (AxC)∪ (BxC)

L.H.S = (A∪B)xC

(A∪B)= {0,1}∪{2,3,4} = {0,1,2,3,4}

(A∪B)xC={0,1,2,3,4}x{3,5}

$$=\left\{\begin{matrix}(0,3)(0,5)(1,3)(1,5)\\(2,3)(2,5)(3,3)(3,5)\\(4,3)(4,5)\end{matrix}\right\} \rightarrow (1)$$

R.H.S = (AxC)∪ (BxC)

$$\text{AxC} = \left\{\begin{matrix}(0,3)(0,5)\\(1,3)(1,5)\end{matrix}\right\}$$

BxC={(2,3) (2,5) (3,3) (3,5) (4,3) (4,5)}

$$(\text{AxC})\cup(\text{BxC})=\left\{\begin{matrix}(0,3)(0,5)(1,3)(1,5)\\(2,3)(2,5)(3,3)(3,5)\\(4,3)(4,5)\end{matrix}\right\} \rightarrow (2)$$

From (1) and (2) L.H.S = R.H.S

Hence proved.

7.Let A=Set of natural numbers less than 8

B=Set of all prime numbers less than 8

C= Set of even prime numbers

Verify (i) (A∩B)xC=(AxC) ∩(BxC)

(ii) Ax(B-C)=(AxB)-(AxC)

Sol: A={1,2,3,4,5,6,7} ; B={2,3,5,7} ; C={2}

(i) (A∩B)xC=(AxC) ∩(BxC)

L.H.S= A∩B={1,2,3,4,5,6,7}∩{2,3,5,7}

A∩B={2,3,5,7}

(A∩B)xC = {2,3,5,7}x{2}

={(2,2) (3,2) (5,2) (7,2)} → (1)

$$\text{R.H.S} = \quad \text{AxC}=\left\{\begin{matrix}(1,2)(2,2)(3,2)(4,2)\\(5,2)(6,2)(7,2)\end{matrix}\right\}$$

BxC = {(2,2) (3,2) (5,2) (7,2)}

$$(\text{AxC})\cap(\text{BxC}) = \left\{\begin{matrix}(2,2)(3,2)\\(5,2)\ (7,2)\end{matrix}\right\} \rightarrow \qquad (2)$$

From (1) and (2) L.H.S=R.H.S

Hence proved.

(ii) Ax(B-C)=(AxB)-(AxC)

L.H.S : B-C ={2,3,5,7} –{2}={3,5,7}

$$\text{Ax(B-C)}=\left\{\begin{matrix}(1,3)(1,5)(1,7)(2,3)\\(2,5)(2,7)(3,3)(3,5)\\(3,7)(4,3)(4,5)(4,7)\\(5,3)(5,5)(5,7)(6,3)\\(6,5)(6,7)(7,3)(7,5)\\(7,7)\end{matrix}\right\} \rightarrow \quad (1)$$

$$\text{R.H.S} : A \times B = \left\{\begin{matrix}(1,2)\ (1,3)\ (1,5)\ (1,7)\\(2,2)\ (2,3)\ (2,5)\ (2,7)\\(3,2)\ (3,3)\ (3,5)\ (3,7)\\(4,2)\ (4,3)\ (4,5)\ (4,7)\\(5,2)\ (5,3)\ (5,5)\ (5,7)\\(6,2)\ (6,3)\ (6,5)\ (6,7)\\(7,2)\ (7,3)\ (7,5)\ (7,7)\end{matrix}\right\}$$

$$A \times C = \left\{\begin{matrix}(1,2)\ (2,2)(3,2)(4,2)\\(5,2)\ (6,2)\ (7,2)\end{matrix}\right\}$$

$$(A \times B)-(A \times C) = \left\{\begin{matrix}(1,3)\ (1,5)\ (1,7)\\(2,3)\ (2,5)\ (2,7)\\(3,3)\ (3,5)\ (3,7)\\(4,3)\ (4,5)\ (4,7)\\(5,3)\ (5,5)\ (5,7)\\(6,3)\ (6,5)\ (6,7)\\(7,3)\ (7,5)\ (7,7)\end{matrix}\right\} \longrightarrow \text{(ii)}$$

from (i) & (ii) L.H.S = R.H.S.

Try: Eg ~~1~~3: Let A={x∈N1<$x<4$}
B={x∈W/ 0≤x<2} C={x∈N/x<3}
Verify that (i) Ax(B∪C)=(AxB)∪(AxC)
(ii) Ax(B∩C)=(AxB)∩(AxC)

Eg. 1.18: If the function f:R→R is defined by

f(x)=$\begin{cases} 2x+7 & , x<-2 \\ x^2-2 & ,-2\le x<3 \\ 3x-2 & , x\ge 3 \end{cases}$ then find the values of

(i)f(4) (ii) f(-2) (iii) f(4)+2f(1) (iv) $\frac{f(1)-3f(4)}{f(-3)}$

f(x)=$\begin{cases} 2x+7 & , x<-2 \\ x^2-2 & ,-2\le x<3 \\ 3x-2 & , x\ge 3 \end{cases}$

Sol:

$$f(x)=\begin{cases} 2x+7 & , x\in\{-3,-4,-5,\cdots\} \\ x^2-2 & , x\in\{-2,-1,0,1,2\} \\ 3x-2 & , x\in\{3,4,5,\cdots\} \end{cases}$$

(i) f(4) =3x-2 = 3(4)-2 =12-2 =10
(ii) f(-2) =x^2-2 = $(-2)^2$-2 = 4-2 =2
(iii) f(4) =3x-2= 3(4) -2 =10
f(1) = x^2-1= $(-1)^2$-1 =1-1 =0
f(4)+2f(1) =10+2(0) =10+0 =10
(iv) f(1) =0 f(4) =10
f(-3)=2x+7 = 2(-3)+7 = -6+7 =1
$\frac{f(1)-3f(4)}{f(-3)}=\frac{0-3(10)}{1}=\frac{-30}{1}$=-30

Exercise-1.2

1. Let A={1,2,3,7} B={3,0,-1,7} which of the following are relation from A to B?

(i) R_1 = {(2,1) (7,1)} (ii) R_2 = {(-1,1)}
(iii) R_3 = {(2,-1) (7,7) (1,3)}
(iv) R_4 = {(7,-1) (0,3) (3,3) (0,7)}

Sol: AxB =$\begin{Bmatrix} (1,3)(1,0)(1,-1)(1,7) \\ (2,3)(2,0)(2,-1)(2,7) \\ (3,3)(3,0)(3,-1)(3,7) \\ (7,3)(7,0)(7,-1)(7,7) \end{Bmatrix}$

(i) R_1 ={(2,1) , (7,1)}
(2,1) and (7,1) ∈ AxB
∴ R_1 is not relation.
(ii) R_2 = {(-1,1} (-1,1)} ∉ AxB
∴ R_2 is not relation.
(iii) R_3 ={(2.-1) (7,7) (1,3)}
(2,-1) (7,7) (1,3) ∈ AxB
∴ R_3 is relation.

(iv) R_4={(7,-1) (0,3) (3,3) (0,7)}
(7,-1) and (3,3) ∈ AxB
But (0,3) and (0,7) ∉ AxB
∴ R_4 is not relation.

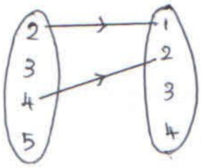
4. Represent each of the given relation by 'a' an arrow diagram 'b' a graph 'c' a set in roaster form wherever possible.

(i) {(x,y)/ x=2y, x∈{2,3,4,5} ; y∈{1,2,3,4}}

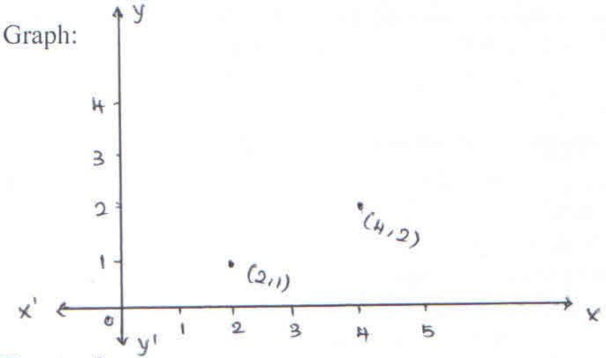
Sol: (i) R=$\begin{Bmatrix} (x,y)/ & x=2y \\ & x\in\{2,3,4,5\} \\ & y\in\{1,2,3,4\} \end{Bmatrix}$

x	2	3	4	5
$y=\frac{x}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$

a) Arrow diagram:



b) Graph:

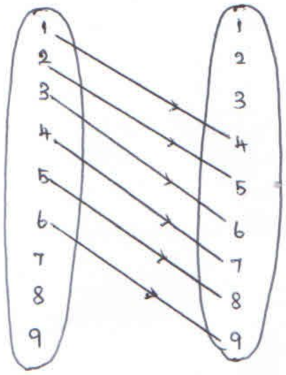


c) Roaster form:
R={(2,1) (4,2)}

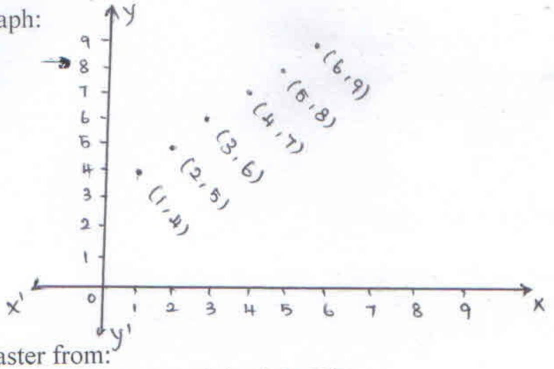
(ii) R=$\begin{Bmatrix} (x,y)\ / & y=x+3 \\ & x,y\ are\ natural \\ & numbers<10 \end{Bmatrix}$
x and y ∈ {1,2,3,4,5,6,7,8,9}

x	1	2	3	4	5	6
y=x+3	4	5	6	7	8	9

a) Arrow diagram:



b) Graph:



c)Roaster from:

R={(1,4)(2,5)(3,6)(4,7)(5,8)(6,9)}

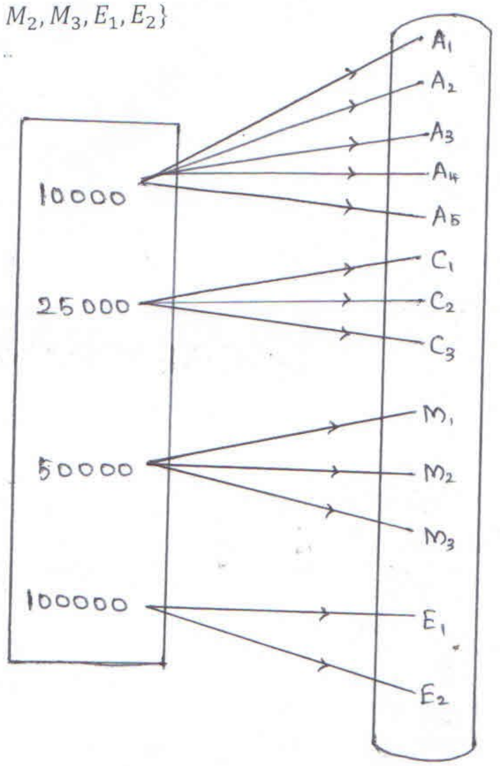
5. A company has four categories of employees given by Assistants(A), Clerks (c), Managers(M) and an Executive officers(E).The company provide Rs.10,000 , Rs.25,000 , Rs.50,000 , and Rs.100000 as salaries to the people who work in the categories A,C,M and E respectively. If A_1,A_2,A_3,A_4 and A_5 were Assistants; C_1,C_2,C_3,C_4 were clerks; M_1,M_2,M_3 were Managers and E_1,E_2 were Executive officers and if the relations R is defined by $_xR_y$ where x is salary given to a person y, express the relation R through an ordered pair and an arrow diagram.

Sol:

Salary(x)={10000, 25000, 50000, 100000}

Person(y)= $\{A_1,A_2,A_3,A_4,A_5,C_1,C_2,C_3,C_4,M_1,$

$M_2,M_3,E_1,E_2\}$



$$R=\left\{\begin{array}{c}(10000,A_1)(10000,A_2)(10000,A_3)\\(10000,A_4)(10000,A_5)(25000,C_1)\\(25000,C_2)(25000,C_3)(25000,C_4)\\(50000,M_1)(50000,M_2)(50000,M_3)\\(100000,E_1)(100000,E_2)\end{array}\right\}$$

Exercise-1.3

1. Let f={(x,y)|x,y∈N and y=2x} be a relation on N. Find domain, Co-domain, Range. Is this relation a function?

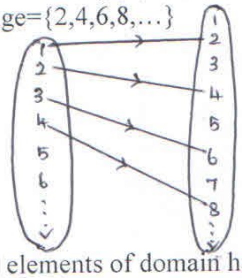
Sol: x∈{1,2,3,4,5,6,…} ; y∈{1,2,3,4,5,6,7,8,…}

x	1	2	3	4	…
y=2x	2	4	6	8	…

Domain={1,2,3,4,5,6,…}

Co-domain={1,2,3,4,5,6,7,8,…}

Range={2,4,6,8,…}



Since all elements of domain have image in co-domain. So it is function.

Exercise-1.4

2.Let f:A→B be a function defined by f(x)=$\frac{x}{2}$-1 , where A{2,4,6,10,12} and B={0,1,2,4,5,9}. Represent the function(f)

(i)set of ordered pairs (ii) a table (iii) an arrow diagram (iv)graph

Sol f(x) = $\frac{x}{2} - 1$, x ∈ A

x=2, f(2) = $\frac{2}{2}$ -1 =1-1=0

x=4, f(4) = $\frac{4}{2}$ -1 =2-1=1

x=6, f(6) = $\frac{6}{2}$ -1 =3-1=2

x=10, f(10) = $\frac{10}{2}$ -1 =5-1=4

x=12, f(12) = $\frac{12}{2}$ -1 =6-1=5

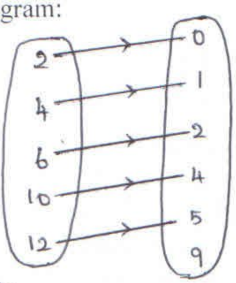
(i)Set of ordered pairs

f = {(2,0)(4,1)(6,2)(10,4) (12,5)}

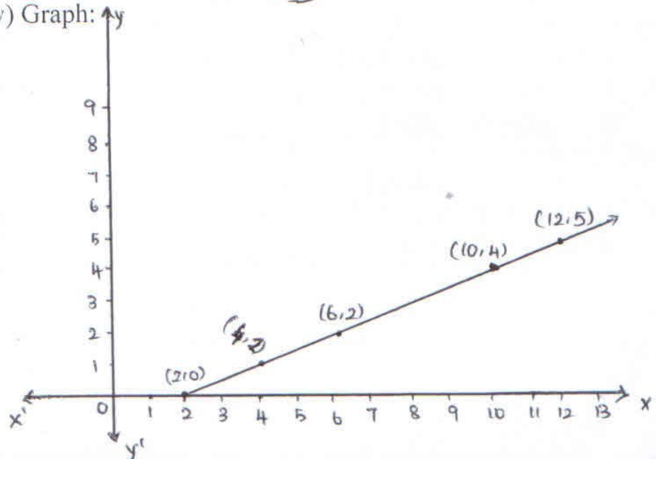
(ii)Table

x	2	4	6	10	12
f(x)	0	1	2	4	5

(i) Arrow diagram:



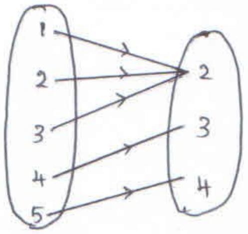
(iv) Graph:



3. Represent the function f ={(1,2)(2,2)(3,2)(4,3)(5,4)} through (i) an arrow diagram (ii) a table form (iii) a graph.

Sol: f={(1,2)(2,3) (3,2) (4,3) (5,4)}

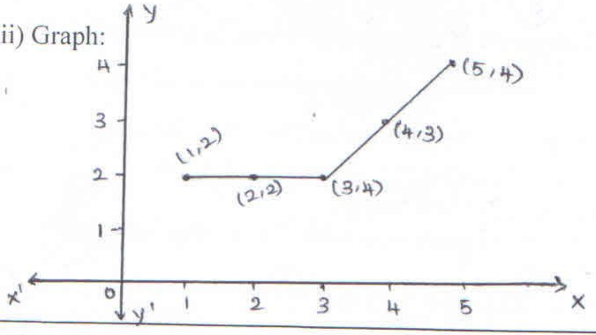
(i) Arrow diagram:



(ii) Table form:

x	1	2	3	4	5
f(x)	2	2	2	3	4

(iii) Graph:



9. If the function f is defined by

$$f(x)=\begin{cases} x+2 \quad , \quad if\ x>1 \\ 2 \quad ,if -1\le x\le 1 \\ x-1 \quad ,if -3<x<-1 \end{cases}$$

Find values of (i) f(3) (ii) f(0) (iii) f(-1.5) (iv) f(2)+f(-2)

Solution: (i) f(3) =3+2 =5

(ii) f(0) = 0

(iii) f(-1.5) =-1.5-1 =-2.5

(iv) f(2) = 2+2 =4

f(-2)= -2-1= -3

f(2) + f(-2) = 4-3= 1

10. A function f:[-5,9]→R is define as follows:

$$f(x)=\begin{cases} 6x+1\ ,\ if -5\le x<2 \\ 5x^2-1\ ,\ \ if\ 2\le x<6 \\ 3x-4\ ,\ if\ 6\le x\le 9 \end{cases}$$

Find (i) f(-3) +f(2) (ii) f(7)-f(1)

(iii) 2f(4)+f(8) (iv) $\frac{2f(-2)-f(6)}{f(4)+f(-2)}$

Sol:

$$f(x)=\begin{cases} 6x+1\ ; \quad x=-5,-4,-3,-2,-1,0,1 \\ 5x^2-1\ ; \quad x=2,3,4,5 \\ 3x-4\ ; \quad x=6,7,8,9 \end{cases}$$

(i) f(-3)+f(2)

f(-3) =6(-3)+1= -18+1 =-17

f(2) = 5(4)-1 =20-1 =19

f(-3)+f(2)= -17+19 =2

(ii) f(7)-f(1)

f(7)= 3(7)-4= 21-4 =17

f(1)= 6(1)+1= 6+1 =7

f(7)-f(1)= 17-7 =10

(iii) 2f(4)+f(8)

f(4)= $5(4)^2$-1 =80-1= 79

f(8)= 3(8)-4 =24-4 =20

2f(4)+f(8)=2(79)+20 =158+20= 178

(iv) $\frac{2f(-2)-f(6)}{f(4)+f(-2)}$

f(-2)= 6(-2)+1= -12+1 = -11

f(6) = 3(6)-4 =18-4 =14

f(4)=79

$$\frac{2f(-2)-f(6)}{f(4)+f(-2)}=\frac{2(-11)-14}{79-11}=\frac{-22-14}{68}=\frac{-36}{68}=\frac{-9}{17}$$

Try: Eg.1.18 :If f: R→R is defined by

$$f(x)=\begin{cases} 2x+7 \quad ,x<-2 \\ x^2-2 \quad ,-2\le x<3 \\ 3x+2 \quad , \quad x\ge 3 \end{cases}$$

then find values of (i) f(4) (ii) f(-2)

(iii) f(4)+2f(1) (iv)$\frac{f(1)-3f(4)}{f(-3)}$

Eg.1.11 : Let A={1,2,3,4} and B={2,5,8,11,14} be two sets, let f:A→B be a function given by f(x)=3x-1.

Represent this function :

(i) By arrow diagram (ii) In a table form

(iii) As set of ordered pairs (iv) In a graphical form

Exercise-1.5

8. (i) Show that (fog)oh=fo(goh) where f(x)=x-1; g(x)=3x+1; h(x)=x^2

Sol: L.H.S : (fog)oh

fog=f[g(x)] =f[3x+1]

fog= (3x+1)-1 =3x

(fog)oh =3h(x) =$3x^2$ → (i)

R.H.S : fo(goh)

goh= g[h(x)] =g(x^2)= $3x^2$+1

fo(goh) =f($3x^2$+1)

=($3x^2$+1)-1 =$3x^2$ → (ii)

From (i) & (ii) L.H.S=R.H.S

Hence proved.

8. (ii) Show that (fog)oh=fo(goh) where f(x)=x^2 ; g(x)=2x and h(x)=x+4.

Sol: L.H.S : (fog)oh

fog= f[g(x)]= f(2x) =$(2x)^2$=$4x^2$

(fog)oh =$4[(h(x)]^2$

=$4(x+4)^2$ → (i)

R.H.S : fo(goh)

goh= g[h(x)] =g[x+4] =2(x+4)

fo(goh)= f[2(x+4)] =$[2(x+4)]^2$

=$4(x+1)^2$ → (ii)

From (i) & (ii) L.H.S = R.H.S

Hence proved.

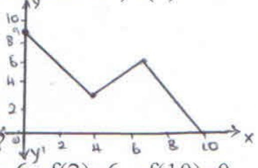
(iii) Show that (fog)oh=fo(goh) where f(x)=x-4 ;
$g(x)=x^2$; h(x)=3x-5.

Sol: L.H.S=(fog)oh
$fog= f[g(x)]= f[x^2] =x^2-4$
$(fog)oh= [h(x)]^2-4 = (3x-5)^2-4$
$= 9x^2-30x+25-4$
$= 9x^2-30x+21 \rightarrow$ (i)
R.H.S : fo(goh)
$goh= g[h(x)] = g[3x-5] =(3x-5)^2$
$fo(goh)= f[(3x-5)^2] =(3x-5)^2-4$
$= 9x^2-30x+25-4$
$= 9x^2-30x+21 \rightarrow$ (ii)
From (i) & (ii) L.H.S = R.H.S
Hence proved.

Exercise-1.3

4. A graph representing the function f(x) is given in figure it is clear that f(9) =2.

(i) Find the following values of the function. a) f(0) b)f(7) c)f(2) d) f(10)
(ii) For what value of x if f(x)=1?
(iii) Describe the following:
a)Domain b)Range
(iv) What is the image of 6 under f?



Sol: (i) From graph, f(0)=9 ; f(7)=6 ; f(2)=6 ; f(10)=0
(ii) f(x)=1 when x=9.5
(iii) Domain ={x|0≤x≤10}
Range ={y | 0≤y≤9}
(iv) Image of 6 is 5.

6. A function f is defined by f(x)=2x-3.
(i) Find $\frac{f(o)+f(1)}{2}$ (ii) Find x such that f(x)=0
(iii) Find x such that f(x)=x
(iv) Find x such that f(x)=f(1-x).

Sol: (i) f(0)= 0-3 =-3
f(1)= 2-3 = -1
$\frac{f(o)+f(1)}{2} = \frac{-3-1}{2} = \frac{-4}{2} = -2$
(ii) f(x)=0 → 2x-3=0
2x=3 → $x=\frac{3}{2}$
(iii) f(x)=x → 2x-3=x
2x-x=3 → x=3
(iv) f(x)=f(1-x) → 2x-3 = 2(1-x)-3
2x-3 = 2-2x-3
2x+2x= -1+3
4x= 2
$x=\frac{2}{4}=\frac{1}{2}$

8. A function f is defined by f(x)=3-2x. Find x such that $f(x^2)=[f(x)]^2$

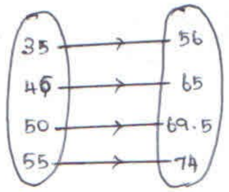
Sol: f(x)=3-2x
$[f(x)]^2= (3-2x)^2= 9-12x+4x^2$
$f(x^2)=3-2x^2$
Given: $f(x^2)=[f(x)]^2$
$3-2x^2=9-12x+4x^2$
$9-12x+4x^2+2x^2-3=0$
Divide by '6'
$x^2-2x+1=0$
$(x-1)^2=0 \rightarrow x-1=0 \rightarrow x=1$

10. The data in the adjacent table depicts the length of a person forehand and their corresponding height. Based on this data, a student finds a relationship between the height(y) and the forehand length(x) as y-ax+b, where a,b are constants.

Length 'x' of forehand (cm)	Height(y) (inches)
35	56
45	65
50	69.5
55	74

(i) Check if this relation a function (ii) Find a and b
(iii) Find the height of a person whose Forehand length is 40cm.
(iv) Find the length of forehand of a person if the height is 53.5 inches.

Sol: y= ax+b → (1)
(i) R= {(35,56) (45,65) (50,69.5) (55,74)}



All elements from (x) have image in (y) So, Relation is function.
(ii) (35,56) in (1) → 35a+b=56 → (2)
(45,65) in (1) → 45a+b=65 → (3)
Solve (2)&(3) 35a + b = 56
45a + b = 65
(-) (-) (-)
-10a =-9
$a=\frac{9}{10} = 0.9$
Substitute a=0.9 in (2),
35(0.9)+b=56
b=56-31.5
b=24.5
Y=0.9x+24.5 → (4)
(iii) Forehand length (x)=40cm ; Height(y)=?
(4)→ y=0.9(40) + 24.5
y= 36+24.5 = 60.5 inches
(iv) Height(y)= 53.3 inches ; Length of forehand(x)=?
(4) → 53.3 =0.9x+24.5
53.3-24.5 =0.9x
$x=\frac{28.8}{0.9} = \frac{288}{9} = 32$cm.
Length of forehand (x)=32cm.

12. The function 't' which maps temperature in celsius(c) into temperature in Fahrenheit (F) is defined by t(c) =F where $F=\frac{9}{5}C+ 32$. Find (i) t(0) (ii) t(28) (iii) t(-10) (iv) value of C when t(c) = 212 (v) the temperature when the Celsius value is equal to the Fahrenheit value.

Solution: (i) $t(c) = F =\frac{9}{5}C+ 32$

t(0) = $\frac{9}{5}$ (0)+ 32= 32°F

(ii) t (28) = $\frac{9}{5}$ (28)+ 32

= $\frac{252}{5}$ + 32

= 50.4+32 = 82.4°F

(iii) t (-10) = $\frac{9}{5}$ (-10)+ 32

= -18 +32 = 14°F

(iv) t(c) = 212

$\frac{9}{5}$ C+ 32 = 212

$\frac{9}{5}$ C = 212 – 32 = 180

C = 180 x $\frac{9}{5}$ = 100℃

(v) F= C

$\frac{9}{5}$ C+ 32 = C

$\frac{9}{5}$ C –C = -32

$\frac{9c-5c}{5}$ = - 32

4C = -160

C = 40°

Exercise 1.5

3. If f(x)= 2x -1, g(x) = $\frac{x+1}{2}$, show that fog = gof=x.

Solution

fog = f [g(x)]= f $\left[\frac{x+1}{2}\right]$

= 2($\frac{x+1}{2}$) -1

= x + 1 – 1= x → (i)

gof = g [f(x)]= g $[2x-1]$

= $\frac{(2x-1)+1}{2}$ = $\frac{2x}{2}$ = x → (ii)

from (i) and (ii) $\boxed{\text{Fog = gof = x}}$

Hence proved.

5. Let A,B,C ⊆ N and a function f :A → B be defined by f(x) = 2 x H and g : B → C be defined g(x) =x^2. Find the range of fog and gof.

Solution : fog = f [g(x)] = f [x^2]= $2x^2 + 1$

gof = g [f(x)]= g [$2x+1$]=$2x+1^2$

f :A → B , g : B → C

⟹ $fog : C \rightarrow$ A , where A,B,C ⊆ N

range of fog ={ y/y = $2x^2 + 1, x\epsilon\, c$}

⟹ $gof : A \rightarrow C$

range of fog ={ y/y = $(2x+1)^2$ } where $x\epsilon\, A$

6. If f(x) = x^2-1 find (i) fof (ii) fofof

Solution: (i) fof =$f[f(x)]$

= $f[x^2-1)]$

=$(x^2-1)^2 - 1$

= $x^4 - 2x^2 + 1 - 1$

= $x^4 - 2x^2$

(ii) fofof = $f[fof]$

= f ($x^4 - 2x^2$)

=($x^4 - 2x^2)^2 - 1$

= $x^8 - 4x^6 + 4x^4 - 1$.

8. f: R → R and g : R → R are defined by f(x)=x^5 and g(x)=x^4 then check if f,g are one –one and fog is one –one?

Check f is 1-1

f: R → R

f(x)=x^5

f(x_1) = f(x_2)

→ $x_1^5 = x_2^5$

→ $x_1 = x_2$

→ f is 1-1

Check g is 1-1

g: R → R

g(x)=x^4

→ g(x_1) = g(x_2)

→ $x_1^4 = x_2^4$

→ $x_1 = \pm\, x_2$

→ g is not 1-1.

fog = f [g(x)]= f [x^4]

→ $fog = (x^4)^5$

→ $fog = x^{20}$

(fog) (x_1) = (fog) (x_2)

→ ($x_1)^{20}$ = ($x_2)^{20}$

→ $x_1 = \pm\, x_2$

→ fog is not 1-1

9. Let f = { (-1,3),(0,-1) (2, -9)} be a linear function from Z to Z. find f(x).

Sol : let f (x)= y = ax +b ________(1)

Be a linear function.

(-1,3) in (1) → 3= a(-1) +b

→ -a +b =3 ________ (2)

(0,-1) in (1) → -1= a(0) +b

b = -1

Sub b = -1 in (2) → -a-1=3

-a = 4 → a = -4

sub a= -4, b = -1 in equation (1)

y=-4x – 1 is the required linear function.

10.In electrical circuit theory a circuit c(t) is called a linear circuit if it satisfies the superposition Principle given by c (at_1+ at_2) = ac(t_1) + b(ct_2) where a, b are constants. Show that the circuit (ct) = 3t is linear.

Sol: Given

c (at_1 + at_2) = ac(t_1) + bc(t_2)

LHS = c (at_1 + bt_2)

= 3 (at_1 + bt_2)

= 3at_1 +3bt_2

= a(3t_1) + b(3t_2)

= ac(t_1)+b(ct_2) = RHS

→ LHS = RHS

→ Hence c(t) is linear. Hence proved.

Problems to be practice

Example 1.4 let A ={3,4,7,8} and B={1,7,10} Which of the following sets are relations from A to B?

(i) R_1 ={(3,7)(4,7)(7,10)(8,11)} (ii) R_2={(3,11)(4,12)}

(iii) R_3 = {(3,7)(4,10)(7,7)(7,8)(8,11)(8,7)(8,10)}

Eg 1.8: If x= {-5,1,3,4} and y = {a,b,c} then which of the following relations are function from x to y?

(i) R_1 = {(-5,a)(1,a)(3,b)} (ii) R_2 ={(-5,b)(1,b)(3,a)(4,c)}

(iii) {(-5,a)(1,a)(3,b)(4,c)(1,2)}

Eg:1.5 let f be a function f:N → N be defined by f(x)=3x+2, x ∈ N

(i) find the images of 1,2,3

(ii) find the pre images of 29,53

(iii) identify the type of function:-

Eg 1.16: forensic scientists can determine the height (in cm) of a person based on the length of the thigh bone. They usually do so using the function t_1(b) = 2.47b +54.10 where b is the length of the thigh bone.

(i) Verify the function t_1 is one –one or not

(ii) Also find the height of a person if the height of his thigh bone is 50cm.

(iii) Find the length of the thigh bone if the height of a person is 14.96cm.

Eg:1.11 let A ={1,2,3,4} and B={2,5,8,11,14} be two sets. Let f : A →B be a function given by f(x)= 3x -1. Represent this function.

(i) by arrow diagram (ii) in a table form

(iii) as a set of ordered pairs (iv) in a graphical form.

Eg. 1.23 : If f(x)=2x +3, g(x)= 1-2x and h(x) = 3x prove that fo(goh) = (fog)oh

Eg : 1. 24 : Find x

If gof (x) = fog(x), given that f(x)= 3x + 1 and g(x) = x+3.

Chapter – 3

Algebra:

(2 Marks)

1. (i) Find the excluded value:

$\frac{y}{y^2-25}$

$y^2 - 25 = 0$

$y^2 = 25$

$y^x = 5^x$

$y = \pm 5$

The excluded values are +5 & -5.

(ii) $\frac{t}{t^2-5t+6}$

$t^2 - 5t + 6 = 0$

$(t-2)\ (t-3) = 0$

$t = 2 \qquad t = 3$

The excluded values are 2 & 3

(iii) $\frac{x^3-27}{x^3+x^2-6x}$

$x^3 + x^2 - 6x = 0$

$x(x^2 + x - 6) = 0$

$x(x+3)\ (x-2) = 0$

$x = 0 \mid x + 3 = 0 \mid x - 2 = 0$

$x = -3 \qquad x = 2$

the excluded values are 0, -3, 2.

(iv) $\frac{x^2+6x+8}{x^2+x-2}$

$x^2 + x - 2 = 0$

$(x+2)\ (x-1) = 0$

$x = -2, \quad x = 1$

The excluded values are -2 & 1.

(v) $\frac{x+10}{8x}$

$8x = 0$

$x = 0$

The excluded value is 0.

(vi) $\frac{7p+2}{8P^2+13P+5}$

$8P^2 + 13P + 5 = 0$

$(8P+5)\ (P+1) = 0$

P=$\frac{-5}{8}$, P = -1

The excluded values are $\frac{-5}{8}$ & -1.

(vii) $\frac{x}{x^2+1}$

$x^2 + 1 \geq 0,$ for all x

$x^2 + 1 \geq 0 + 1 = 1$

$x^2 + 1 \neq 0$ for any x

There can be no real excluded values for the given rational expression.

---

2. Find the square root of

(i) $\frac{400x^4y^{12}z^{16}}{100x^8y^4z^4}$

$\sqrt{\frac{400x^4y^{12}z^{16}}{100x^8y^4z^4}} = \frac{20x^2y^6z^8}{10x^4y^2z^2}$ =2 $\left|\frac{y^4z^6}{x^2}\right|$

---

ii)$\frac{121(a+b)^8(x+y)^8(b-c)^8}{81\ (b-c)^4(a-b)^{12}(b-c)^4}$

$$\sqrt{\frac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^4}}$$

$$= \frac{11}{9}\left|\frac{(a+b)^4(x+y)^4(b-c)^4}{(b-c)^2(a-b)^6(b-c)^2}\right|$$

$= \frac{11}{9}\ \left|\frac{(a+b)^4\ \ (x+y)^4}{(a-b)^6}\right|$

---

iii) $256\ (x-a)^8(x-b)^4(x-c)^{16}(x-d)^{20}$

$\sqrt{256\ (x-a)^8(x-b)^4(x-c)^{16}(x-d)^{20}}$

= $16|(x-a)^4(x-b)^2(x-c)^8(x-d)^{10}|$

---

iv) $\frac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}$

$\sqrt{\frac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}} = \frac{4}{3}\left|\frac{a^4b^6c^8}{f^6g^2h^7}\right|$

---

3 Simplify:

i) $\frac{4x^2y}{2z^2} \times \frac{6xz^3}{20y^4} = \frac{3x^3z}{5y^3}$

---

ii) $\frac{5t^3}{4t-8} \times \frac{6t-12}{10t} = \frac{5t^3}{4(t-2)} \times \frac{6(t-2)}{10t}$

$= \frac{3t^2}{4}$

---

iii) $\frac{x^3}{9y^2} \times \frac{27y}{x^5} = \frac{3}{x^2y}$

---

iv) $\frac{14x^4}{y} \div \frac{7x}{3y^4} = \frac{14x^4}{y} \times \frac{3y^4}{7x} = 6x^3y^3$

---

4. Find the zeros of the quadratic expression

$x^2 + 8x + 12$

Let $P(x) = x^2 + 8x + 12$

P(-2) = $(-2)^2 + 8(-2) + 12 = 0$

P(-6) = $(-6)^2 + 8(-6)$+12 = 0

-2 & -6 are zeros of P(x)

---

5. Write down the quadratic equation in general form for which sum and product of the roots are given below.

(i) 9, 14

Sum of the roots = 9

Product of the roots = 14

Equation: x^2 −(sum of the roots) x + (Product of the roots) = 0

= $x^2 - 9x + 14 = 0$

---

ii) $\frac{-7}{2}$, $\frac{5}{2}$

Sum of the roots = $\frac{-7}{2}$

Product of the roots = $\frac{5}{2}$

Equation: x^2 −(sum of the roots) x + (Product of the roots) = 0

$x^2 - \left(\frac{-7}{2}\right)x + \frac{5}{2} = 0$

$2x^2 + 7x + 5 = 0$

---

iii) $\frac{-3}{5}$, $\frac{-1}{2}$

Sum of the roots = $\frac{-3}{5}$

Equation: x^2 −(sum of the roots) x + (Product of the roots) = 0

$x^2 - \left(\frac{-3}{5}\right) x + \left(-\frac{1}{2}\right) = 0$

$\frac{10x^2+6x-5}{10} = 0$

$10x^2 + 6x - 5 = 0$

iv) -9, 20

Sum of the roots = -9

Product of the roots = 20

Equation: x^2 −(sum of the roots) x + (Product of the roots) = 0

$x^2 - (-9)x + 20 = 0$

$x^2 + 9x + 20 = 0$

v) $\frac{5}{3}$,4

Sum of the roots = $\frac{5}{3}$

Product of the roots = 4

Equation: x^2 −(sum of the roots) x + (Product of the roots) = 0

$x^2 - \frac{5}{3}x + 4 = 0$

$3x^2 - 5x + 12 = 0$

vi) $\frac{-3}{2}$, -1

Sum of the roots = $\frac{-3}{2}$

Product of the roots = -1

Equation: x^2 −(sum of the roots) x + (Product of the roots) = 0

$x^2 - (\frac{-3}{2})x - 1 = 0$

$2x^2 + 3x - 2 = 0$

6. Find the sum and product of the roots for each of the following quadratic equations.

(i) $x^2 + 8x - 65 = 0$

a = 1 ; b = 8; c = -65

$\alpha + \beta = \frac{-b}{a} = \frac{-8}{1} = -8$

$\alpha\beta = \frac{c}{a} = \frac{-65}{1} = -65$

ii) $2x^2 + 5x + 7 = 0$

a = 2 ; b = 5; c = 7

$\alpha + \beta = \frac{-b}{a} = \frac{-5}{2}$

$\alpha\beta = \frac{c}{a} = \frac{7}{2}$

iii) $Kx^2 - K^2x - 2K^3 = 0$

a = K ; b = $-K^2$; c = $-2K^3$

$\alpha + \beta = \frac{-b}{a} = \frac{-(-K^2)}{K}$ = +K

$\alpha\beta = \frac{c}{a} = \frac{-2K^3}{K} = -2K^2$

iv) $x^2 + 3x - 28 = 0$

a = 1 ; b = 3 ; c = -28

$\alpha + \beta = \frac{-b}{a} = \frac{-3}{1}$ = -3

$\alpha\beta = \frac{c}{a} = \frac{-28}{1}$ = -28

v) $3 + \frac{1}{a} = \frac{10}{a^2}$

$3a^2 + a - 10 = 0$

a = 3; b = 1; c = -10

$\alpha + \beta = \frac{-b}{a} = \frac{-1}{3}$

$\alpha\beta = \frac{c}{a} = \frac{-10}{3}$

vi) $3y^2 - y - 4 = 0$

a = 3 ; b = -1 ; c = -4

$\alpha + \beta = \frac{-b}{a} = \frac{-(-1)}{3} = \frac{1}{3}$

$\alpha\beta = \frac{c}{a} = \frac{-4}{3}$

7. Determine the nature of roots for the following quadratic equations:

i) $x^2 - x - 20 = 0$

a = 1 ; b = -1 ; c = -20

Δ = $b^2 - 4ac$

= $(-1)^2 - 4(1)(-20)$

= 1+80

Δ = 81 > 0, real and unequal roots.

ii) $9x^2 - 24x + 16 = 0$

a = 9 ; b = -24 ; c = 16

Δ = $b^2 - 4ac$

= $(-24)^2 - 4(9)(16)$

= 576 – 576

Δ = 0, real and equal roots.

iii) $2x^2 - 2x + 9 = 0$

a = 2 ; b = -2 ; c = 9

Δ = $b^2 - 4ac$

= $(-2)^2 - 4(2)(9)$

= 4 – 72

Δ = -68 < 0, No real roots.

iv) $15x^2 + 11x + 2 = 0$

a = 15 ; b = 11 ; c = 2

Δ = $b^2 - 4ac$

= $(11)^2 - 4(15)(2)$

= 121 − 120

Δ = 1 > 0, real and unequal roots.

v) $1x^2 - x - 1 = 0$

a = 1 ; b = -1 ; c = -1

$\Delta = b^2 - 4ac$

$= (-1)^2 - 4(1)(-1)$

$= 1 + 4$

$\Delta = 5 > 0$, real and unequal roots.

---

vi) $\sqrt{2}t^2 - 3t + 3\sqrt{2} = 0$

a = $\sqrt{2}$; b = -3 ; c = $3\sqrt{2}$

$\Delta = b^2 - 4ac$

$= (-3)^2 - 4(\sqrt{2})\,(3\sqrt{2})$

$= 9 - (12 \times 2)$

$= 9 - 24$

$\Delta = -15 < 0$, No real roots.

---

vii) $9y^2 - 6\sqrt{2}y + 2 = 0$

a = 9 ; b = $-6\sqrt{2}$; c = 2

$\Delta = b^2 - 4ac$

$= (-6\sqrt{2})^2 - 4(9)(2)$

$= (+36 \times 2) - 72$

$= 72 - 72$

$\Delta = 0$, real and equal roots.

---

8. Consider the following information

regarding the number of men and women

workers in three factories I, II and III.

Factory	Men	Women
I	23	18
II	47	36
III	15	16

Represent the above information in the form of a matrix. What does the entry in the 2nd row and 1st coloumn represent?

Matrix form of 3 × 2 is as follows

$$A = \begin{pmatrix} 23 & 18 \\ 47 & 36 \\ 15 & 16 \end{pmatrix}$$

g_{21}(Entry) 47 men workers in factory II.

---

9. If a matrix has 16 elements, what are the possible orders it can have?

1× 16 2× 8 4× 4

8× 2 16× 1

---

10. Construct a 3 × 3 matrix whose elements are $a_{ij} = i^2 j^2$

$$3 \times 3 \text{ Matrix } A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$a_{11} = 1^2 \times 1^2 = 1$	$a_{21} = 2 \times 1^2 = 4$	$a_{31} = 3^2 \times 1^2 = 9$
$a_{12} = 1^2 \times 2^2 = 4$	$a_{22} = 2^2 \times 2^2 = 16$	$a_{32} = 3^2 \times 2^2 = 36$
$a_{13} = 1^2 \times 3^2 = 9$	$a_{23} = 2^2 \times 3^2 = 36$	$a_{33} = 3^2 \times 3^2 = 81$

$$A = \begin{pmatrix} 1 & 4 & 9 \\ 4 & 16 & 36 \\ 9 & 36 & 81 \end{pmatrix}$$

---

11. In the matrix $A = \begin{pmatrix} 8 & 9 & 4 & 3 \\ -1 & \sqrt{7} & \sqrt{\frac{3}{2}} & 5 \\ 1 & 4 & 3 & 0 \\ 6 & 8 & -11 & 1 \end{pmatrix}$ write

(i) The number of elements = 16.

(ii) The order of the matrix = 4 × 4

(iii) Write the elements

$a_{22} = \sqrt{7}$

$a_{23} = \frac{\sqrt{3}}{2}$

$a_{24} = 5$

$a_{34} = 0$

$a_{43} = -11$

$a_{44} = 1$

---

12. If a matrix has 18 elements, what are the possible orders it can have? What if it has 6 elements?

18	6
1 × 18	1 × 6
2 × 9	2 × 3
3 × 6	3 × 2
6 × 3	6 × 1
9 × 2	
18 × 1	

---

13. Construct a 3 × 3 matrix whose elements are given by (i) $a_{ij} = |i - 2j|$ (ii) $a_{ij} = \frac{(i+j)^3}{3}$

$$\text{i) } 3 \times 3 \text{ matrix } A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$a_{11} = |1 - 2(1)| = |1 - 2| = |-1| = 1$

$a_{12} = |1 - 2(2)| = |1 - 4| = |-3| = 3$

$a_{13} = |1 - 2(3)| = |1 - 6| = |-5| = 5$

$a_{21} = |2 - 2(1)| = |2 - 2| = 0$

$a_{22} = |2 - 2(2)| = |2 - 4| = |-2| = 2$

$a_{23} = |2 - 2(3)| = |2 - 6| = |-4| = 4$

$a_{31} = |3 - 2(1)| = |3 - 2| = |1| = 1$

$a_{32} = |3 - 2(2)| = |3 - 4| = |-1| = 1$

$a_{33} = |3 - 2(3)| = |3 - 6| = |-3| = 3$

$$A = \begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 4 \\ 1 & 1 & 3 \end{pmatrix}$$

ii) $a_{11} = \frac{(1+1)^3}{3} = \frac{2^3}{3} = \frac{8}{3}$

$a_{12} = \frac{(1+2)^3}{3} = \frac{3^3}{3} = \frac{27}{3} = 9$

$a_{13} = \frac{(1+3)^3}{3} = \frac{4^3}{3} = \frac{64}{3}$

$a_{21} = \frac{(2+1)^3}{3} = \frac{3^3}{3} = \frac{27}{3} = 9$

$a_{22} = \frac{(2+2)^3}{3} = \frac{4^3}{3} = \frac{64}{3}$

$a_{23} = \frac{(2+3)^3}{3} = \frac{5^3}{3} = \frac{125}{3}$

$a_{31} = \frac{(3+1)^3}{3} = \frac{4^3}{3} = \frac{64}{3}$

$a_{32} = \frac{(3+2)^3}{3} = \frac{5^3}{3} = \frac{125}{3}$

$a_{33} = \frac{(3+3)^3}{3} = \frac{6^3}{3} = \frac{216}{3} = 72$

$$A = \begin{pmatrix} \frac{8}{3} & 9 & \frac{64}{3} \\ 9 & \frac{64}{3} & \frac{125}{3} \\ \frac{64}{3} & \frac{125}{3} & 72 \end{pmatrix}$$

---

14. If A = $\begin{pmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{pmatrix}$ then find the transpose of A.

$$A^T = \begin{pmatrix} 5 & 1 & 3 \\ 4 & -7 & 8 \\ 3 & 9 & 2 \end{pmatrix}$$

---

15. If A = $\begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}$ then find the transpose of –A.

$$-A = \begin{pmatrix} -\sqrt{7} & 3 \\ \sqrt{5} & -2 \\ -\sqrt{3} & 5 \end{pmatrix}$$

$$(-A)^T = \begin{pmatrix} -\sqrt{7} & \sqrt{5} & -\sqrt{3} \\ 3 & -2 & 5 \end{pmatrix}$$

---

16. If A = $\begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix}$ then verify $(A^T)^T = A$

LHS:- $A^T = \begin{pmatrix} 5 & -\sqrt{17} & 8 \\ 2 & 0.7 & 3 \\ 2 & \frac{5}{2} & 1 \end{pmatrix}$

$$(A^T)^T = \begin{pmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{pmatrix} = A$$

Hence verified.

---

17. Find the values of $x, y\ and\ z$.

(i) $\begin{pmatrix} 12 & 3 \\ x & \frac{3}{2} \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix}$

$x = 3 \qquad y = 12 \qquad z = 3$

(ii) $\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$

$$\begin{array}{l|l|l} 5+z=5 & x+y=6 & xy=8 \\ z=5-5 & x=6-y & (6-y)y=8 \\ z=0 & x=6-4=2 & 6y-y^2=8 \\ & x=6-2=4 & y^2-6y+8=0 \\ & & (y-4)\ (y-2)=0 \end{array}$$

y=4; y=2

---

18. If A = $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$, B = $\begin{pmatrix} 1 & 7 & 0 \\ 1 & 3 & 1 \\ 2 & 4 & 0 \end{pmatrix}$ find A+B.

$$A+B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} + \begin{pmatrix} 1 & 7 & 0 \\ 1 & 3 & 1 \\ 2 & 4 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 9 & 3 \\ 5 & 8 & 7 \\ 9 & 12 & 9 \end{pmatrix}$$

---

19. $$A = \begin{array}{l|cccc|} & \text{Tamil} & \text{English} & \text{Science} & \text{Mathematics} \\ \text{Group1} & 22 & 15 & 14 & 23 \\ \text{Group2} & 50 & 62 & 21 & 30 \\ \text{Group3} & 53 & 80 & 32 & 40 \end{array}$$

$$B = \begin{array}{l|cccc|} & \text{Tamil} & \text{English} & \text{Science} & \text{Mathematics} \\ \text{Group1} & 20 & 38 & 15 & 40 \\ \text{Group2} & 18 & 12 & 17 & 80 \\ \text{Group3} & 81 & 47 & 52 & 18 \end{array}$$

Find the total marks.

$$A+B = \begin{vmatrix} 42 & 53 & 29 & 63 \\ 68 & 74 & 38 & 110 \\ 134 & 127 & 84 & 58 \end{vmatrix}$$

---

20. If A = $\begin{pmatrix} 1 & 3 & -2 \\ 5 & -4 & 6 \\ -3 & 2 & 9 \end{pmatrix}$ B = $\begin{pmatrix} 1 & 8 \\ 3 & 4 \\ 9 & 6 \end{pmatrix}$, find A+B

It is not possible to add A and B. Because they have different orders.

---

21. If A = $\begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$

then find 2A+B.

$$2A+B = 2\begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix} + \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 16 & 12 \\ 2 & 6 & 18 \\ -8 & 6 & -2 \end{pmatrix} + \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 18 & 27 & 9 \\ 1 & 8 & 22 \\ -1 & 11 & -2 \end{pmatrix}$$

22. If A = $\begin{pmatrix} 5 & 4 & -2 \\ \frac{1}{2} & \frac{3}{4} & \sqrt{2} \\ 1 & 9 & 4 \end{pmatrix}$, B = $\begin{pmatrix} -7 & 4 & -3 \\ \frac{1}{4} & \frac{7}{2} & 3 \\ 5 & -6 & 9 \end{pmatrix}$

find 4A-3B.

$$4A\text{-}3B = 4\begin{pmatrix} 5 & 4 & -2 \\ \frac{1}{2} & \frac{3}{4} & \sqrt{2} \\ 1 & 9 & 4 \end{pmatrix} - 3\begin{pmatrix} -7 & 4 & -3 \\ \frac{1}{4} & \frac{7}{2} & 3 \\ 5 & -6 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 20 & 16 & -8 \\ 2 & 3 & 4\sqrt{2} \\ 4 & 36 & 16 \end{pmatrix} - \begin{pmatrix} -21 & 12 & -9 \\ \frac{3}{4} & \frac{21}{2} & 9 \\ 15 & -18 & 27 \end{pmatrix}$$

$$= \begin{pmatrix} 41 & 4 & 1 \\ \frac{5}{4} & \frac{-15}{2} & 4\sqrt{2} - 9 \\ -11 & 54 & -11 \end{pmatrix}$$

---

23. Find the value of a, b, c, d from the matrix equation.

$$\begin{pmatrix} d & 8 \\ 3b & a \end{pmatrix} + \begin{pmatrix} 3 & a \\ -2 & -4 \end{pmatrix} = \begin{pmatrix} 2 & 2a \\ b & 4c \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ -5 & 0 \end{pmatrix}$$

$$\begin{pmatrix} d+3 & 8+a \\ 3b-2 & a-4 \end{pmatrix} = \begin{pmatrix} 2 & 2a+1 \\ b-5 & 4c \end{pmatrix}$$

$$\begin{array}{l|l|l} d + 3 = 2 & 3b - 2 = b - 5 & 8 + a = 2a + 1 \\ d = 2 - 3 & 3b - b = -5 + 2 & 8 - 1 = 2a - a \\ d = -1 & 2b = -3 & 7 = a \end{array}$$

b = $\frac{-3}{2}$ or $a = 7$

also $a - 4 = 4c$

$7 - 4 = 4c$

$3 = 4c$

$\frac{3}{4} = c$ or c = $\frac{3}{4}$

---

24. If A = $\begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix}$, B = $\begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix}$ find

(i) B – 5A = $\begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix} - 5\begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix}$

$$= \begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix} - \begin{pmatrix} 0 & 20 & 45 \\ 40 & 15 & 35 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & -17 & -37 \\ -39 & -11 & -26 \end{pmatrix}$$

(ii) 3A – 9B = $3\begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix} - 9\begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix}$

$$= \begin{pmatrix} 0 & 12 & 27 \\ 24 & 9 & 21 \end{pmatrix} - \begin{pmatrix} 63 & 27 & 72 \\ 9 & 36 & 81 \end{pmatrix}$$

$$= \begin{pmatrix} -63 & -15 & -45 \\ 15 & -27 & -60 \end{pmatrix}$$

---

25. If A = $\begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{pmatrix}$ then, verify

(i) A + B = B + A (ii) A + (-A) = (-A) + A = 0

(i) L.H.S: A + B = $\begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix} + \begin{pmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 6 & 16 \\ 6 & 7 \\ 9 & -3 \end{pmatrix}$

R.H.S: B + A = $\begin{pmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix} = \begin{pmatrix} 6 & 16 \\ 6 & 7 \\ 9 & -3 \end{pmatrix}$

LHS = RHS, Hence verified.

ii) LHS: A + (-A) = $\begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix} + \begin{pmatrix} -1 & -9 \\ -3 & -4 \\ -8 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$

RHS: (-A) + A = $\begin{pmatrix} -1 & -9 \\ -3 & -4 \\ -8 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$

LHS = RHS, Hence Verified.

---

26. If A = $\begin{pmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 8 & 3 & 1 \\ 2 & 4 & 1 \\ 5 & 3 & 1 \end{pmatrix}$, find AB

AB = $\begin{pmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \end{pmatrix} \begin{pmatrix} 8 & 3 & 1 \\ 2 & 4 & 1 \\ 5 & 3 & 1 \end{pmatrix}$

$$= \begin{pmatrix} (1 \;\; 2 \;\; 0)\begin{pmatrix} 8 \\ 2 \\ 5 \end{pmatrix} & (1 \;\; 2 \;\; 0)\begin{pmatrix} 3 \\ 4 \\ 3 \end{pmatrix} & (1 \;\; 2 \;\; 0)\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \\ (3 \;\; 1 \;\; 5)\begin{pmatrix} 8 \\ 2 \\ 5 \end{pmatrix} & (3 \;\; 1 \;\; 5)\begin{pmatrix} 3 \\ 4 \\ 3 \end{pmatrix} & (3 \;\; 1 \;\; 5)\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 8+4+0 & 3+8+0 & 1+2+0 \\ 24+2+25 & 9+4+15 & 3+1+5 \end{pmatrix}$$

AB = $\begin{pmatrix} 12 & 11 & 3 \\ 51 & 28 & 9 \end{pmatrix}$

---

27. If A = $\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$, B = $\begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$ find AB & BA

check if A B = B A.

AB = $\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 4+1 & 0+3 \\ 2+3 & 0+9 \end{pmatrix}$

$= \begin{pmatrix} 5 & 3 \\ 5 & 9 \end{pmatrix}$

BA = $\begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 4+0 & 2+0 \\ 2+3 & 1+9 \end{pmatrix}$

$= \begin{pmatrix} 4 & 2 \\ 5 & 10 \end{pmatrix}$

A B $\neq BA$

---

28. Find the order of the product matrix AB if

	Order of A	Order of B	Order of AB
(i)	3× 3	3 × 3	3 × 3
(ii)	4× 3	3 × 2	4× 2
(iii)	4× 2	2 × 2	4 × 2
(iv)	4× 5	5 × 1	4 × 1
(v)	1× 1	1 × 3	1 × 3

29. If A is of order p× q and B is of order q× r what is the order of AB and BA?

(i)	Order of A	Order of B	Order of AB
	p× q	q× r	p× r
(ii)	Order of B	Order of A	Order of BA
	q× r	p× q	not defined

---

30. A has 'a' rows and 'a+3' columns.

B has 'b' rows and '17-b' columns.

If both products AB and BA exist, find a,b?

Order of A	Order of B	Order of AB exists if
a× $a+3$	$b\times 17-b$	$a+3=b$
		$a-b=-3\rightarrow(1)$
Order of B	Order of A	Order of BA exists if
$b\times 17-b$	a× $a+3$	$a=17-b$
		$a+b\ =\ 17\rightarrow$ (2)

(1) + (2)

$a-b=-3$

$a+b=17$

2a = 14

a = $\frac{14}{2}$ $=7$ a = 7

sub a = 7 in (2) 7 + b = 17

b = 17 – 7 , b = 10

a = 7, b = 10.

---

31. If A = $\begin{pmatrix}2 & 5\\4 & 3\end{pmatrix}$, B = $\begin{pmatrix}1 & -3\\2 & 5\end{pmatrix}$ find AB, BA

check if AB = BA.

$$AB=\begin{pmatrix}2 & 5\\4 & 3\end{pmatrix}\begin{pmatrix}1 & -3\\2 & 5\end{pmatrix}=\begin{pmatrix}2+10 & -6+25\\4+6 & -12+15\end{pmatrix}$$

$$=\begin{pmatrix}12 & 19\\10 & 3\end{pmatrix}$$

$$BA=\begin{pmatrix}1 & -3\\2 & 5\end{pmatrix}\begin{pmatrix}2 & 5\\4 & 3\end{pmatrix}=\begin{pmatrix}2-12 & 5-9\\4+20 & 10+15\end{pmatrix}$$

$$=\begin{pmatrix}-10 & -4\\24 & 25\end{pmatrix}$$

AB $\neq$ BA.

---

32. S.T. the matrices A = $\begin{pmatrix}1 & 2\\3 & 1\end{pmatrix}$, B = $\begin{pmatrix}1 & -2\\-3 & 1\end{pmatrix}$ satisfy commutative property AB = BA.

$$AB=\begin{pmatrix}1 & 2\\3 & 1\end{pmatrix}\begin{pmatrix}1 & -2\\-3 & 1\end{pmatrix}=\begin{pmatrix}1-6 & -2+2\\3-3 & -6+1\end{pmatrix}$$

$$=\begin{pmatrix}-5 & 0\\0 & -5\end{pmatrix}$$

$$BA=\begin{pmatrix}1 & -2\\-3 & 1\end{pmatrix}\begin{pmatrix}1 & 2\\3 & 1\end{pmatrix}=\begin{pmatrix}1-6 & 2-2\\-3+3 & -6+1\end{pmatrix}$$

$$=\begin{pmatrix}-5 & 0\\0 & -5\end{pmatrix}$$

AB = BA.

---

33. IF A = $\begin{pmatrix}cos\theta & sin\theta\\-sin\theta & cos\theta\end{pmatrix}$ prove that A$A^T=I$.

LHS: $A^T=\begin{pmatrix}cos\theta & -sin\theta\\sin\theta & cos\theta\end{pmatrix}$

$$A.\,A^T=\begin{pmatrix}cos\theta & sin\theta\\-sin\theta & cos\theta\end{pmatrix}\begin{pmatrix}cos\theta & -sin\theta\\sin\theta & cos\theta\end{pmatrix}$$

$$=\begin{pmatrix}cos^2\theta+sin^2\theta & -cos\theta\ sin\theta+cos\theta\ sin\theta\\-\ sin\theta\ cos\theta+cos\theta sin\theta & sin^2\theta+cos^2\theta\end{pmatrix}$$

$$=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$$

= I

= RHS

LHS = RHS

---

34. Verify that $A^2=I$, when A = $\begin{pmatrix}5 & -4\\6 & -5\end{pmatrix}$

$$A^2=\begin{pmatrix}5 & -4\\6 & -5\end{pmatrix}\begin{pmatrix}5 & -4\\6 & -5\end{pmatrix}$$

$$=\begin{pmatrix}25-24 & -20+20\\30-30 & -24+25\end{pmatrix}$$

$$=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$$

$A^2=I$ Hence Proved

---

35. If A = $\begin{pmatrix}7 & 8 & 6\\1 & 3 & 9\\-4 & 3 & -1\end{pmatrix}$, B = $\begin{pmatrix}4 & 11 & -3\\-1 & 2 & 4\\7 & 5 & 0\end{pmatrix}$ then, find 2A+B.

$$2A+B=2\begin{pmatrix}7 & 8 & 6\\1 & 3 & 9\\-4 & 3 & -1\end{pmatrix}+\begin{pmatrix}4 & 11 & -3\\-1 & 2 & 4\\7 & 5 & 0\end{pmatrix}$$

$$=\begin{pmatrix}14 & 16 & 12\\2 & 6 & 18\\-8 & 6 & -2\end{pmatrix}+\begin{pmatrix}4 & 11 & -3\\-1 & 2 & 4\\7 & 5 & 0\end{pmatrix}$$

$$=\begin{pmatrix}18 & 27 & 9\\1 & 8 & 22\\-1 & 11 & -2\end{pmatrix}$$

---

36. If A = $\begin{pmatrix}5 & 4 & -2\\\frac{1}{2} & \frac{3}{4} & \sqrt{2}\\1 & 9 & 4\end{pmatrix}$ B = $\begin{pmatrix}-7 & 4 & -3\\\frac{1}{4} & \frac{7}{2} & 3\\5 & -6 & 9\end{pmatrix}$

find 4A – 3B.

$$4A-3B=4\begin{pmatrix}5 & 4 & -2\\\frac{1}{2} & \frac{3}{4} & \sqrt{2}\\1 & 9 & 4\end{pmatrix}-3\begin{pmatrix}-7 & 4 & -3\\\frac{1}{4} & \frac{7}{2} & 3\\5 & -6 & 9\end{pmatrix}$$

$$=\begin{pmatrix}20 & 16 & -8\\2 & 3 & 4\sqrt{2}\\4 & 36 & 16\end{pmatrix}-\begin{pmatrix}-21 & 12 & -9\\\frac{3}{4} & \frac{21}{2} & 9\\15 & -18 & 27\end{pmatrix}$$

$$=\begin{pmatrix}41 & 4 & 1\\5/4 & -15/2 & 4\sqrt{2}-9\\-11 & 54 & -11\end{pmatrix}$$

5 Marks

1. Find the square root of $64x^4 - 16x^3 + 17x^2 - 2x + 1$

	$8x^2 - x + 1$	
$8x^2$	$64x^4 - 16x^3 + 17x^2 - 2x + 1$ $64x^4$	(−)
$16x^2 - x$	$-16x^3 + 17x^2$ $-16x^3 + x^2$	(−)
$16x^2 - 2x + 1$	$16x^2 - 2x + 1$ $16x^2 - 2x + 1$	(−)
	0	

$$\sqrt{64x^4 - 16x^3 + 17x^2 - 2x + 1} = \left|8x^2 - x + 1\right|$$

2. Find the square root of $x^4 - 12x^3 + 42x^2 - 36x + 9$

	$x^2 - 6x + 3$	
x^2	$x^4 - 12x^3 + 42x^2 - 36x + 9$ x^4 (−)	
$2x^2 - 6x$	$-12x^3 + 42x^2$ $-12x^3 + 36x^2$ (+) (−)	$\frac{-12x^3}{2x^2} = -6x$
$2x^2 - 12x + 3$	$6x^2 - 36x + 9$ $6x^2 - 36x + 9$ (−) (+) (0)	$\frac{6x^2}{2x^2} = 3$

$$\sqrt{x^4 - 12x^3 + 42x^2 - 36x + 9} = |x^2 - 6x + 3|$$

3. Find the square root of $37x^2 - 28x^3 + 4x^4 + 42x + 9$

	$2x^2 - 7x - 3$	
$2x^2$	$4x^4 - 28x^3 + 37x^2 + 42x + 9$ $4x^4$ (−)	
$4x^2 - 7x$	$-28x^3 + 37x^2$ $-28x^3 + 49x^2$ (+) (−)	$\frac{-28x^3}{4x^2} = -7x$
$4x^2 - 14x - 3$	$-12x^2 + 42x + 9$ $-12x^2 + 42x + 9$ (+) (−) (−)	$\frac{-12x^2}{4x^2} = -3$
	0	

$$\sqrt{4x^4 - 28x^3 + 37x^2 + 42x + 9} = |2x^2 - 7x - 3|$$

4. $16x^4 + 8x^2 + 1$

	$4x^2 + 1$
$4x^2$	$16x^4 + 0x^3 + 8x^2 + 0x + 1$ $16x^4$ (−)
$8x^2 + 1$	$8x^2 + 0x + 1$ $8x^2 + 0 + 1$ (−) (−) (−)
	0

$$\sqrt{16x^4 + 8x^2 + 1} = |4x^2 + 1|$$

5. $121x^4 - 198x^3 - 183x^2 + 216x + 144$

	$11x^2 - 9x - 12$
$11x^2$	$121x^4 - 198x^3 - 183x^2 + 216x + 144$ $121x^4$ (-)
$22x^2 - 9x$	$-198x^3 - 183x^2$ $-198x^3 + 81x^2$ (+) (−)
$22x^2 - 18x - 12$	$-264x^2 + 216x + 144$ $-264x^2 + 216x + 144$ (+) (−) (−)
	0

$$\sqrt{121x^4 - 198x^3 - 183x^2 + 216x + 144} = |11x^2 - 9x - 12|$$

6. If $9x^4 + 12x^3 + 28x^2 + ax + b$ is a perfect square find the values of a & b.

	$3x^2 + 2x + 4$	
$3x^2$	$9x^4 + 12x^3 + 28x^2 + ax + b$ $9x^4$	(−)
$6x^2 + 2x$	$12x^3 + 28x^2$ $12x^3 + 4x^2$	(−)
$6x^2 + 4x + 4$	$24x^2 + ax + b$ $24x^2 + 16x + 16$	(−)
	0	

$a - 16 = 0$	$b - 16 = 0$
$a = 16$	$b = 16$

7. Find the values of 'a' and 'b' if the following polynomials are perfect squares.

$4x^4 - 12x^3 + 37x^2 + bx + a$

	$2x^2 - 3x + 7$	
$2x^2$	$4x^4 - 12x^3 + 37x^2 + bx + a$ $4x^4$ (−)	
$4x^2 - 3x$	$-12x^3 + 37x^2$ $-12x^3 + 9x^2$ (+) (−)	$\frac{-12x^3}{4x^2} = -3x$
$4x^2 - 6x + 7$	$28x^2 + bx + a$ $28x^2 - 42x + 49$ (−) (+) (−)	$\frac{28x^2}{4x^2} = 7$
	0	

$b + 42 = 0$

$b = -42$

$a - 49 = 0$

$a = 49$

∴ The value of a = 49 and b = – 42

8. $ax^4 + bx^3 + 361x^2 + 220x + 100$

$10 + 11x + 12x^2$

10 | $100+220x + 361x^2 + bx^3 + ax^4$

100

(-)

$20+11x$ | $220x + 361x^2$

$220x + 121x^2$

(-) (-)

$20+22x+12x^2$ | $240x^2 + bx^3 \quad + ax^4$

$240x^2 + 264x^3 + 144x^4$

(-) (-) (-)

0

$b - 264 = 0$

$b = 264$

$a - 144 = 0$

$a = 144$

∴ The value of a = 144 and b = 264

9. $36x^4 - 60x^3 + 61x^2 - mx + n$

$6x^2 - 5x + 3$

$6x^4$ | $36x^4 - 60x^3 + 61x^2 - mx + n$

36

(-)

$12x^2 - 5x$ | $-60x^3 + 61x^2$

$-60x^3 + 25x^2$

(+) (-)

$12x^2+10x+3$ | $36x^2 - m x + n$

$36 x^2 - 30 + 9$

(-) (+) (-)

$-m + 30 = 0 \qquad n - 9 = 0$

-m = -30 **n = 9**

m = 30

10. $x^4 - 8x^3 + mx^2 + nx + 16$

$x^2 - 4x + 4$

x^2 | $x^4 - 8x^3 + mx^2 + nx + 16$

x^4

(-)

$2x^2 - 4x$ | $-8x^3 + mx^2$

$-8x^3 + 16x^2$

(+) (–)

$\frac{-8x^3}{2x^2} = -4x$

$2x^2 - 8x + 4$ | $x^2(m - 16) + nx + 16$

$8x^2 - 32x + 16$

(–) (+) (–)

0

$m - 16 - 8 = 0 \qquad n + 32 = 0$

$m - 24 = 0 \qquad n = -32$

$m = 24$

(2 Marks)

11. Solve $x^2 + 2x - 2 = 0$ by formula method.

a = 1; b = 2; c = -2

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{2^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{12}}{2}$$

$$= \frac{-2 \pm 2\sqrt{3}}{2}$$

$$= \frac{2(-1 \pm \sqrt{3})}{2}$$

$$x = -1 \pm \sqrt{3}$$

12. solve $2x^2 - 3x - 3 = 0$ by formula method.

$a = 2, \quad b = -3, \quad c = -3$ (2 Marks)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-3)}}{2(2)}$$

$$= \frac{3 \pm \sqrt{9+24}}{4} \qquad x = \frac{3 \pm \sqrt{33}}{4}$$

13. $2x^2 - 5x + 2 = 0$ by formula method

a = 2, b = -5, c = 2 (2 Marks)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{5 \pm \sqrt{25 - 4(2)(2)}}{4}$$

$$= \frac{5 \pm \sqrt{25 - 16}}{4}$$

$$= \frac{5 \pm \sqrt{9}}{4}$$

$$= \frac{5 \pm 3}{4}$$

$$= \frac{5+3}{4} \text{ or } \frac{5-3}{4}$$

$$= \frac{8}{4} \text{ or } \frac{2}{4} \quad = 2 \text{ or } \frac{1}{2}$$

The solution set is $\frac{1}{2}$ and 2

14. Find the value of a, b, c, d from the equation

$$\begin{pmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{pmatrix} = \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}$$

a – b = 1	2a+c=5	2a-b=0	3c+d=2
	sub a=-1 2(-1)+c=5	2a= b sub a=-1	sub c=7 3(7)+d=2
sub 2a=b	-2+c=5	2(-1)=b	21+d=2
a-2a=1	c=5+2	b = -2	d=2-21
–a=1	c = 7		d=-19

$a = -1$

---

15. Find x, y, z

$$\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$$

$x + y + z = 9 \longrightarrow$ (1)

$x + z = 5 \longrightarrow$ (2)

$y + z = 7 \longrightarrow$ (3)

$$\begin{array}{ccc} x+y+z & = & 9 \\ x \quad\; +z & = & 5 \\ (-) \quad (-) & & (-) \\ \hline & y = 4 & \end{array}$$

sub $y = 4$ in (3)   sub z = 3 in (2)

$y + z = 7$ $x + 3 = 5$

$4 + z = 7$ $x = 5 - 3$

$z = 7 - 4$ $x = 2$

$z = 3$

$x = 2,$ $y = 4,$ $z = 3.$

---

16. If A = $\begin{pmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{pmatrix}$, B = $\begin{pmatrix} 8 & -6 & -4 \\ 2 & 11 & -3 \\ 0 & 1 & 5 \end{pmatrix}$ c = $\begin{pmatrix} 5 & 3 & 0 \\ -1 & -7 & 2 \\ 1 & 4 & 3 \end{pmatrix}$

Compute (i) 3A+2B-C   (ii) $\frac{1}{2}A - \frac{3}{2}B$

(i) 3A+2B-C = $3\begin{pmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{pmatrix} + 2\begin{pmatrix} 8 & -6 & -4 \\ 2 & 11 & -3 \\ 0 & 1 & 5 \end{pmatrix} - \begin{pmatrix} 5 & 3 & 0 \\ -1 & -7 & 2 \\ 1 & 4 & 3 \end{pmatrix}$

$$= \begin{pmatrix} 3 & 24 & 9 \\ 9 & 15 & 0 \\ 24 & 21 & 18 \end{pmatrix} + \begin{pmatrix} 16 & -12 & -8 \\ 4 & 22 & -6 \\ 0 & 2 & 10 \end{pmatrix} - \begin{pmatrix} 5 & 3 & 0 \\ -1 & -7 & 2 \\ 1 & 4 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 9 & 1 \\ 14 & 44 & -8 \\ 23 & 19 & 25 \end{pmatrix}$$

---

ii) $\frac{1}{2}A - \frac{3}{2}B = \frac{1}{2}\ (A - 3B)$

$$= \frac{1}{2}\left[\begin{pmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{pmatrix} - \begin{pmatrix} 24 & -18 & -12 \\ 6 & 33 & -9 \\ 0 & 3 & 15 \end{pmatrix}\right]$$

$$= \frac{1}{2}\begin{pmatrix} -23 & 26 & 15 \\ -3 & -28 & 9 \\ 8 & 4 & -9 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{-23}{2} & 13 & \frac{15}{2} \\ \frac{-3}{2} & -14 & \frac{9}{2} \\ 4 & 2 & \frac{-9}{2} \end{pmatrix}$$

---

17. If A = $\begin{pmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{pmatrix}$ B= $\begin{pmatrix} 2 & 3 & 4 \\ 1 & 9 & 2 \\ -7 & 1 & -1 \end{pmatrix}$

C = $\begin{pmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{pmatrix}$ then verify that

A + (B+C) = (A+B) + C

LHS = A + (B+C)

(B+C) = $\begin{pmatrix} 2 & 3 & 4 \\ 1 & 9 & 2 \\ -7 & 1 & -1 \end{pmatrix} + \begin{pmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{pmatrix}$

$$= \begin{pmatrix} 10 & 6 & 8 \\ 2 & 7 & 5 \\ -5 & 7 & 5 \end{pmatrix}$$

A+ (B+C) = $\begin{pmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{pmatrix} + \begin{pmatrix} 10 & 6 & 8 \\ 2 & 7 & 5 \\ -5 & 7 & 5 \end{pmatrix}$

$$= \begin{pmatrix} 14 & 9 & 9 \\ 4 & 10 & -3 \\ -4 & 5 & -6 \end{pmatrix}$$

RHS = (A+B)+C

A+B = $\begin{pmatrix} 4 & 3 & 1 \\ 2 & 3 & -8 \\ 1 & 0 & -4 \end{pmatrix} + \begin{pmatrix} 2 & 3 & 4 \\ 1 & 9 & 2 \\ -7 & 1 & -1 \end{pmatrix}$

$$= \begin{pmatrix} 6 & 6 & 5 \\ 3 & 12 & -6 \\ -6 & 1 & -5 \end{pmatrix}$$

(A+B) + C = $\begin{pmatrix} 6 & 6 & 5 \\ 3 & 12 & -6 \\ -6 & 1 & -5 \end{pmatrix} + \begin{pmatrix} 8 & 3 & 4 \\ 1 & -2 & 3 \\ 2 & 4 & -1 \end{pmatrix}$

$$= \begin{pmatrix} 14 & 9 & 9 \\ 4 & 10 & -3 \\ -4 & 5 & -6 \end{pmatrix}$$

LHS = RHS   Verified.

---

18. Find x and y if $x+y = \begin{pmatrix} 7 & 0 \\ 3 & 5 \end{pmatrix}$ and

$x-y = \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix}$

$x+y = \begin{pmatrix} 7 & 0 \\ 3 & 5 \end{pmatrix} \longrightarrow$ ①

$x-y = \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix} \longrightarrow$ ②

$2x = \begin{pmatrix} 7 & 0 \\ 3 & 5 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix}$

$2x = \begin{pmatrix} 10 & 0 \\ 3 & 9 \end{pmatrix}$

$x = \frac{1}{2}\begin{pmatrix} 10 & 0 \\ 3 & 9 \end{pmatrix}$

$= \begin{pmatrix} \frac{10}{2} & \frac{0}{2} \\ \frac{3}{2} & \frac{9}{2} \end{pmatrix}$

$$x = \begin{pmatrix} 5 & 0 \\ \frac{3}{2} & \frac{9}{2} \end{pmatrix}$$

sub x in (1)

$$\begin{pmatrix} 5 & 0 \\ \frac{3}{2} & \frac{9}{2} \end{pmatrix} + y = \begin{pmatrix} 7 & 0 \\ 3 & 5 \end{pmatrix}$$

$$y = \begin{pmatrix} 7 & 0 \\ 3 & 5 \end{pmatrix} - \begin{pmatrix} 5 & 0 \\ \frac{3}{2} & \frac{9}{2} \end{pmatrix}$$

$$y = \begin{pmatrix} 2 & 0 \\ \frac{3}{2} & \frac{1}{2} \end{pmatrix}$$

$$x = \begin{pmatrix} 5 & 0 \\ \frac{3}{2} & \frac{9}{2} \end{pmatrix} \quad y = \begin{pmatrix} 2 & 0 \\ \frac{3}{2} & \frac{1}{2} \end{pmatrix}$$

---

19. Find the values of x, y, z if

(i) $\begin{pmatrix} x-3 & 3x-z \\ x+y+7 & x+y+z \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 6 \end{pmatrix}$

$x-3=1 \quad 3x-z=0 \quad x+y+7=1$

$x=1+3 \quad sub\ x=4 \quad sub\ x=4$

$x=4 \quad 3(4)-z=0 \quad 4+y+7=1$

$12-z=0 \quad 11+y=1$

$z=12 \quad y=1-11$

$y=-10$

$x=4, \ y=-10, \quad z=12.$

---

ii) $(x \quad y-z \quad z+3) + (y \quad 4 \quad 3) = (4 \quad 8 \quad 16)$

$x+y=4 \longrightarrow$ (1)

$y-z+4=8 \longrightarrow$ (2)

$z+3+3=16 \longrightarrow$ (3)

z+6=16

z = 16-6

z=10

sub z in (2)

$y-10+4=8$

$y-6=8$

$y=8+6$

$y=14$

sub y in (1)

$x+14=4$

$x=4-14$

$x=-10$; y = 14; z=10

---

20. Find x and y if x $\begin{pmatrix} 4 \\ -3 \end{pmatrix} + y\begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

$4x-2y=4$ (1)

$-3x+3y=6$ (2)

(1) x (3) = $12x-6y = 12$

(2) x (4) = $-12x+12y = 24$

+6y = 36

y = $\frac{36}{6}$

y = 6

sub y in (1)

$4x-2(6)=4$

$4x-12=4$

$4x=4+12$

$4x=16$

$x=\frac{16}{4}$

$x=4$

$x=4, \ \ y=6.$

---

21. Find the non zero values of x satisfying the matrix equation.

$$x\begin{pmatrix} 2x & 2 \\ 3 & x \end{pmatrix} + 2\begin{pmatrix} 8 & 5x \\ 4 & 4x \end{pmatrix} = 2\begin{pmatrix} x^2+8 & 24 \\ 10 & 6x \end{pmatrix}$$

$$\begin{pmatrix} 2x^2 & 2x \\ 3x & x^2 \end{pmatrix} + \begin{pmatrix} 16 & 10x \\ 8 & 8x \end{pmatrix} = \begin{pmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{pmatrix}$$

$$\begin{pmatrix} 2x^2+16 & 2x+10x \\ 3x+8 & x^2+8x \end{pmatrix} = \begin{pmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{pmatrix}$$

$$\begin{pmatrix} 2x^2+16 & 12x \\ 3x+8 & x^2+8x \end{pmatrix} = \begin{pmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{pmatrix}$$

$12x=48$

$x=\frac{48}{12}$

$x=4$

---

22. Solve for x, y: $\begin{pmatrix} x^2 \\ y^2 \end{pmatrix} + 2\begin{pmatrix} -2x \\ -y \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$

$x^2-4x=5$ (1)

$y^2-2y=8$ (2)

$x^2-4x-5=0$ $\quad (-5)$

$(x-5)\ (x+1) = 0$ (-5) (1)

$x - 5 = 0 \quad | \quad x + 1 = 0$

$x = 5 \quad | \quad x = -1$

$x = 5, -1$

$y^2 - 2y - 8 = 0$ (-8)

$(y - 4)\,(y + 2) = 0$ (-4) (2)

$y - 4 = 0 \quad | \quad y + 2 = 0$

$y = 4 \quad | \quad y = -2$

$y = 4,\ -2$

---

23. Solve: $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$

$2x + y = 4 \qquad (1)$

$x + 2y = 5 \qquad (2)$

$2x + y = 4$

$(2) \times 2 \quad 2x + 4y = 10$

$(-) \quad (-) \quad (-)$

$-3y = -6$

$y = \frac{-6}{-3}$

$y = 2$

sub y = 2 in (1) $2x + 2 = 4$

$2x = 4 - 2, \quad 2x = 2$

$x = \frac{2}{2}, \ x = 1$

$x = 1, \quad y = 2.$

---

24. If A = $(1 \quad -1 \quad 2)$, $B = \begin{pmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 3 \end{pmatrix}$

$C = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$ show that (AB)C = A(BC)

LHS: (AB)C

AB = $(1 \quad -1 \quad 2) \begin{pmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 3 \end{pmatrix}$

$= \left((1 \quad -1 \quad 2) \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \quad (1 \quad -1 \quad 2) \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix} \right)$

$= (1 - 2 + 2 \quad -1 - 1 + 6)$

AB = $(1 \quad 4)$

(AB)C = $(1 \quad 4) \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$

$= \left((1 \quad 4) \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (1 \quad 4) \begin{pmatrix} 2 \\ -1 \end{pmatrix} \right)$

$(1 + 8 \quad 2 - 4)$

(AB)C = $(9 \quad -2)$

RHS: A(BC)

BC = $\begin{pmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$

$= \begin{pmatrix} (1 \quad -1) \begin{pmatrix} 1 \\ 2 \end{pmatrix} & (1 \quad -1) \begin{pmatrix} 2 \\ -1 \end{pmatrix} \\ (2 \quad 1) \begin{pmatrix} 1 \\ 2 \end{pmatrix} & (2 \quad 1) \begin{pmatrix} 2 \\ -1 \end{pmatrix} \\ (1 \quad 3) \begin{pmatrix} 1 \\ 2 \end{pmatrix} & (1 \quad 3) \begin{pmatrix} 2 \\ -1 \end{pmatrix} \end{pmatrix}$

$= \begin{pmatrix} 1 - 2 & 2 + 1 \\ 2 + 2 & 4 - 1 \\ 1 + 6 & 2 - 3 \end{pmatrix}$

BC = $\begin{pmatrix} -1 & 3 \\ 4 & 3 \\ 7 & -1 \end{pmatrix}$

A(BC) = $(1 \quad -1 \quad 2) \begin{pmatrix} -1 & 3 \\ 4 & 3 \\ 7 & -1 \end{pmatrix}$

$= \left((1 \quad -1 \quad 2) \begin{pmatrix} -1 \\ 4 \\ 7 \end{pmatrix} \quad (1 \quad -1 \quad 2) \begin{pmatrix} 3 \\ 3 \\ -1 \end{pmatrix} \right)$

$= (-1 - 4 + 14 \quad 3 - 3 - 2)$

A(BC) = (9 -2)

LHS = RHS verified.

---

25. If A = $\begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix}$,

C = $\begin{pmatrix} -7 & 6 \\ 3 & 2 \end{pmatrix}$ verify that A(B+C) = AB + AC

LHS: A(B+C)

$$B+C = \begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix} + \begin{pmatrix} -7 & 6 \\ 3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} -6 & 8 \\ -1 & 4 \end{pmatrix}$$

$$A(B+C) = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} -6 & 8 \\ -1 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} (1 \quad 1) \begin{pmatrix} -6 \\ -1 \end{pmatrix} & (1 \quad 1) \begin{pmatrix} 8 \\ 4 \end{pmatrix} \\ (-1 \quad 3) \begin{pmatrix} -6 \\ -1 \end{pmatrix} & (-1 \quad 3) \begin{pmatrix} 8 \\ 4 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} -6-1 & 8+4 \\ 6-3 & -8+12 \end{pmatrix}$$

$$A(B+C) = \begin{pmatrix} -7 & 12 \\ 3 & 4 \end{pmatrix}$$

RHS: AB + AC

$$AB = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} (1 \quad 1) \begin{pmatrix} 1 \\ 4 \end{pmatrix} & (1 \quad 1) \begin{pmatrix} 2 \\ 2 \end{pmatrix} \\ (-1 \quad 3) \begin{pmatrix} 1 \\ 4 \end{pmatrix} & (-1 \quad 3) \begin{pmatrix} 2 \\ 2 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 1-4 & 2+2 \\ -1-12 & -2+6 \end{pmatrix}$$

$$AB = \begin{pmatrix} -3 & 4 \\ -13 & 4 \end{pmatrix}$$

$$AC = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} -7 & 6 \\ 3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} (1 \quad 1) \begin{pmatrix} -7 \\ 3 \end{pmatrix} & (1 \quad 1) \begin{pmatrix} 6 \\ 2 \end{pmatrix} \\ (-1 \quad 3) \begin{pmatrix} -7 \\ 3 \end{pmatrix} & (-1 \quad 3) \begin{pmatrix} 6 \\ 2 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} -7+3 & 6+2 \\ 7+9 & -6+6 \end{pmatrix}$$

$$AC = \begin{pmatrix} -4 & 8 \\ 16 & 0 \end{pmatrix} \quad AB+AC = \begin{pmatrix} -3 & 4 \\ -13 & 4 \end{pmatrix} + \begin{pmatrix} -4 & 8 \\ 16 & 0 \end{pmatrix}$$

$$AB + AC = \begin{pmatrix} -7 & 12 \\ 3 & 4 \end{pmatrix}$$

LHS = RHS , Verified.

---

26. If A = $\begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{pmatrix}$

show that $(AB)^T = B^T A^T$

LHS: $(AB)^T$

$$AB = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} (1 \quad 2 \quad 1) \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} & (1 \quad 2 \quad 1) \begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix} \\ (2 \quad -1 \quad 1) \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} & (2 \quad -1 \quad 1) \begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix} \end{pmatrix}$$

$$AB = \begin{pmatrix} 2-2+0 & -1+8+2 \\ 4+1+0 & -2-4+2 \end{pmatrix}$$

$$AB = \begin{pmatrix} 0 & 9 \\ 5 & -4 \end{pmatrix}$$

$$(AB)^T = \begin{pmatrix} 0 & 5 \\ 9 & -4 \end{pmatrix}$$

RHS: $B^T \ A^T$

$$B^T = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 1 & 2 \\ 2 & -1 \\ 1 & 1 \end{pmatrix}$$

$$B^T A^T = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & -1 \\ 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} (2 \quad -1 \quad 0) \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} & (2 \quad -1 \quad 0) \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \\ (-1 \quad 4 \quad 2) \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} & (-1 \quad 4 \quad 2) \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 2-2+0 & 4+1+0 \\ -1+8+2 & -2-4+2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 5 \\ 9 & -4 \end{pmatrix}$$

LHS = RHS, verified.

27. Given that A = $\begin{pmatrix}1 & 3\\ 5 & -1\end{pmatrix}$, B = $\begin{pmatrix}1 & -1 & 2\\ 3 & 5 & 2\end{pmatrix}$

C = $\begin{pmatrix}1 & 3 & 2\\ -4 & 1 & 3\end{pmatrix}$ verify that A(B+C) = AB + AC.

LHS: A(B+C)

B+C = $\begin{pmatrix}1 & -1 & 2\\ 3 & 5 & 2\end{pmatrix} + \begin{pmatrix}1 & 3 & 2\\ -4 & 1 & 3\end{pmatrix}$

= $\begin{pmatrix}2 & 2 & 4\\ -1 & 6 & 5\end{pmatrix}$

A(B+C) = $\begin{pmatrix}1 & 3\\ 5 & -1\end{pmatrix}\begin{pmatrix}2 & 2 & 4\\ -1 & 6 & 5\end{pmatrix}$

= $\begin{pmatrix}(1\ \ 3)\begin{pmatrix}2\\-1\end{pmatrix} & (1\ \ 3)\begin{pmatrix}2\\6\end{pmatrix} & (1\ \ 3)\begin{pmatrix}4\\5\end{pmatrix}\\ (5\ \ -1)\begin{pmatrix}2\\-1\end{pmatrix} & (5\ \ -1)\begin{pmatrix}2\\6\end{pmatrix} & (5\ \ -1)\begin{pmatrix}4\\5\end{pmatrix}\end{pmatrix}$

= $\begin{pmatrix}2-3 & 2+18 & 4+15\\ 10+1 & 10-6 & 20-5\end{pmatrix}$

A(B+C) = $\begin{pmatrix}-1 & 20 & 19\\ 11 & 4 & 15\end{pmatrix}$

RHS: AB + AC

AB = $\begin{pmatrix}1 & 3\\ 5 & -1\end{pmatrix}\begin{pmatrix}1 & -1 & 2\\ 3 & 5 & 2\end{pmatrix}$

= $\begin{pmatrix}(1\ \ 3)\begin{pmatrix}1\\3\end{pmatrix} & (1\ \ 3)\begin{pmatrix}-1\\5\end{pmatrix} & (1\ \ 3)\begin{pmatrix}2\\2\end{pmatrix}\\ (5\ \ -1)\begin{pmatrix}1\\3\end{pmatrix} & (5\ \ -1)\begin{pmatrix}-1\\5\end{pmatrix} & (5\ \ -1)\begin{pmatrix}2\\2\end{pmatrix}\end{pmatrix}$

= $\begin{pmatrix}1+9 & -1+15 & 2+6\\ 5-3 & -5-5 & 10-2\end{pmatrix}$

AB = $\begin{pmatrix}10 & 14 & 8\\ 2 & -10 & 8\end{pmatrix}$

AC = $\begin{pmatrix}1 & 3\\ 5 & -1\end{pmatrix}\begin{pmatrix}1 & 3 & 2\\ -4 & 1 & 3\end{pmatrix}$

= $\begin{pmatrix}(1\ \ 3)\begin{pmatrix}1\\-4\end{pmatrix} & (1\ \ 3)\begin{pmatrix}3\\1\end{pmatrix} & (1\ \ 3)\begin{pmatrix}2\\3\end{pmatrix}\\ (5\ \ -1)\begin{pmatrix}1\\-4\end{pmatrix} & (5\ \ -1)\begin{pmatrix}3\\1\end{pmatrix} & (5\ \ -1)\begin{pmatrix}2\\3\end{pmatrix}\end{pmatrix}$

= $\begin{pmatrix}1-12 & 3+3 & 2+9\\ 5+4 & 15-1 & 10-3\end{pmatrix}$

AC = $\begin{pmatrix}-11 & 6 & 11\\ 9 & 14 & 7\end{pmatrix}$

AB+AC = $\begin{pmatrix}10 & 14 & 8\\ 2 & -10 & 8\end{pmatrix} + \begin{pmatrix}-11 & 6 & 11\\ 9 & 14 & 7\end{pmatrix}$

AB + AC = $\begin{pmatrix}-1 & 20 & 19\\ 11 & 4 & 15\end{pmatrix}$

LHS = RHS, Verified.

28. Let A = $\begin{pmatrix}1 & 2\\ 1 & 3\end{pmatrix}$, $B = \begin{pmatrix}4 & 0\\ 1 & 5\end{pmatrix}$, C = $\begin{pmatrix}2 & 0\\ 1 & 2\end{pmatrix}$ show that

(i) A(BC) = (AB)C (ii) (A-B)C = AC – BC

(iii) $(A-B)^T = A^T - B^T$

(i) A(BC) = (AB) C

LHS: A(BC)

BC = $\begin{pmatrix}4 & 0\\ 1 & 5\end{pmatrix}\begin{pmatrix}2 & 0\\ 1 & 2\end{pmatrix}$

= $\begin{pmatrix}(4\ \ 0)\begin{pmatrix}2\\1\end{pmatrix} & (4\ \ 0)\begin{pmatrix}0\\2\end{pmatrix}\\ (1\ \ 5)\begin{pmatrix}2\\1\end{pmatrix} & (1\ \ 5)\begin{pmatrix}0\\2\end{pmatrix}\end{pmatrix}$

= $\begin{pmatrix}8+0 & 0+0\\ 2+5 & 0+10\end{pmatrix}$

= $\begin{pmatrix}8 & 0\\ 7 & 10\end{pmatrix}$

A(BC) = $\begin{pmatrix}1 & 2\\ 1 & 3\end{pmatrix}\begin{pmatrix}8 & 0\\ 7 & 10\end{pmatrix}$

= $\begin{pmatrix}(1\ \ 2)\begin{pmatrix}8\\7\end{pmatrix} & (1\ \ 2)\begin{pmatrix}0\\10\end{pmatrix}\\ (1\ \ 3)\begin{pmatrix}8\\7\end{pmatrix} & (1\ \ 3)\begin{pmatrix}0\\10\end{pmatrix}\end{pmatrix}$

= $\begin{pmatrix}8+14 & 0+20\\ 8+21 & 0+30\end{pmatrix}$

A(BC) = $\begin{pmatrix}22 & 20\\ 29 & 30\end{pmatrix}$

RHS = (AB) C AB = $\begin{pmatrix}1 & 2\\ 1 & 3\end{pmatrix}\begin{pmatrix}4 & 0\\ 1 & 4\end{pmatrix}$

= $\begin{pmatrix}(1\ \ 2)\begin{pmatrix}4\\1\end{pmatrix} & (1\ \ 2)\begin{pmatrix}0\\5\end{pmatrix}\\ (1\ \ 3)\begin{pmatrix}4\\1\end{pmatrix} & (1\ \ 3)\begin{pmatrix}0\\5\end{pmatrix}\end{pmatrix}$

= $\begin{pmatrix}4+2 & 0+10\\ 4+3 & 0+15\end{pmatrix}$

= $\begin{pmatrix}6 & 10\\ 7 & 15\end{pmatrix}$

(AB) C = $\begin{pmatrix}6 & 10\\ 7 & 15\end{pmatrix}\begin{pmatrix}2 & 0\\ 1 & 2\end{pmatrix}$

= $\begin{pmatrix}(6\ \ 10)\begin{pmatrix}2\\1\end{pmatrix} & (6\ \ 10)\begin{pmatrix}0\\2\end{pmatrix}\\ (7\ \ 15)\begin{pmatrix}2\\1\end{pmatrix} & (7\ \ 15)\begin{pmatrix}0\\2\end{pmatrix}\end{pmatrix}$

= $\begin{pmatrix}12+10 & 0+20\\ 14+15 & 0+30\end{pmatrix}$

(AB) C = $\begin{pmatrix}22 & 20\\ 29 & 30\end{pmatrix}$

LHS = RHS, Verified.

(ii) (A-B) C = AC – BC

LHS: (A-B) C

(A-B) = $\begin{pmatrix}1 & 2\\ 1 & 3\end{pmatrix} - \begin{pmatrix}4 & 0\\ 1 & 5\end{pmatrix}$

= $\begin{pmatrix}-3 & 2\\ 0 & -2\end{pmatrix}$

(A-B) C = $\begin{pmatrix} \mathbf{-3} & \mathbf{2} \\ \mathbf{0} & \mathbf{-2} \end{pmatrix} \begin{pmatrix} \mathbf{2} & \mathbf{0} \\ \mathbf{1} & \mathbf{2} \end{pmatrix}$

$$= \begin{pmatrix} (-3 \quad 2)\begin{pmatrix} 2 \\ 1 \end{pmatrix} & (-3 \quad 2)\begin{pmatrix} 0 \\ 2 \end{pmatrix} \\ (0 \quad -2)\begin{pmatrix} 2 \\ 1 \end{pmatrix} & (0 \quad -2)\begin{pmatrix} 0 \\ 2 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} -6+2 & 0+4 \\ 0-2 & 0-4 \end{pmatrix}$$

(A-B) C = $\begin{pmatrix} -4 & 4 \\ -2 & -4 \end{pmatrix}$

RHS: AC – BC

AC = $\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$

$$= \begin{pmatrix} (1 \quad 2)\begin{pmatrix} 2 \\ 1 \end{pmatrix} & (1 \quad 2)\begin{pmatrix} 0 \\ 2 \end{pmatrix} \\ (1 \quad 3)\begin{pmatrix} 2 \\ 1 \end{pmatrix} & (1 \quad 3)\begin{pmatrix} 0 \\ 2 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 2+2 & 0+4 \\ 2+3 & 0+6 \end{pmatrix}$$

AC = $\begin{pmatrix} 4 & 4 \\ 5 & 6 \end{pmatrix}$

BC = $\begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$

$$= \begin{pmatrix} (4 \quad 0)\begin{pmatrix} 2 \\ 1 \end{pmatrix} & (4 \quad 0)\begin{pmatrix} 0 \\ 2 \end{pmatrix} \\ (1 \quad 5)\begin{pmatrix} 2 \\ 1 \end{pmatrix} & (1 \quad 5)\begin{pmatrix} 0 \\ 2 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 8+0 & 0+0 \\ 2+5 & 0+10 \end{pmatrix}$$

BC = $\begin{pmatrix} 8 & 0 \\ 7 & 10 \end{pmatrix}$

AC – BC = $\begin{pmatrix} 4 & 4 \\ 5 & 6 \end{pmatrix}$ – $\begin{pmatrix} 8 & 0 \\ 7 & 10 \end{pmatrix}$

AC – BC = $\begin{pmatrix} -4 & 4 \\ -2 & -4 \end{pmatrix}$

LHS = RHS, Verified .

(iii) $(A-B)^T$ = A^T – B^T

LHS: $(A-B)^T$

(A-B) = $\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix} - \begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix}$

$$= \begin{pmatrix} -3 & 2 \\ 0 & -2 \end{pmatrix}$$

$(A-B)^T = \begin{pmatrix} -3 & 0 \\ 2 & -2 \end{pmatrix}$

RHS: $A^T - B^T$

$A^T = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$

$B^T = \begin{pmatrix} 4 & 1 \\ 0 & 5 \end{pmatrix}$

$A^T - B^T = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix} - \begin{pmatrix} 4 & 1 \\ 0 & 5 \end{pmatrix}$

$A^T - B^T = \begin{pmatrix} -3 & 0 \\ 2 & -2 \end{pmatrix}$

LHS = RHS, Verified.

29. If A = $\begin{pmatrix} cos\theta & 0 \\ 0 & cos\theta \end{pmatrix}$, B = $\begin{pmatrix} sin\theta & 0 \\ 0 & sin\theta \end{pmatrix}$

then show that A^2+B^2= I.

LHS: A^2+B^2

$$A^2 = A \times A = \begin{pmatrix} cos\theta & 0 \\ 0 & cos\theta \end{pmatrix} \begin{pmatrix} cos\theta & 0 \\ 0 & cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} cos^2\theta & 0 \\ 0 & cos^2\theta \end{pmatrix}$$

$$B^2 = B \times B = \begin{pmatrix} sin\theta & 0 \\ 0 & sin\theta \end{pmatrix} \begin{pmatrix} sin\theta & 0 \\ 0 & sin\theta \end{pmatrix}$$

$$= \begin{pmatrix} sin^2\theta & 0 \\ 0 & sin^2\theta \end{pmatrix}$$

$$A^2+B^2 = \begin{pmatrix} cos^2\theta & 0 \\ 0 & cos^2\theta \end{pmatrix} + \begin{pmatrix} sin^2\theta & 0 \\ 0 & sin^2\theta \end{pmatrix}$$

$$= \begin{pmatrix} sin^2\theta + cos^2\theta & 0 \\ 0 & cos^2\theta + sin^2\theta \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

= I

= RHS. Hence proved.

30. If A = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and I = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ show that

$$A^2 - (a+d)A = (bc-ad)I_2$$

LHS: $A^2 - (a+d)A$

$$A^2 = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a^2+bc & ad+bd \\ ac+cd & bc+d^2 \end{pmatrix}$$

(a+d)A= (a+d) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$= \begin{pmatrix} (a+d)a & (a+d)b \\ (a+d)c & (a+d)d \end{pmatrix} = \begin{pmatrix} a^2+ad & ab+bd \\ ac+dc & ad+d^2 \end{pmatrix}$$

$$A^2 - (a+d)A = \begin{pmatrix} a^2+bc & ad+bd \\ ac+cd & bc+d^2 \end{pmatrix} - \begin{pmatrix} a^2+ad & ab+bd \\ ac+dc & ad+d^2 \end{pmatrix}$$

$$= \begin{pmatrix} bc-ad & 0 \\ 0 & bc-ad \end{pmatrix}$$

$$= (bc-ad) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= (bc-ad)I_2$$

= RHS. Hence proved.

31. If A = $\begin{pmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{pmatrix}$, B = $\begin{pmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{pmatrix}$

verify that $(AB)^T = B^T A^T$

LHS: $(AB)^T$

$$AB = \begin{pmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{pmatrix} \begin{pmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 5+2+45 & 35+4-9 \\ 1+2+40 & 7+4-8 \end{pmatrix}$$

$$AB = \begin{pmatrix} 52 & 30 \\ 43 & 3 \end{pmatrix}$$

$$(AB)^T = \begin{pmatrix} 52 & 43 \\ 30 & 3 \end{pmatrix}$$

RHS: $B^T A^T$

$$B^T = \begin{pmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{pmatrix}$$

$$B^T A^T = \begin{pmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{pmatrix} \begin{pmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} \begin{pmatrix} 1 & 1 & 5 \end{pmatrix} \begin{pmatrix} 5 \\ 2 \\ 9 \end{pmatrix} & \begin{pmatrix} 1 & 1 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 8 \end{pmatrix} \\ \begin{pmatrix} 7 & 2 & -1 \end{pmatrix} \begin{pmatrix} 5 \\ 2 \\ 9 \end{pmatrix} & \begin{pmatrix} 7 & 2 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 8 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 5+2+45 & 1+2+40 \\ 35+4-9 & 7+4-8 \end{pmatrix}$$

$$B^T A^T = \begin{pmatrix} 52 & 43 \\ 30 & 3 \end{pmatrix}$$

LHS = RHS, verified.

---

32. If A= $\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ show that $A^2 - 5A + 7I_2 = 0$.

LHS: $A^2 - 5A + 7I_2$

$$A^2 = A \times A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix}$$

$$5A = 5\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 15 & 5 \\ -5 & 10 \end{pmatrix}$$

$$7I_2 = 7\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$$

$$A^2 - 5A + 7I_2 = \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} - \begin{pmatrix} 15 & 5 \\ -5 & 10 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

= 0, RHS verified.

Chapter – 5

Co-ordinate geometry

Formulae:

1. Area of triangle=

$$\tfrac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\}sq.units$$

2. Three points A,B,C are collinear then

(i) Area of ΔABC=0

(ii) $x_1y_2 + x_2y_3 + x_3y_1 = x_2y_1 + x_3y_2 + x_1y_3$

(iii) Slope of AB=Slope of BC=Slope of AC

3. Area of Quadrilateral

$$= \tfrac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1) - (x_2y_1 + x_3y_2 + x_4y_3 + x_1y_4)\}sq.units$$

4. (i) Slope m=$\tan\theta$ (ii) Slope=$\frac{y_2-y_1}{x_2-x_1}$

(iii) Slope=m=$\frac{-a}{b}$

5. (i) Equation of straight line general form

ax+by+c=0.

(ii) Equation of straight line one point-slope form

$$y - y_1 = m(x - x_1)$$

(iii) Equation of straight line slope and

y – Intercept form y=mx+c

(iv) Equation of straight line Two point form

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

(v) Equation of straight line Intercept form

$$\frac{x}{a} + \frac{y}{b} = 1$$

(vi) Equation of line parallel to Y-axis

x=c

(vii) Equation of line parallel to

X-axis y=b.

6. Slopes of 2 parallel lines are equal

7. When 2 lines are perpendicular

$m_1 \text{x}\, m_2 = -1$

8. Line: $ax + by + c = 0$

Parallel line: $ax + by + k = 0$

Perpendicular line: $bx - ay + k = 0$

2 Marks

1. Find the area of the triangle whose vertices are (1,-1), (-4,6) and (-3,-5)

Let vertices be $A(1,-1), B(-4,6), C(-3,-5)$

Area of ΔABC

$= \frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1)$
$-(x_2y_1 + x_3y_2 + x_1y_3)\} sq.units$

$= \frac{1}{2}\{(6+20+3)-(4-18-5)\}$

$= \frac{1}{2}[29-(-19)] = \frac{1}{2}[29+19]$

$= \frac{1}{2}$ x 48 = 24 sq. units.

---

2. Find the area of triangle formed by The points (-10,-4), (-8,-1) and (-3,-5).

Let vertices be A(-10,-4), B(-8, -1), C(-3,-5)

$\downarrow \quad \downarrow \qquad \downarrow \quad \downarrow \qquad \downarrow \quad \downarrow$

$x_1 \quad y_1 \qquad x_2 \quad y_2 \qquad x_3 \quad y_3$

Area of ΔABC

$= \frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1)$
$-(x_2y_1 + x_3y_2 + x_1y_3)\} sq.units$

$= \frac{1}{2}[\{(-8\text{x}-4) + (-10\text{x}-5) + (-3\text{x}-1)\}$
$-\{(-1\text{x}-10) + (-4\text{x}-3) + (-5\text{x}-8)\}$

$= \frac{1}{2}[(32+50+3)-(10+12+40)]$

$= \frac{1}{2}[85-62] = \frac{1}{2}$ x 23 = 11.5 sq.units.

---

3. Determine whether the set of points $(-\frac{1}{2},3)$, (-5,6) and (-8,8) are collinear?

We know that when points are collinear

$x_1y_2 + x_2y_3 + x_3y_1 = x_2y_1 + x_3y_2 + x_1y_3$

Let $(-\frac{1}{2}, 3)$ (-5, 6) (-8, 8)

LHS = $x_1y_2 + x_2y_3 + x_3y_1$

$= (-\frac{1}{2} \text{ x } 6)+(-5 \text{ x } 8)+(-8 \text{ x } 3)$

= -3-40-24

= -67

RHS = $(3\text{x}-5)+(6\text{x}-8)+(8\text{x} -\frac{1}{2})$

= -15-48-4

= -67

LHS=RHS ∴ Points are collinear.

---

4. If the area of the triangle formed by the vertices (0,0), (p, 8) and (6,2) taken in order is 20 sq. units find the value of p.

Let A(0,0) B(p,8) C(6,2).

Given that, Area of ΔABC = 20 sq.units

$\frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1)$
$-(x_2y_1 + x_3y_2 + x_1y_3)\}=20$

(0+2p+0) – (0+48+0) = 20 x 2

2p-48 = 40

2p =40+48

$P=\frac{88}{2} = 44$

P=44.

---

5. Find the value of ‘a’ for which the points (2,3), (4,a) and (6,-3) are collinear.

Let A(2,3), B(4,a), C(6,-3)

When points are collinear we have

$x_1y_2 + x_2y_3 + x_3y_1 = x_2y_1 + x_3y_2 + x_1y_3$

2a+ 4(-3) +6(3) = 4(3)+6a+2(-3)

2a-12+18= 12+6a-6

2a-6a= 12-6+12-18

-4a= 24-24

-4a= 0

$a= \frac{0}{-4}$

a= 0

---

6. What is the slope of line whose inclination with positive direction of x-axis is 90° .

Here $\theta = 90^\circ$

Slope m = $\tan\theta$

$= \tan 90^\circ$

$= \infty$

7. What is the slope of line whose inclination with positive direction of x-axis is 0°

Here $\theta = 0^\circ$

Slope m = $\tan\theta$

$= \tan 0^\circ$

m = 0

---

8. What is the inclination of line whose slope is 0.

Given m=0, Let θ be the inclination of the line.

m=0

$\tan\theta = 0$

$\tan\theta = \tan 0^\circ$

$\therefore \theta = 0^\circ$

9. What is the inclination of a line whose slope is 1.

Given m=1, let θ be the inclination of the line.

m=1

$\tan\theta = 1$

$\tan\theta = \tan 45^{\circ}$

$\therefore\ \theta = 45^{\circ}$

---

10. Find the slope of a line joining the points $(5,\sqrt{5})$ and origin (0,0).

$$\text{Slope}=\frac{y_2-y_1}{x_2-x_1}=\frac{0-\sqrt{5}}{0-5}=\frac{-\sqrt{5}}{-5}=\frac{\sqrt{5}}{\sqrt{5}\sqrt{5}}=\frac{1}{\sqrt{5}}$$

---

11. Find the slope of a line joining the points $(\sin\theta, -\cos\theta)$ and $(-\sin\theta, \cos\theta)$.

Points $(\sin\theta, -\cos\theta)$ and $(-\sin\theta, \cos\theta)$

$$\text{Slope}=\frac{y_2-y_1}{x_2-x_1}=\frac{\cos\theta-(-\cos\theta)}{-\sin\theta-\sin\theta}=\frac{2\cos\theta}{-2\sin\theta}$$

$= -\cot\theta$

---

12. What is the slope of a line Perpendicular to the line joining A(5,1) and P where P is the mid-point of the segment joining (4,2) and (-6,4).

P is midpoint of the segment joining (4,2) and (-6,4).

$$P(x,y)=\left(\frac{4+(-6)}{2},\frac{2+4}{2}\right)=(-1,3)$$

A(5,1) P(-1,3)

$$\text{slope of AP} =\frac{y_2-y_1}{x_2-x_1}=\frac{3-1}{-1-5}=\frac{2}{-6}=\frac{-1}{3}$$

Slope of the perpendicular to AP

$$=\frac{-1}{slope\ of\ AP}=\frac{-1}{\left(\frac{-1}{3}\right)}=3.$$

---

13. Show that the given points are Collinear. (-3,-4), (7,2) and (12,5)

Let A(-3,-4), B(7,2) and C(12,5)

$$\text{Slope of AB} = \frac{y_2-y_1}{x_2-x_1}=\frac{2-(-4)}{7-(-3)}$$

$$=\frac{6}{10}=\frac{3}{5}$$

$$\text{Slope of BC} = \frac{y_2-y_1}{x_2-x_1}=\frac{5-2}{12-7}=\frac{3}{5}.$$

Slope of AB = Slope of BC

∴ The points are collinear.

---

14. If the three points (3,-1), (a,3), (1,-3) are collinear. Find the value of a.

When the points are collinear we have

Slope of AB=Slope of BC

$$\frac{3-(-1)}{a-3}=\frac{-3-3}{1-a}$$

$$\frac{4}{a-3}=\frac{-6}{1-a}$$

$4(1-a) = -6(a-3)$

$2(1-a) = -3(a-3)$

$2-2a = -3a+9$

$-2a+3a = 9-2 \qquad \therefore\ a = 7$

---

15. The line through the points (-2,a) and (9,3) has slope $\frac{-1}{2}$. Find the value of a.

Given that,

Slope of (-2,a) and (9,3) $=\frac{-1}{2}$

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-1}{2}$$

$$\frac{3-a}{9-(-2)}=\frac{-1}{2}$$

2(3-a) = -1 x 11

6-2a =-11

-2a = -11-6

-2a =-17

$a =\frac{17}{2}$

---

16. The line through the points (-2,6) and (4,8) is perpendicular to the line. Through the points (8,12) and $(x, 24)$. Find the value of x.

Let line through A(-2,6) B(4,8)

$$\text{Slope of AB} =m_1=\frac{y_2-y_1}{x_2-x_1}=\frac{8-6}{4-(-2)}=\frac{2}{6}$$

$$=\frac{1}{3}$$

Another line through C(8, 12) and D(x, 24)

$$\text{Slope of CD} =m_2=\frac{y_2-y_1}{x_2-x_1}=\frac{24-12}{x-8}=\frac{12}{x-8}$$

AB and CD are perpendicular then

m_1 x m_2 = -1

$$\frac{1}{3}\ \text{x}\ \frac{12}{x-8}=-1 \longrightarrow \frac{4}{x-8}=-1$$

4 = (x-8) x (-1) = 8-x

$x = 8-4 = 4$

---

17. Determine whether the sets of points are collinear?

(a, b+c), (b, c+a) and (c, a+b)

Let A(a, b+c) B(b, c+a), C(c, a+b)

Area of ΔABC

$$=\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}\ \text{sq. units}$$

$$=\frac{1}{2}\begin{vmatrix} a & b & c & a \\ b+c & c+a & a+b & b+c \end{vmatrix}$$

$$=\frac{1}{2}[\{a(c+a)+b(a+b)+c(b+c)\}$$

$$-\{b(b+c)+c(c+a)+a(a+b)\}]$$

$$=\frac{1}{2}[(\text{ac}+a^2+\text{ab}+b^2+\text{bc}+c^2)-$$

$$(b^2+\text{bc}+c^2+\text{ac}+a^2+\text{ab})]$$

$$=\frac{1}{2}[\text{ac}+a^2+\text{ab}+b^2+\text{bc}+c^2-b^2\text{-bc}-c^2\text{-ac-}a^2\text{-ab}]$$

$=\frac{1}{2}(0) = 0$ sq. units

∴The given points are collinear

18. Vertices of given triangle are taken in order (p,p), (5,6) and (5,-2) and the area is 32 sq. units. Find the value of p

Let A(p,p) B(5,6) C(5,-2)

Area of ΔABC =32 sq.units

$= \frac{1}{2}\begin{vmatrix} p & 5 & 5 & p \\ p & 6 & -2 & p \end{vmatrix} = 32$

[(6p-10+5p)-(5p-30+2p)] = 64

11p-10-3p-30=64

8p-40=64

8p=64+40

8p=104

$p=\frac{104}{8} = 13$

---

19. Find the value of 'a' for which the given points (a, 2-2a), (-a+1, 2a) and (-4-a, 6-2a) are collinear.

Let A(a, 2-2a), B(-a+1, 2a) C(-4-a, 6-2a)

Given that points are collinear

Area of ΔABC = 0

$\frac{1}{2}\begin{vmatrix} a & -a+1 & -4-a & a \\ 2-2a & 2a & 6-2a & 2-2a \end{vmatrix} = 0$

{($2a^2$+[(-a+1)x(6-2a)]+[(-4-a)x(2-2a)]}

-{[(2-2a)(-a+1)]+2a(-4-a)+a(6-2a)} =0

$2a^2$-6a+6+$2a^2$-2a+(-8)-2a+8a+$2a^2$

-[-2a+$2a^2$+2-2a-8a-$2a^2$+6a-$2a^2$ =0

$8a^2$+4a-4=0 $2a^2$+a-1=0

$2a^2$+2a-a-1=0

2a (a+1)-1(a+1) = 0 (2a-1) (a+1)=0

2a-1=0 (or) a+1=0

$a = \frac{1}{2}$ (or) a=-1

---

20. Find the equation of a straight line passing through the midpoint of a line segment joining the points (1, -5), (4, 2) and parallel to X – axis.

Midpoint of (1,-5) and (4,2)

$M(x_1, y_1) = \left(\frac{1+4}{2}, \frac{-5+2}{2}\right) = \left(\frac{5}{2}, \frac{-3}{2}\right)$

Equation of line slope point form

$y - y_1 = m(x - x_1)$

When line parallel to X-axis, m = ∞

$y\text{-}\left(-\frac{3}{2}\right) = m\left(x - \frac{5}{2}\right)$

$y + \frac{3}{2} = 0\left(x - \frac{5}{2}\right)$

$y + \frac{3}{2} = 0$

$y = -\frac{3}{2}$

2y+3=0

---

21. Find the equation of straight line passing through the midpoints of a line segment joining the points. (1,-5), (4,2) and parallel to Y-axis.

Midpoint of (1,-5) and (4,2)

$M(x_1, y_1) = \left(\frac{1+4}{2}, \frac{-5+2}{2}\right) = \left(\frac{5}{2}, \frac{-3}{2}\right)$

Equation of line slope point form

$y - y_1 = m(x - x_1)$

When line parallel to Y-axis, m = 0

$y + \frac{3}{2} = \infty\left(x - \frac{5}{2}\right)$

$y + \frac{3}{2} = \frac{1}{0}\left(x - \frac{5}{2}\right)$

$0 \text{ x } \left(y + \frac{3}{2}\right) = \left(x - \frac{5}{2}\right)$

$\left(x - \frac{5}{2}\right) = 0$

$x = \frac{5}{2}$

2x-5=0

---

22. Find the equation of a line through (2,3) and (-7,-1).

Let (x_1,y_1)=(2,3) and (x_2,y_2)=(-7,-1)

Equation of line 2 point form

$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

$\frac{y-3}{-1-3} = \frac{x-2}{-7-2}$

$\frac{y-3}{-4} = \frac{x-2}{-9}$

9(y-3)=4(x-2)

9y-27=4x-8

4x-9y+19=0

---

23. Find the equation of a line through the given pair of points $(2, \frac{2}{3})$ and $(\frac{-1}{2}, 2)$.

Let $(x_1,y_1)=(2, \frac{2}{3})$ and $(x_2,y_2)=(\frac{-1}{2}, 2)$

Equation of line 2 point form

$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

$\frac{y-\frac{2}{3}}{-2-\frac{2}{3}} = \frac{x-2}{-\frac{1}{2}-2}$

$\frac{y-\frac{2}{3}}{-\frac{8}{3}} = \frac{x-2}{\frac{-5}{2}}$

$\frac{5}{2}\left(y - \frac{2}{3}\right) = \frac{8}{3}(x-2)$

5(3y-2)= 16(x-2)

15y-10 = 16x-32

16x-15y-22=0.

---

24. A cat is located at the point (-6,-4) in xy plane. A bottle of milk is kept at (5,11). The cat wish to consume the milk travelling through shortest possible distance. Find the equation of the path it needs to take its milk.

Equation of the line joining the point is,

$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

$\frac{y+4}{11+4} = \frac{x+6}{5+6}$

$\frac{y+4}{15} = \frac{x+6}{11}$

15$(x+6)$=11(y+4)

15x +90 = 11y+44

15x-11y+90-44=0

15x-11y+46=0

The equation of the path is
15x-11y+46=0.

---

25. Find the equation of a straight line which has slope $\frac{-5}{4}$ and passing through the point (-1,2).

m=$\frac{-5}{4}$, (x_1,y_1)=(-1,2)

$y - y_1 = m(x - x_1)$

y-2= $\frac{-5}{4}$ $(x$-(-1))

4(y-2)=-5(x+1)

4y-8= -5x-5

5x+4y-3=0.

---

26. Find the equation of a line whose intercepts on the x and y axes are4,-6.

Equation of line intercept form

$\frac{x}{a}+\frac{y}{b} = 1$

$\frac{x}{4}+\frac{y}{-6} = 1$

$\frac{x}{4}-\frac{y}{6} = 1$

$\frac{3x-2y}{12}$ =1

$3x - 2y$=12

$3x - 2y - 12 = 0$

---

27. Find the equation of a line whose intercepts on the x and y axes are -5, $\frac{3}{4}$

Ans: 3x-20y+15=0

---

28. Find the slope of the line 5y-3=0.

Slope m = $\frac{-co.efficient\ of\ x}{co.efficient\ of\ y}$

= $\frac{0}{5}$ = 0

---

29. Find the slope of the line $7x - \frac{3}{17} = 0$.

Ans: ∞

---

30. Find the slope of the line which is Parallel to y=$0.7x - 11 = 0$

Given line is $0.7x - y - 11 = 0$

Slope m= $\frac{-0.7}{-1}$ = 0.7

Slope of parallel lines are equal.

∴ Slope of parallel line to given line is 0.7

---

31. Find the slope of the line which is perpendicular to the line $x = -11$.

Slope of $x + 11 = 0$ is = $\frac{-1}{0} = \infty$

Slope of line perpendicular to $x + 11 = 0$ is

$m_2 = \frac{-1}{m_2} = \frac{-1}{\infty} = 0$.

---

32. Check whether the given lines are parallel or perpendicular.

$\frac{x}{3}+\frac{y}{4}+\frac{1}{7} = 0$ and $\frac{2x}{3}+\frac{y}{2}+\frac{1}{10} = 0$

$m_1 = \frac{\left(\frac{-1}{3}\right)}{\left(\frac{1}{4}\right)} = \frac{-1}{3}$ x $\frac{4}{1} = \frac{-4}{3}$

$m_2 = \frac{\left(\frac{-2}{3}\right)}{\left(\frac{1}{2}\right)} = \frac{-2}{3}$ x $\frac{2}{1} = \frac{-4}{3}$

$m_1 = m_2$

∴ Lines are parallel.

---

33. Check whether the given lines are parallel or perpendicular.

$5x + 23y$+14=0 and 23x-5y+9=0

m_1=$\frac{-5}{23}$, m_2=$\frac{-23}{-5}$ =$\frac{23}{5}$

$m_1 \neq m_2$

then, m_1 x m_2= $\frac{-5}{23}$ x $\frac{23}{5}$

m_1 x m_2 = -1

∴ Lines are perpendicular.

---

34. If the straight lines 12y=-(p+3)x+12 , 12x-7y=16 are perpendicular then find ‘p’.

L_1 is (p+3)x+12y-12=0

$m_1 = \frac{-(p+3)}{12}$

L_2 is 12x-7y=16

$m_2 = \frac{-12}{-7} = \frac{12}{7}$

Lines are perpendicular.

m_1 x m_2 = -1

$\frac{-(p+3)}{12}$ x $\frac{12}{7}$ = -1

-(p+3)=-7

p+3=7

p=7-3

p=4.

Practice:

35. Find the area of the triangle whose Vertices are (-3,5), (5,6) and (5,-2).

---

36. Show that the points P(-1.5 , 3), Q(6,-2) R(-3,4) are collinear.

37. If the area of the triangle formed by the vertices A(-1,2), B(k,-2) and C(7,4) taken in order is 22 sq. units, find the value of k.

38. The line r passes through the points (-2,2) and (5,8) and the line s passes through the points (-8,7) and (-2,0). Is the line perpendicular to s?

39. The line p passes through the points (3,-2), (12,4) and the line q passes through the points (6,-2) and (12,2). Is p parallel to q?

40. Show that the points (-2,5), (6,-1) and (2,2) are collinear.

41. Find the equation of a straight line Whose slope is 5 and y intercept is -9.

42. Find the equation of a straight line whose inclination is 45° and y intercept is 11.

43. Calculate the slope and y intercept of the straight line $8x$-7y+6=0.

44. Find the equation of a line passing through the point (3,-4) and having slope $\frac{-5}{7}$.

45. Find the equation of a line passing through the point A(1,4) and perpendicular to the line joining points (2,5) and (4,7).

46. Find the equation of a line which passes through (5,7) and makes intercepts on the axes equal in magnitude but opposite in sign.

47. Find the intercepts made by the line $4x$-9y+36=0 on the coordinate axes.

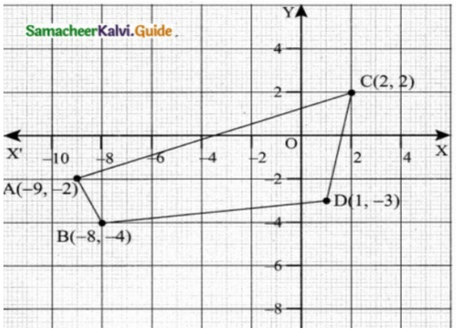
48. Show that the straight lines $2x$+3y-8=0 and $4x$+6y+18=0 are parallel.

49. Show that the straight lines x-2y+3=0 and $6x$+3y+8=0 are perpendicular.

Five Marks:

1. Find the area of the quadrilateral whose vertices are at (-9,-2), (-8,-4), (2,2) and (1,-3).

Let A(-9,2) B(-8,-4) C(1,-3) D(2,2)



Area of quadrilateral

$=\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix}$ sq. units

$=\frac{1}{2}\begin{vmatrix} -9 & -8 & 1 & 2 & -9 \\ -2 & -4 & -3 & 2 & -2 \end{vmatrix}$

$=\frac{1}{2}$ [(36+24+2-4)-(16-4-6-18)]

$=\frac{1}{2}$ [58-(-12)] = $\frac{1}{2}$ (70) = 35 sq. units.

2. Find the area of the quadrilateral whose vertices are at (-9,0), (-8,6), (-1,-2) and (-6,-3).

Ans: 34 sq. units

3. Find the area of the quadrilateral formed by the points (8,6), (5,11), (-5,12) and (-4,3).

Ans: 79 sq. units

4. Find the value of k, if the area of a quadrilateral is 28 sq. units, whose vertices are taken in order (-4,-2), (-3,k), (3,-2) and (2,3).

Solution:

Let $(\underset{x_1}{-4}, \underset{y_1}{-2}), (\underset{x_2}{-3}, \underset{y_2}{k}), (\underset{x_3}{3}, \underset{y_3}{-2}), (\underset{x_4}{2}, \underset{y_4}{3})$

Area of quadrilateral

$=\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix}$ sq. units

$\frac{1}{2}\begin{vmatrix} -4 & -3 & 3 & 2 & -4 \\ -2 & k & -2 & 3 & -2 \end{vmatrix}$=28 sq. units

[(-4k+6+9-4)-(6+3k-4-12)]=28x2

11-4k-(-10+3k) = 56

-7k+21=56

-7k=56-21=35

k=$\frac{35}{-7}$ = -5

5. If the points A(-3,9) B(a,b) and C(4,-5) are collinear and if a+b=1, then find a and b.

Since the three points A(-3,9), B(a,b) and C(4,-5) are collinear.

Area of ΔABC = 0

$x_1y_2 + x_2y_3 + x_3y_1$

$=x_2y_1 + x_3y_2 + x_1y_3$

-3b-5a+36=9a+4b+15

9a+5a+4b+3b+15-36=0

14a+7b-21 = 0

2a+b = 3 ⟶ ①

Also a+b = 1 ⟶ ②

b = 1-a Substituting in ①

2a+1-a = 3

a = 3-1

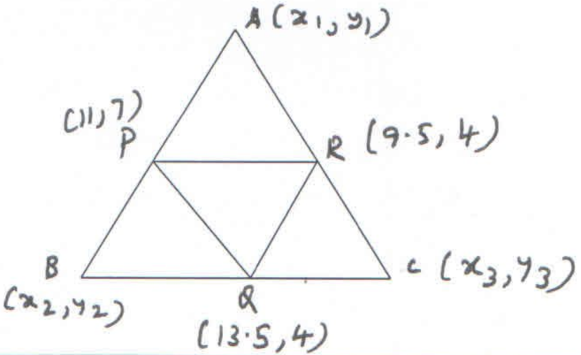
a = 2

and b = 1-2 = -1

b = -1

6. If the points P(-1,-4), Q(b,c) and R(5,-1) are collinear and if 2b+c = 4, then find the values of b and c.

- 7. Let P(11,7), Q(13.5,4) and R(9.5,4) be the mid-points of the sides AB, BC and AC respectively of ΔABC. Find the co-ordinates of the vertices A, B and C. Hence find the area of ΔABC and Compare this with area of ΔPQR.



Mid point of AB = P ⇒ $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = (11,7)$

Mid point of BC = Q ⇒ $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right) = (13.5,4)$

Mid point of AC = R ⇒ $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right) = (9.5,4)$

$x_1 + x_2 = 22 \rightarrow (1)$ $y_1 + y_2 = 14 \rightarrow (4)$

$x_2 + x_3 = 27 \rightarrow (2)$ $y_2 + y_3 = 8 \rightarrow (5)$

$x_1 + x_3 = 19 \rightarrow (3)$ $y_1 + y_3 = 8 \rightarrow (6)$

From (1) & (2) $x_1 - x_3 = -5 \rightarrow (7)$

From (3) & (7)

$$\begin{array}{l} x_1 + x_3 = 19 \\ \underline{x_1 - x_3 = -5} \\ 2x_1 \quad = 14 \end{array}$$

$x_1 = 7$

$x_2 = 22 - x_1 = 22 - 7 = 15$

$x_3 = 19 - x_1 = 19 - 7 = 12$

From (4) & (5) $y_1 - y_3 = 6 \rightarrow (8)$

From (6) & (8)

$$\begin{array}{l} y_1 + y_3 = 8 \\ \underline{y_1 - y_3 = 6} \\ 2y_1 \quad = 14 \end{array}$$

$y_1 = 7$

$y_2 = 14 - y_1 = 14 - 7 = 7$

$y_3 = 8 - y_1 = 8 - 7 = 1$

$\therefore A(x_1, y_1) = (7, 7)$

$B(x_2, y_2) = (15,7)$

$C(x_3, y_3) = (12,1)$

Area of ΔABC $= \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} 7 & 15 & 12 & 7 \\ 7 & 7 & 1 & 7 \end{vmatrix}$

$= \frac{1}{2}|(49 + 15 + 84) - (105 + 84 + 7)|$

$= \frac{1}{2}|148 - 196|$

$= \frac{1}{2}|-48|$

$= \frac{1}{2}$ x 48 = 24 sq. units

Area of ΔPQR $= \frac{1}{2}\begin{vmatrix} 11 & 13.5 & 9.5 & 11 \\ 7 & 4 & 4 & 7 \end{vmatrix}$

$= \frac{1}{2}|(44 + 54 + 66.5)$

$-(94.5 + 38 + 44)|$

$= \frac{1}{2}|164.5 - 176.5|$

$= \frac{1}{2}$ x 12 = 6 sq. units.

Area of ΔABC = 4 x Area of ΔPQR

8. A triangular shaped glass with vertices at A(-5, -4) B(1,6) and C(7,-4) has to be painted. If one bucket of paint covers 6 sq. feet, how many buckets of paint will be required to paint the whole glass, if only one coat of paint is applied.

Solution:

Area of ΔABC=$\frac{1}{2}\{(x_1y_2+x_2y_3+x_3y_1)$

$-(x_2y_1+x_3y_2+x_1y_3)\}$

=$\frac{1}{2}$ {(-62)-(58)}

=$\frac{1}{2}${-62-58}

=$\frac{1}{2}|-120|$

=$\frac{1}{2}$ x 120 =60

No. of buckets of paint can required=$\frac{60}{6}$ =10 cans.

9. Eg.5.5: The floor of a hall is covered with identical tiles which are in the shapes of triangles has the vertices at (-3,2), (-1,-1) and (1,2). If the floor of the hall is completely covered by 110 tiles. Find the area of the floor.

10. Show that the given points form a right angled triangle and check whether they satisfies Pythagoras theorem.

(i) A(1,-4), B(2,-3) and C(4,-7)

Slope of AB =$\frac{y_2-y_1}{x_2-x_1}=\frac{-3+4}{2-1}=\frac{1}{1}$ =1

Slope of BC =$\frac{-7-(-3)}{4-2}=\frac{-7+3}{2}=\frac{-4}{2}$ =-2

Slope of AC =$\frac{-7-(-4)}{4-1}=\frac{-7+4}{3}=\frac{-3}{3}$ =-1

Slope of AB x Slope of AC=1 x (-1)

=-1

⟹ AB perpendicular to AC

∴ ∟A=90°

Hence ABC is a right angle Δ

Verification of Pythagoras theorem.

BC is hypotenuse

$BC^2=(x_2-x_1)^2+(y_2-y_1)^2$

$=(-4)^2+2^2$ =16+4 =20.

$AB^2+AC^2=[(2-1)^2+(-3+4)^2]+$

$[(-7+4)^2+(4-1)^2]$

$=(1^2+1^2)+(-3)^2+3^2$

=(1+1)+9+9 =20.

They satisfy Pythagoras theorem.

11. Show that the points L(0,5), M(9,12) and N(3,14) form a right angled triangle and check whether they satisfies Pythagoras theorem.

12. Without using Pythagoras theorem, show that the points (1,-4), (2,-3) and (4,-7) form a right angled triangle.

13. Show that the given points form a parallelogram A(2.5,3.5), B(10,-4), C(2.5,-2.5) and D(-5,5).

Solution:

Slope of AD=$\frac{y_2-y_1}{x_2-x_1}$=$\frac{5-3.5}{-5-2.5}$ =$\frac{1.5}{-7.5}$= $\frac{-15}{75}$

=$\frac{-1}{5}$

Slope of BC=$\frac{y_2-y_1}{x_2-x_1}$=$\frac{-2.5-(-4)}{2.5-10}$ =$\frac{-2.5+4}{-7.5}$

= $\frac{1.5}{-7.5}$ =$\frac{-1}{5}$

Slope of AD = Slope of BC

∴ AD parallel to BC

Slope of AB=$\frac{-4-3.5}{10-2.5}$ =$\frac{-7.5}{7.5}$= $\frac{-75}{75}$ =-1

Slope of CD=$\frac{5-(-2.5)}{-5-2.5}$ =$\frac{7.5}{-7.5}$= $\frac{75}{-75}$ =-1

Slope of AB = Slope of CD

∴ AB parallel to CD.

We found opposite sides of quadrilateral are parallel. Hence ABCD is a parallelogram.

14. If the points A(2,2), B(-2,-3) C(1,-3) and D(x,y) Form a parallelogram then find the value of x and y.

Solution:

A(2,2), B(-2,-3) C(1,-3) and D(x,y)

ABCD is parallelogram

Diagonals bisects each other.

Midpoint of BD = Midpoint of AC

$$\left(\frac{x+(-2)}{2}, \frac{y+(-3)}{2}\right) = \left(\frac{2+1}{2}, \frac{2+(-3)}{2}\right)$$

$\frac{x-2}{2}=\frac{3}{2}$	$\frac{y-3}{2}=\frac{2-3}{2}$
x-2= 3	y-3=-1
x=3+2	y=-1+3
x=5	y=2

15. Find the equation of the median and altitude of ΔABC through A where the vertices are A(6,2) ,

B(-5,-1) and C (1,9)

let AP be altitude

AP perpendicular BC

slope of AP =$\frac{-1}{slope\ of\ BC}$

= $\frac{1}{\frac{g-(-1)}{1-(-5)}}$

m = - $\frac{6}{10}$ = $\frac{-3}{5}$

Equation of AP through A(6,2) m=$\frac{-3}{5}$

y-y_1 = m(x-x_1)

Y-2 = $\frac{-3}{5}$ (x -6)

5(y-2) = -3(x-6)

5y-10 = -3x+18

3x + 5y-10-18=0

3x+5y-28 = 0. is equation altitude

let AQ be median

Q(x,y) = mid point of BC

=$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

= $\left(\frac{-5+1}{2}, \frac{-5+9}{2},\right)$= (-2 , 4)

Equation of AQ A(6,2) Q(-2,4)

$\frac{y-y_1}{y_2-y_1}$ =$\frac{x-x_1}{x_2-x_1}$

$\frac{y-2}{4-2}$ =$\frac{x-6}{-2-6}$

$\frac{y-2}{2}$ =$\frac{x-6}{-8}$ = $\frac{y-2}{1}$ = $\frac{x-6}{-4}$

-4(y-2) = x-6

-4y +8 =x-6

x+4y-6-8 = 0

x+4y-14 = 0

16. find the equation of a straight line Passing through (1,-4) and has intercepts which are in the ratio 2 : 5

Ratio of intercept = 2 : 5 =

coefficient of y : coefficient of x

M = slope= $\frac{-coefficient\ of\ x}{coefficient\ of\ y}$ = $\frac{-5}{2}$

Point (1, -4)

Equation line point slope form

$y-y_1 = m(x-x_1)$

$y+4 = \frac{-5}{2}(x-1)$

2(y+4) = -5(x-1)

2y + 8 = -5x+5

5x+2y-5+8 = 0

5x+2y+3 =0

17. Find the equation of a straight line passing through (-8,4) and making equal intercepts on the co-ordinates.

Intercepts are equal a = b

$\frac{a}{b} = \frac{1}{1}$

M = slope = $\frac{-coefficient\ of\ x}{coefficient\ of\ y}$ = $\frac{-a}{b}$ = -1

Point (-8 , 4)

Equation of line point – slope form

$y-y_1 = m(x-x_1)$

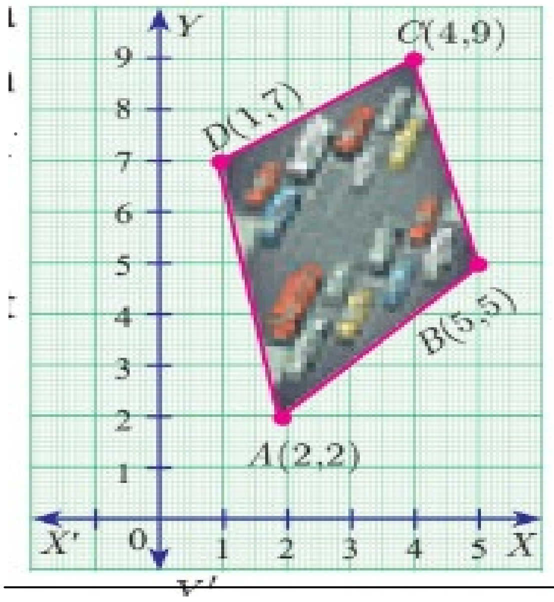
y-4 = -1 (x+8)

y-4 = (-x-8)

x+y-4+8=0

x+y+4=0

18. The given diagram shows a plan for constructing a new parking lot at a campus. It is estimated that such construction would cost Rs.1300 per square feet. What will be the total cost for making the parking lot ?



19. A line makes positive intercept on coordinate axes whose sum is 7 and it passes through (-3,8) find its equation.

20. A(-3,0) B(10,-2) and C(12,3) are vertices of ΔABC. Find the equation of altitude through A and B.

21. Find the equation of the perpendicular bisector of the line joining the points A(-4,2) and B(6,-4)

let CD perpendicular AB and D is Mid point of AB. A(-4,2) B(6,-4)

point D (x,y) = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{-4+6}{2}, \frac{2-4}{2}\right)$

$= \left(\frac{2}{2}, \frac{-2}{2}\right)$ = (1, -1)

Slope of CD = $\frac{-1}{slope\ of\ AB}$ = $\frac{-1}{\frac{y_2-y_1}{x_2-x_1}}$

$= \frac{-1}{\frac{-4-2}{6+4}} = \frac{-10}{-6} = \frac{5}{3}$

Equation of line point – slope form

D(1,-1) m = $\frac{-4}{3}$

$y-y_1 = m(x-x_1)$

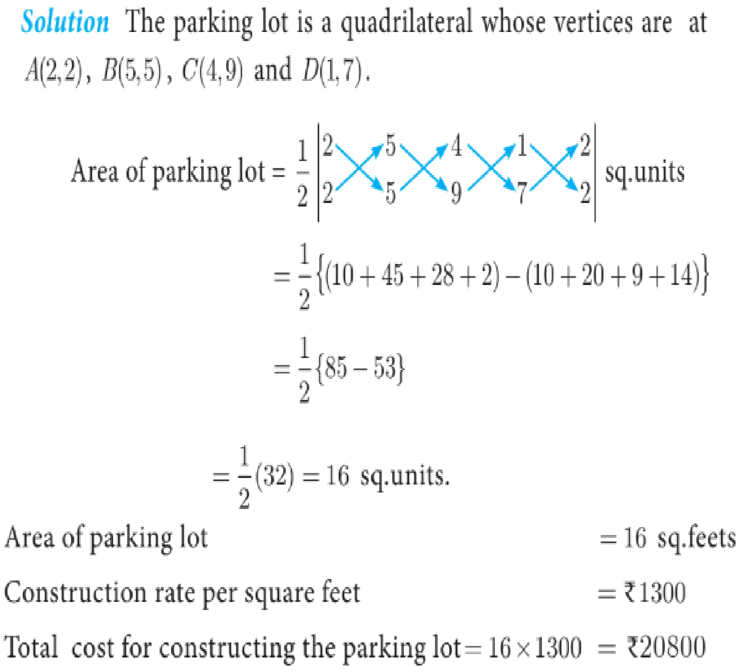
$(y+1) = \frac{5}{3}(x-1)$

3(y+1) = 5 (x-1)

3y + 3 = 5x -5

5x-3y-5-3 = 0

5x-3y-8 = 0



Solution The parking lot is a quadrilateral whose vertices are at $A(2,2)$, $B(5,5)$, $C(4,9)$ and $D(1,7)$.

$$\text{Area of parking lot} = \frac{1}{2}\begin{vmatrix} 2 & 5 & 4 & 1 & 2 \\ 2 & 5 & 9 & 7 & 2 \end{vmatrix} \text{ sq.units}$$

$$= \frac{1}{2}\{(10+45+28+2)-(10+20+9+14)\}$$

$$= \frac{1}{2}\{85-53\}$$

$$= \frac{1}{2}(32) = 16 \text{ sq.units.}$$

Area of parking lot	= 16 sq.feets
Construction rate per square feet	= ₹1300
Total cost for constructing the parking lot = 16 × 1300	= ₹20800

Chapter 8 : Statistics and Probability

2 Marks:- Examples and Exercise 8.1

(*) Range, R=L-S

(*) Coefficient of range = $\frac{L-S}{L+S}$

Example 8.1: Find the range and coefficient of Range of the following data : 25,67,48,53,18,39,44

Solution: L=67, S=18

R=L-S = 67-18 = 49

Coefficient of range = $\frac{L-S}{L+S}$ = $\frac{67-18}{67+18}$

= $\frac{49}{85}$ = 0.576

Example 8.2: Find the range of the following distribution.

Age(in years)	16-18	18-20	20-22	22-24	24-26	26-28
Number of students	0	4	6	8	2	2

Solution: L=28, S = 18, R = L-S

R = 28-18 = 10 years

Example 8.3 : The range of a set of data is 13.67 an the largest value is 70.08. Find the smallest value.

Solution: R = 13.67 ; L= 70.08

R=L-S

13.67 = 70.08 – S

S = 70.08 – 13.67 = 56.41

Exercise 8.1

Find the range and coefficient of range of the following data.

(i) 63, 89, 98, 125, 79, 108, 117, 68

Solution : L = 125 ; S = 63

R = L –S = 125 -63 = 62

Coefficient of range = $\frac{L-S}{L+S}$

= $\frac{125-63}{125+63}$ = $\frac{62}{188}$ = 0.329 = 0.33

(ii) 43.5, 13.6, 18.9, 38.4, 61.4 , 29.8

Solution :

L = 61.4 ; S=13.6

R = L –S = 61.4 – 13.6 = 47.8

Coefficient of range = $\frac{L-S}{L+S}$

= $\frac{61.4-13.6}{61.4+13.6}$ = $\frac{47.8}{75}$ = 0.637 = 0.64

---

2 . If the range and the smallest value of a set of data are 36.8 and 13.4 respectively , then find the largest value.

Solution: R = L – S

36.8 = L – 13.4

36.8 + 13.4 = L

L = 50.2

---

3. Calculate the range of the following data

Income	400-450	450-500	500-550	550-600	600-650
Number of workers	8	12	30	21	6

Solution :

L = 650 ; S =400

R = L – S = 650 – 400 = 250

R = 250

---

4. Find the standard deviation of first 21 natural numbers

Solution:

Standard deviation of first n natural numbers

= $\sqrt{\frac{n^2-1}{12}}$

Standard deviation of first 21 natural numbers

= $\sqrt{\frac{21^2-1}{12}}$ = $\sqrt{\frac{441-1}{12}}$ = $\sqrt{\frac{440}{12}}$

= $\sqrt{36.6666}$ = 6.05

---

5. If the standard deviation of a data is 4.5 and if each value of the data is decreased by 5 , then find the new standared deviation

Solution :

Standard deviation of the data = 4.5

Each data is decreased by 5

The new standard deviation = 4.5

---

6. If the standard deviation of a data is 3.6 and each value of the data is divided by 3, then find the new variance and new standard deviation.

Solution:

Standard deviation of the data=3.6

Each data is divided by 3

The new standard deviation = $\frac{3.6}{3}$ = 1.2

The new variance = $(1.2)^2$ = 1.41

Examples & Exercise 8.2

(*) Coefficient of variation

C.V = $\frac{\sigma}{\bar{x}}$ x 100%

Example 8.15 : The mean of a data is 25.6 and it's Coefficient of variation is 18.75. Find the standard deviation.

Solution : Mean $\bar{x}$ = 25.6, C.V = 18.75

C . V = $\frac{\sigma}{\bar{x}}$ x 100%

18.75 = $\frac{\sigma}{25.6}$ x 100 → σ = $\frac{18.75 X 25.6}{100}$ = 4.8

σ = 4.8

Exercise 8.2 :

1. The standard deviation and mean of a data are 6.5 and 12.5 respectively. Find the coefficient of variation.

Solution: σ =6.5 , $\bar{x}$ = 12.5

$$C.V = \frac{\sigma}{\bar{x}} \text{ x } 100\%$$

$$= \frac{6.5}{12.5} \text{ x } 100\% = 52\%$$

C. V = 52%

---

2. The standard deviation and coefficient of variation of a data are 1.2 and 25.6 respectively. Find the value of mean.

Solution:

σ = 1.2 ; C. V = 25.6

$$C.V = \frac{\sigma}{\bar{x}} \text{ x } 100\%$$

$$25.6 = \frac{1.2}{\bar{x}} \text{ x } 100 \Longrightarrow \bar{x} = \frac{1.2}{25.6} \text{ x } 100 = 4.6875$$

$\bar{x}$ = 4.69

---

3. If the mean and coefficient of variation of a data are 15 and 48 respectively, then find the value of standard deviation.

Solution: $\bar{x}$ = 15, C.V = 48

$$C.V = \frac{\sigma}{\bar{x}} \text{ x } 100\%$$

$$48 = \frac{\sigma}{15} \text{ x } 100$$

$$\sigma = \frac{48 \text{ X } 15}{100} = 7.2 \rightarrow \sigma = 7.2$$

---

4. If n = 5 , $\bar{x}$ = 6, Σx^2 = 765 then calculate the coefficient of variation.

Solution:

$$\sigma = \sqrt{\left(\frac{\Sigma x^2}{n}\right) - \left(\frac{\Sigma x}{n}\right)^2}$$

$$= \sqrt{\frac{765}{5} - (6)^2}$$

$$= \sqrt{153 - 36} = \sqrt{117} = 10.82$$

$$c.v = \frac{\sigma}{\bar{x}} \text{ x } 100\%$$

$$= \frac{10.82}{6} \text{ x } 100 = 180.28\%$$

Example & Exercise 8.3

(*) $P(E) = \frac{n(E)}{n(S)}$

(*) $P(S) = \frac{n(S)}{n(S)} = 1$

(*) $0 \leq P(E) \leq 1$

(*) $P(\bar{E}) = 1 - P(E)$

(*) $P(E) + P(\bar{E}) = 1$

(*) Coin :

1 Coin : S= {H, T},n(S)=2

2 Coins: S= {HH,HT,TH,TT} ; n(S)=4

3 Coins :S={HHH,HHT,HTT,TTT,TTH,THH,THT, HTH}

n(S)=8

(*) Dice :

1 Die: S= {1, 2, 3, 4, 5, 6} ; n(S)=6

2 Dice: S ={ (1,1),(1,2),(1,3),(1,4),(1,5),(1,6)
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

n(S)=36

(*) Cards:

n(S)=52

Spade :	13	Heart :	13
Clavor :	13	Diamond :	13

Black cards : 26 Red cards : 26

(*) No. of kings = 4
No. of Queens = 4
No. of Jacks = 4

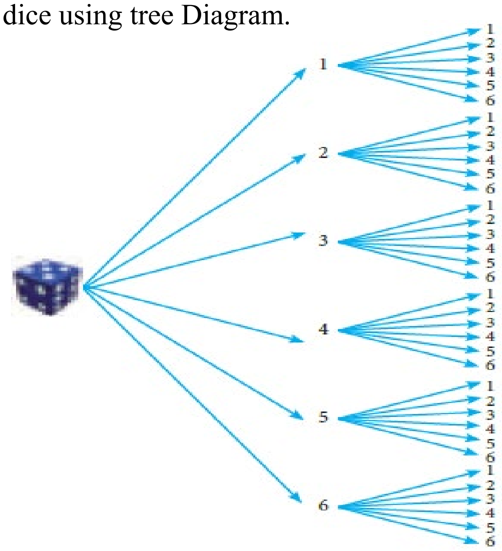
(*) No. of black kings = 2
No. of black queens = 2
No. of black Jacks = 2

(*) No. of Red kings = 2
No. of Red queens = 2
No. of Red Jacks = 2

(*) Numbered of cards are : 2,3,4,5,6,7,8,9,10

(*) face card 4K, 4R, 4J $\Longrightarrow$ 12 cards.

Example 8.17 : Express the sample space for rolling two dice using tree Diagram.



S= {(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

n(S) = 36.

Example 8.18 : A bag contains 5 blue balls and 4 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is (i) blue (ii) not blue.

Solution: n(S) = 5+4=9

(i) Let A be the event of getting a blue ball.

n(A) = 5

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{9}$$

(ii) Let $\bar{A}$ be the event of not getting a blue ball.

$$P(\bar{A}) = 1 - P(A) = 1 - \frac{5}{9} = \frac{4}{9}$$

Example 8.20: Two coins are tossed together. What is the probability of getting different faces on the coins?

Solution:

$S = \{HH, HT, TH, TT\}$

$n(S) = 4$

Let A be the event of getting different faces on the coins.

$A = \{HT, TH\}$

$n(A)=2$

$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

Example 8.22 :

What is the probability that a leap year selected at random will contain 53 Saturdays.

Solution : leap year =366 days = 52 full weeks and 2 days

$$S = \begin{Bmatrix} Sun-Mon, Mon-Tue, Tue-wed, Wed-Thu \\ Thu-Fri, Fri-Sat, Sat-Sun \end{Bmatrix}$$

$n(S) = 7$

Let A be the event of getting 53^{rd} Sunday.

$A = \{Fri-Sat, Sat-Sun\}$

$n(A) = 2$

$P(A) = \frac{n(A)}{n(S)} = \frac{2}{7}$

Example 8.23 :

A die is rolled and a coin is tossed simultaneously. Find the probability that the die shows an odd number and the coin shows a head.

Solution:

$S = \{1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T, 5H, 5T, 6H, 6T\}$

$n(S) = 12$

Let A be the event of getting an odd number and a head.

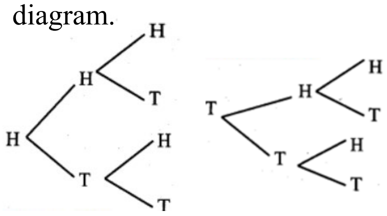
$A = \{1H, 3H, 5H\}$

$n(A) = 3$

$P(A) = \frac{n(A)}{n(S)} = \frac{3}{12} = \frac{1}{4}.$

Exercise 8.3

1. Write the sample space for tossing three coins using tree diagram.

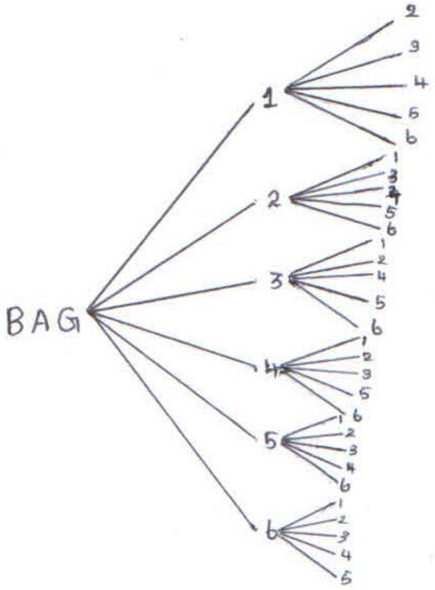


S = {HHH, HHT, HTH, HTT,THH, THT,TTH, TTT}; n(S)=8

2. Write the sample space for selecting two balls at a time from a bag containing 6 balls numbered 1 to 6

(Using tree diagram)

Solution:



$$S = \begin{Bmatrix} (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,3)(2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5) \end{Bmatrix} ; \; n(S)=30$$

3. If A is an event of a random experiment such that $P(A) : P(\bar{A}) = 17 : 15$ and $n(S) = 640$ then find (i) $P(\bar{A})$ (ii) n(A).

Solution:

(i) $\frac{P(A)}{P(\bar{A})} = \frac{17}{15}$

$\frac{p(A)}{1-P(A)} = \frac{17}{15}$

$15P(A) = 17\,[1 - P(A)]$

$= 17-17\,P(A).$

$17\,P(A) + 15\,P(A) = 17$

$32\,P(A) = 17$

$P(A) = \frac{17}{32}$

$P(\bar{A}) = 1- P(A) = 1-\frac{17}{32} = \frac{32-17}{32}$

$P(\bar{A}) = \frac{15}{32}$

(ii) $P(A) = \frac{17}{32}$

$\frac{n(A)}{n(S)} = \frac{17}{32} \rightarrow \frac{n(A)}{640} = \frac{17}{32}$

$n(A) = \frac{17}{32} \times 640 = 17 \times 20 = 340$

$n(A) = 340$

4. A coin is tossed thrice. What is the probability of getting two consecutive tails?

Solution:

$S = \{HHH, HHT, HTT, TTT, TTH, THH, THT, HTH\}$

$n(S) = 8$

Let A be the event of getting two consecutive tails.

$A = \{HTT, TTH, TTT\}$

$n(A) = 3$

$P(A) = \frac{n(A)}{n(S)} = = \frac{3}{8}$

Examples & Exercise 8.4

(*) $A \cap \bar{A} = \phi$

(*) $A \cup \bar{A} = S$

(*) $P(A\cup B) = P(A) + P(B)$

If A, B are mutually exclusive events.

(*) $P(A\cap \bar{B}) = P(only\ A)$

$= P(A) - P(A \cap B)$

(*) $P(\bar{A} \cap B) = P(only\ B)$

$= P(B) - P(A \cap B)$

(*) Addition theorem of probability:

(i) $P(A\cup B) = P(A) + P(B) - P(A \cap B)$

(ii) $P(A\cup B \cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C).$

(*)$P(A\cap B) = \frac{n(A \cap B)}{n(S)}$ $P(A) = \frac{n(A)}{n(S)}$ $P(B) = \frac{n(B)}{n(S)}$

Example 8.26:

If $P(A) = 0.37$, $P(B) = 0.42$, $P(A\cap B) = 0.09$ then, find $P(A\cup B)$.

Solution:

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= 0.37 + 0.42 - 0.09$

$= 0.79 - 0.09$

$= 0.70$.

Example 8.27:

What is the probability of drawing either a king or a queen in a single draw from a well shuffled pack of 52 cards?

Solution:

$n(S)=52$

let A be the event of drawing a king card

$n(A) = 4$

$P(A) = \frac{n(A)}{n(S)} = \frac{4}{52}$

Let B be the event of drawing a queen card

$n(B) = 4$

$P(B) = \frac{n(B)}{n(S)} = \frac{4}{52}$

A, B are mutually exclusive events,

$P(A\cap B) = 0$

$P(A\cup B) = P(A) + P(B) = \frac{4}{52} + \frac{4}{52}$

$= \frac{8}{52} = \frac{2}{13}$

Example 8.29:

If A and B are two events such that $P(A) = \frac{1}{4}$ $P(B) = \frac{1}{2}$ and $P(A \text{ and } B) = \frac{1}{8}$. find (i) P(A or B) (ii) P (not A and not B)

Solution:

(i) $P(A \text{ or } B) = P(A\cup B)$

$= P(A) + P(B) - P(A\cap B)$

$= \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{2+4-1}{8}$

$= \frac{6-1}{8} = \frac{5}{8}$

(ii) P (not A and not B)

$= P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$

$= 1 - P(A\cup B) = 1 - \frac{5}{8} = \frac{8-5}{8} = \frac{3}{8}$

Exercise 8.4

1. $P(A) = \frac{2}{3}$, $P(B) = \frac{2}{5}$, $P(A\cup B) = \frac{1}{3}$, then find $P(A\cap B)$

Solution:

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$\frac{1}{3} = \frac{2}{3} + \frac{2}{5} - P(A\cap B)$

$P(A\cap B) = \frac{2}{3} + \frac{2}{5} - \frac{1}{3} = \frac{1}{3} + \frac{2}{5}$

$= \frac{5+6}{15} = \frac{11}{15}$

---

2. If A and B are two events such that , P(A) = 0.42 P(B) = 0.48 and $P(A\cap B) = 0.16$ find (i) P (not A) (ii) P (not B) (iii) P (A or B)

Solution:

(i) $P(\text{not } A) = P(\bar{A}) = 1 - P(A)$

$= 1 - 0.42 = 0.58$

(ii) $P(\text{not } B) = P(\bar{B}) = 1 - P(B)$

$= 1 - 0.48 = 0.52$

(iii) $P(A \text{ or } B) = P(A\cup B)$

$= P(A) + P(B) - P(A\cap B)$

$= 0.42 + 0.48 - 0.16$

$= 0.90 - 0.16$

$= 0.74$

---

3. If A and B are two mutually exclusive events of a random experiment and P(not A) = 0.45, $P(A\cup B) = 0.65$, then find P(B).

Solution: A,B are mutually exclusive

$P(A\cap B) = 0$

$P(\text{not } A) = P(\bar{A}) = 0.45$

$P(A) = 1 - P(\bar{A})$

$= 1 - 0.45 = 0.55$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$0.65 = 0.55 + P(B) - 0$

$0.65 - 0.55 = P(B)$

$P(B) = 0.1$

---

4. The Probability that atleast one of A and B occur is 0.6 If A and B occur simultaneously with probability 0.2, then find $P(\bar{A}) + P(\bar{B})$

Solution: $P(A\cup B) = 0.6$, $P(A\cap B) = 0.2$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$0.6 = P(A) + P(B) - 0.2$

$P(A) + P(B) = 0.6 + 0.2 = 0.8$

$P(\bar{A}) + P(\bar{B}) = 1 - P(A) + 1 - P(B)$

$= 2 - [P(A) + P(B)]$

$= 2 - 0.8$

$= 1.2$

5. The probability of happening of an event A is 0.5 and that of B is 0.3. If A and B are mutually exclusive events, then find the probability that neither A nor B happen.

Solution:

P(A) = 0.5

P(B) = 0.3

A, B are mutually exclusive events, $P(A\cap B) = 0$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= 0.5 + 0.3 - 0$

$= 0.8$

P(neither A nor B)

$= P(\overline{A\cup B}) = 1 - P(A\cup B)$

$= 1 - 0.8$

$= 0.2$

Chapter 8 : 5 marks:

(*) Variance, $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$

(*) Standard deviation $\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\overline{x})^2}{n}}$

(*) ungrouped data:

(i) Direct method: $\sigma = \sqrt{\frac{\Sigma {x_i}^2}{n} - (\frac{\Sigma x_i}{n})^2}$

(ii) Mean method:

$d_i = x_i - \bar{x}$

$\sigma = \sqrt{\frac{\Sigma {d_i}^2}{n}}$

(iii) Assumed Mean method :

$d_i = x_i - A$

$\sigma = \sqrt{\frac{\Sigma {d_i}^2}{n} - (\frac{\Sigma d_i}{n})^2}$

(iv) Step deviation method:

$\sigma = C \text{ x } \sqrt{\frac{\Sigma {d_i}^2}{n} - (\frac{\Sigma d_i}{n})^2}$

(*) Grouped data: (i) Mean method:

$\sigma = \sqrt{\frac{\Sigma f_i {d_i}^2}{N}}$, $N = \sum_{i=1}^{n} f_i \quad d_i = x_i - \bar{x}$

(ii) Assumed Mean method

$d_i = x_i - A$

$\sigma = \sqrt{\frac{\Sigma f_i {d_i}^2}{N} - (\frac{\Sigma f_i d_i}{N})^2}$

(*) Continuous frequency distribution:

(i) Mean method:

$\sigma = \sqrt{\frac{\Sigma f_i (x_i-\overline{x})^2}{N}}$

(ii) Step deviation method:

$d_i = \frac{x_i - A}{c}$

$\sigma = \sqrt{\frac{\Sigma f_i {d_i}^2}{N} - (\frac{\Sigma f_i d_i}{N})^2}$

Example 8.10: Find the mean and variance of the first n natural numbers?

Solution

$\bar{x} = \frac{\Sigma x_i}{n} = \frac{1+2+3+\cdots+n}{n}$

$= \frac{n(n+1)}{2 \times n}$

$\bar{x} = \frac{n+1}{2}$

$\sigma^2 = \frac{\Sigma {x_i}^2}{n} - (\frac{\Sigma x_i}{n})^2$

$= \frac{n(n+1)(2n+1)}{6 \times n} - (\frac{n\,(n+1)}{2 \times n})^2$

$= \frac{2n^2+3n+1}{6} - \frac{n^2+2n+1}{4}$

$= \frac{4n^2+6n+2+3n^2-6n-3}{12}$

$\sigma^2 = \frac{n^2-1}{12}$

Exercise 8.1 :

1. Find the variance and standard deviation of the wages of 9 workers given below:

Rs. 310, Rs. 290, Rs. 320, Rs.280 Rs.300, Rs. 290, Rs. 320, Rs. 310, Rs.280.

Solution:

Arranging the numbers in ascending order:

Rs. 280, Rs. 280, Rs.290, Rs.290 Rs.300, Rs 310, Rs. 310, Rs. 320, Rs. 320

$\bar{x} = \frac{280+280+290+290+300+310+310+320+320}{9}$

$\bar{x} = \frac{2700}{9}$

$\bar{x} = 300$

x_i	$d_i = x_i - \bar{x} = x_i - 300$	${d_i}^2$
280	-20	400
280	-20	400
290	-10	100
290	-10	100
300	0	0
310	10	100
310	10	100
320	20	400
320	20	400
		$\Sigma {d_i}^2 = 2000$

$\sigma = \sqrt{\frac{\Sigma {d_i}^2}{n}} = \sqrt{\frac{2000}{9}}$

$\sigma = \sqrt{222.222} = 14.91$

$\sigma^2 = 222.22$

Practice Examples 8.4, 8.5, 8.6, 8.7 and Exercise 8.1: 4

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2. The rainfall recorded in various places of five districts in a week are given below. Find its standard deviation.

Rain fall (in mm)	45	50	55	60	65	70
Number of places	5	13	4	9	5	4

Solution: $\bar{x} = \frac{\Sigma xifi}{N}$

x_i	f_i	$x_i f_i$	$d_i = x_i - \bar{x}$	d_i^2	$f_i d_i^2$
45	5	225	-11	121	605
50	13	650	-6	36	468
55	4	220	-1	1	4
60	9	540	4	16	144
65	5	325	9	81	405
70	4	280	14	196	784
	N=Σf_i=40	$\Sigma\, x_i f_i$= 2240			$\Sigma f_i d_i^2$=2410

$\bar{x} = \frac{\Sigma x_i f_i}{N} = \frac{2240}{40} \quad \rightarrow \quad \bar{x} = 56$

$\sigma = \sqrt{\frac{\Sigma\, f_i d_i^{\,2}}{N}} = \sqrt{\frac{2410}{40}}$

$= \sqrt{\frac{\Sigma f_i d_i^{\,2}}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$

$= \sqrt{60.25}$

$\sigma = 7.76$

Practice Example 8.8, 8.9, 8.11, 8.12

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3. In a study about Viral fever, the number of people affected in a town were noted as follows : Find its standard deviation.

Age in years	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Number of people affected	3	5	16	18	12	7	4

Solution: Assumed Mean, A= 35, C=10

Age	Mid value	f_i	d_i= x_i- A	$d_i = \frac{x_i - A}{c}$	$f_i d_i$	d_i^2	$f_i d_i^2$
0-10	5	3	-30	-3	-9	9	27
10-20	15	5	-20	-2	-10	4	20
20-30	25	16	-10	-1	-16	1	16
30-40	35	18	0	0	0	0	0
40-50	45	12	10	1	12	1	12
50-60	55	7	20	2	14	4	28
60-70	65	4	30	3	12	9	36
		N=65			$\Sigma f_i d_i$ =3		$\Sigma f_i d_i^{\,2}$ = 139

σ =C x $\sqrt{\frac{\Sigma f_{i} d_i^{\,2}}{N} - \left(\frac{\Sigma f_{i} d_i}{N}\right)^2}$

= 10 x $\sqrt{\frac{139}{65} - (\frac{3}{65})^2}$

= 10 x $\sqrt{2.138 - (0.046)^2}$

=10 x $\sqrt{2.138 - 0.002116}$

=10 x$\sqrt{2.136}$ $\simeq$10 $\times$ 1.46 $\rightarrow$ $\sigma \simeq 14.6$

Practice example 8.13, 8.14, Exercise 8.1: 14, 15

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4. The measurements of the diameters (in cms) of the plates prepared in a factory are given below. Find its standard deviation.

Diameter (cm)	21-24	25-28	29-32	33-36	37-40	41-44
Number of plates	15	18	20	16	8	7

Solution: A = 34.5 C= 4

Diameter (cm)	Mid value x_i	f_i	$d_i = \frac{x_i - A}{c}$	$f_i d_i$	d_i^2	$f_i d_i^2$
20.5-24.5	22.5	15	-3	-45	9	135
24.5-28.5	26.5	18	-2	-36	4	72
28.5-32.5	30.5	20	-1	-20	1	20
32.5-36.5	34.5	16	0	0	0	0
36.5-40.5	38.5	8	1	8	1	8
40.5-44.5	42.5	7	2	14	4	28
		N=Σf_i =84		$\Sigma f_i d_i$= -79		$\Sigma f_i d_i^2$ =263

σ =C x $\sqrt{\frac{\Sigma f_{i} d_i^{\,2}}{N} - \left(\frac{\Sigma f_{i} d_i}{N}\right)^2}$

= 4 x $\sqrt{\frac{263}{84} - \left(\frac{-79}{84}\right)^2}$

= 4 x $\sqrt{3.13 - \left(\frac{6241}{7056}\right)}$ = 4 x $\sqrt{3.13 - 0.88}$

= 4 x $\sqrt{2.25}$ = 4 x 1.5 = 6

σ = 6

Practice Exercise 8.1 : 13.

Exercise 8.2

1. Find the coefficient of variation of 24,26,33,37,29,31

Solution:

$\bar{x} = \frac{24+26+33+37+29+31}{6}$

$\bar{x} = \frac{180}{6}$ =30.

x	d= $x - \bar{x}$	d^2
24	-6	36
26	-4	16
33	3	9
37	7	49
29	-1	1
31	1	1
		$\Sigma d^2 = 112$

$\sigma = \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{\frac{112}{6}}$

= $\sqrt{18.66}$ = 4.32

C.V = $\frac{\sigma}{\bar{x}}$ x 100%

= $\frac{4.32}{30}$ x 100% = 14.4%

Practice Exercise 8.2 : 6

2. The total marks scored by two students Sathya and Vidhya in 5 subjects are 460 and 480 with standard deviation 4.6 and 2.4 respectively. Who is more consistent in performance?

Solution : n =5

Sathya	Vidhya
$\Sigma x_1 = 460$	$\Sigma x_2 = 480$
$\sigma_1 = 4.6$	$\sigma_2 = 2.4$
$\overline{x_1} = \frac{\Sigma x_1}{n}$	$\overline{x_2} = \frac{\Sigma x_2}{n}$
$= \frac{460}{5}$	$= \frac{480}{5}$
$\overline{x_1} = 92$	$\overline{x_2} = 96$
$\text{c.}v_1 = \frac{\sigma_1}{\bar{x}_1} \text{ x } 100\%$	$\text{c.}v_2 = \frac{\sigma_2}{\bar{x}_2} \text{ x } 100\%$
$= \frac{4.6}{92} \; x \; 100$	$= \frac{2.4}{96} \text{ x } 100$
$= 5\%$	$= 2.5\%$

$\text{C.}V_2 < \text{C.}V_1$

C.V. of vidhya < C.V of Sathya

∴ Vidhya is more consistent in performance.

Practice Example: 8.16

3. The mean and standard deviation of marks obtained by 40 students of a class in three subjects Mathematics, Science and Social science are given below.

subject	Mean	SD
Mathematics	56	12
Science	65	14
Social science	60	10

Which of the three subjects show more consistent and which shows less consistent in marks?

Solution:

	Mathematics	Science	Social science
Mean, $\bar{x}$	56	65	60
SD, σ	12	14	10
$\text{c.v} = \frac{\sigma}{\bar{x}} \text{ x } 100\%$	$= \frac{12}{56} \text{ x } 100$ $= 21.43\%$	$= \frac{14}{65} \text{ x } 100$ $= 21.54\%$	$= \frac{10}{60} \text{ x } 100$ $= 16.67\%$

C.V of Mathematics = 21.43%
C.V of Science = 21.54%
C.V of Social science = 16.67%
Social Science in more consistent.
Science is less consistent.

Examples & Exercise 8.3

Example 8.21:

From a well shuffled pack of 52 cards, one card is drawn at random. Find the probability of getting (i) Red card (ii) Heart card (iii) Red king (iv) Face card (v) Number card.

Solution: n(S) = 52

(i) let A be the event of getting a red card.

$$\text{n (A)} = 26 \text{ , P(A)} = \frac{\text{n(A)}}{\text{n(s)}}$$

$$\text{P(A)} = \frac{26}{52} = \frac{1}{2}$$

(ii) let B be the event of getting a heart card.

$$\text{n (B)} = 13 \text{ , P(B)} = \frac{\text{n(B)}}{\text{n(s)}}$$

$$\text{P(B)} = \frac{13}{52} = \frac{1}{4}$$

(iii) Let C be the event of getting a red king card. A Red king card can be either a diamond king or a Heart king.

$$\text{n (C)} = 2 \text{ , P(C)} = \frac{\text{n(C)}}{\text{n(S)}}$$

$$\text{P(C)} = \frac{5}{52} = \frac{1}{26}$$

(iv) Let D be the event of getting a face card. The face cards are Jack (J) Queen (Q) and King (K)

$$\text{n(D)} = 4 \text{ x } 3 = 12, \text{ P(D)} = \frac{\text{n(D)}}{\text{n(s)}}$$

$$\text{P(D)} = \frac{12}{52} = \frac{3}{13}$$

(v) Let E be the event of getting a number card. the number cards are 2,3,4,5,6,7,8,9,10.

$$\text{n(E)} = 4 \text{ x } 9 = 36, \text{ P(E)} = \frac{\text{n(E)}}{\text{n(s)}}$$

$$\text{P(E)} = \frac{36}{52} = \frac{9}{13}$$

Example 8.24 :

A bag contains 6 green balls, some black and red balls. Number of black ball is as twice as the number of red balls. Probability of getting green ball is thrice, the probability of getting a red ball. Find

(i) number of black Balls (ii) Total number of balls

Solution:

Number of green balls = n(G) =6
let number of red balls= n(R) = x
number of black balls = n(B) = $2x$
Total number of balls = n(S) = $6 + x + 2x$
$= 6 + 3x$

Given,

$$\text{P(G)} = 3 \text{ x P(R)}$$

$$\frac{6}{6+3x} = 3 \text{ x } \frac{x}{6+3x}$$

$$3x = 6$$

$$x = \frac{6}{3}, \quad \rightarrow \quad x = 2.$$

(i) number of black balls =2 x 2 = 4
(ii) Total number of balls = 6 +(3 x 2) = 12

Practice Exercise 8.3 :6

Exercise 8.3

At a fete, cards bearing numbers 1 to 1000, one number on one card are put in a box. Each player selects one card at random and that card is not replaced. If the selected card has a perfect square number greater than 500, the player wins a prize. What is the probability that,

(i) the first player wins a prize.

(ii) the second player Wins a prize, if the first has won?

Solution:

$S = \{1,2,3, \dots\dots\dots\dots..1000\}$

$n(S) = 1000$

(i) Let A be the event of selecting a card that is perfect square greater than 500.

$A = \{ 23^2, 24^2, 25^2, 26^2, 27^2, 28^2, 29^2, 30^2, 31^2\}$

$= \{529,576,625,676,729,784,841,900,961\}$

$n(A) = 9 \quad \rightarrow \quad P(A) = \frac{n(A)}{n(s)} = \frac{9}{1000}$

(ii) Probability of the second player winning a prize, if the first has won.

Let B be the event of second player winning a prize.

Since the picked card in (i) is not replaced,

$n(B) = 8$

$n(S) = 1000 - 1 = 999$

$P(B) = \frac{n(B)}{n(s)} = \frac{8}{999}$

---

2. Two unbiased dice are rolled once. Find the probability of getting (i) a doublet (equal numbers on both dice)
(ii) the product as a prime number
(iii) the sum as a prime number (iv) the sum as 1

Solution :

$$S = \left\{ \begin{matrix} (1,1)(1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,2), (2,3)(2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{matrix} \right\}$$

$n(S) = 36$

(i) let A be the event of getting a doublet.

$A = \{(1,1)(2,2), (3,3), (4,4), (5,5), (6,6)\}$

$n(A) = 6$

$P(A) = \frac{n(A)}{n(s)} = \frac{6}{36} = \frac{1}{6}$

(ii) let B be the event of getting the product as prime number.

$B = \{(1,2)(1,3), (1,5), (2,1), (3,1), (5,1)\}$

$n(B) = 6, \ P(B) = \frac{n(B)}{n(s)} = \frac{6}{36} = \frac{1}{6}$

(iii) let C be the event of getting the sum as a prime number.

$$C = \left\{ \begin{matrix} (1,1)(1,2), (1,4), (1,6), (2,1), (2,3) \\ (2,5)(3,2), (3,4), (4,1), (4,3), (5,2) \\ (5,6), (6,1), (6,5) \end{matrix} \right\}$$

$n(C) = 15$

$P(C) = \frac{n(C)}{n(s)} = \frac{15}{36} = \frac{5}{12}$

(iv) let D be the event of getting the sum as 1

$n(D) = 0 \quad \rightarrow \quad P(D) = \frac{n(D)}{n(s)} = \frac{0}{36} = 0$

Practice example 8.19, 8.25

---

3. Three fair coins are tossed together. Find the probability of getting (i) all heads (ii) atleast one tail
(iii) atmost one head (iv) atmost two tails.

Solution:

S = { HHH,HHT,HTT,TTT,TTH,THH,THT, HTH}

$n(S) = 8$

(i) Let A be the event of getting all heads.

A = {HHH}

$n(A) = 1$

$P(A) = \frac{n(A)}{n(s)} = \frac{1}{8}$

(ii) let B be the event of getting atleast one tail.

B = {HHT,HTT,TTT,TTH,THH,THT,HTH}

$n(B) = 7$

$P(B) = \frac{n(B)}{n(s)} = \frac{7}{8}$

(iii) Let C be the event of getting atmost one head.

C={HTT,THT,TTH,TTT}

$n(C) = 4, \ P(C) = \frac{n(C)}{n(s)}$

$= \frac{4}{8} = \frac{1}{2}$

(iv) Let D be the event of getting atmost two tails

D = {HHH,HHT,HTH,HTT,THH,THT,TTH}

$n(D) = 7, \ P(D) = \frac{n(D)}{n(s)} = \frac{7}{8}.$

---

4. A bag contains 5 red balls 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is (i) white
(ii) black or red (iii) not white (iv) neither white nor black.

Solution: Total=5red+6 white+7 green+8 black

$n(S) = 26$

(i) Let A be the event of getting a white ball

$n(A) = 6 \ , \quad P(A) = \frac{n(A)}{n(s)} = \frac{6}{26} = \frac{3}{13}$

(ii) let B be the event of getting black or red ball.

$n(B) = 8+5 = 13$

$P(B) = \frac{n(B)}{n(s)} = \frac{13}{26} = \frac{1}{2}$

(iii) A is the event of getting white ball

$\bar{A}$ is the event of not getting white ball

$P(A) = \frac{3}{13}$

$P(\bar{A}) = 1 - P(A) = 1 - \frac{3}{13}$

$\frac{13-3}{13} = \frac{10}{13}$

(iv) let C be the event of getting neither white nor black

C= 5 Red + 7 green

$n(C) = 5+7 = 12$

$P(C) = \frac{n(C)}{n(s)} \quad \rightarrow \quad = \frac{22}{26} = \frac{6}{13}$

5. In a box there are 20 non-defective and some defective bulbs. If the probability that a bulb selected at random from the box found to be defective is $\frac{3}{8}$ then, find the number of defective bulbs.

Solution:

Number of non- defective bulbs = 20

Let the number of defective bulbs = x

Total number of bulbs = Defective bulbs+ non defective Bulbs

$= x + 20$

Let A be the event of getting defective bulbs.

$$P(A) = \frac{x}{x+20} = \frac{3}{8}$$

$$8x = 3(x + 20)$$

$$8x = 3x + 60$$

$$8x - 3x = 60$$

$$5x = 60$$

$$x = \frac{60}{5}$$

$$x = 12$$

∴ Number of defective bulbs = 12

---

6. The king and Queen of diamonds, queen and jack of hearts, jack and king of spades are removed from a deck of 52 playing cards and then well shuffled. Now one card is drawn at random from the remaining cards. Determine the probability that the card is (i) a clavor (ii) a queen of red card. (iii) a king of black card.

Solution:

Number of cards removed= 2+2+2 =6

n(S)=52-6 = 46

(i) Let A be the event of getting a clavor.

n(A) =13

$P(A) = \frac{n(A)}{n(s)} = \frac{13}{46}$

Diamond (2)
K Q
Heart
Q J
Spade
J K

(ii) let B be the event of getting a queen of red card.

n(B) = 0

$P(B) = \frac{0}{46} = 0$

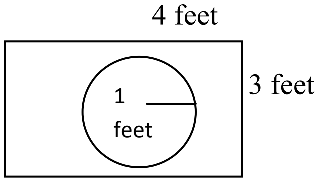
(iii) Let C be the event of getting a king of black card.

n(C) = 1

$P(C) = \frac{n(C)}{n(s)} = \frac{1}{46}$

---

7. Some boys are playing a game in which the stone thrown by them landing in a circular region (given in the figure) is considered as win and landing other than the circular region is considered as loss. What is the probability to win the game.($\pi = 3.14$)



Solution:

Area of rectangle = l x b = 4 x 3

n(S)= 12 square feet

Area of circle = πr^2 sq. units

Let A be the event of winning the game

n(A) = $\pi(1)^2$ square feet.

= π square feet.

$$P(A) = \frac{n(A)}{n(s)} = \frac{\pi}{12} = \frac{3.14}{12} x \frac{100}{100}$$

$$= \frac{314}{1200} = \frac{157}{600}.$$

---

8. Two customers Priya and Amuthan are visiting a particular shop in the same week (Monday to Saturday). Each is equally likely to visit the shop on any one day as on another day. What is the probability that both will visit the shop on (i) the same day (ii) different days (iii) consecutive days?

Solution:

S = { (Mon,Mon),(Mon,Tue),(Mon,Wed),(Mon,Thur), (Mon,Fri),(Mon,Sat), (Tue,Mon),(Tue,Tue), (Tue,Wed), (Tue,Thur),(Tue,Fri),(Tue,Sat), (Wed,Mon),(Wed,Tue),(Wed,Wed),(Wed,Thur), (Wed,Fri), (Wed,Sat), (Thur,Mon),(Thur,Tue), (Thur,Wed), (Thur,Thur), (Thur,Fri), (Thur,Sat), (Fri,Mon), (Fri,Tue), (Fri,Wed), (Fri,Thur), (Fri,Fri), (Fri,Sat), (Sat,Mon), (Sat,Tue),(Sat,Wed), (Sat,Thur), (Sat,Fri), (Sat, Sat)}

n(S)=36

(i) Let A be the event that both will visit the shop on the same day.

A ={(Mon,Mon),(Tue,Tue), (Wed,Wed),(Thur,Thur) (Fri,Fri),(Sat,Sat)}

n(A)=6

$P(A)=\frac{n(A)}{n(s)} = \frac{6}{36} = \frac{1}{6}$

(ii) Let $\bar{A}$ be the event that both will visit the shop in different days.

$P(\bar{A})$ = 1- P(A)

$= 1 - \frac{1}{6} = \frac{6-1}{6} = \frac{5}{6}$

(iii) Let B be the event that both will visit the shop in consecutive days.

B={(Mon,Tue),(Tue, Wed),(Wed,Thur),(Thur,Fri), (Fri,Sat),(Tue,Mon), (Wed,Tue),(Thur,Wed) (Fri,Thur),(Sat,Fri)}

n(B) = 10 → $P(B)=\frac{n(B)}{n(S)} = \frac{10}{36} = \frac{5}{18}$

---

9. In a game , the entry fee is Rs. 150. The game consists of tossing a coin 3 times. Dhana bought a ticket for entry. If one or two heads show, she gets her entry fee back . If she throws 3 heads, she receives double the entry fees. Otherwise she will lose. Find the probability that she (i) Gets double entry fee (ii) Just gets her entry fee (iii) Loses the entry fee

Solution:

S ={ HHH,HHT,HTT,TTT,TTH,THH, HTH,THT}

n(S) = 8

(i) Let A be the event of getting double entry fee

A = {HHH}

n(A) =1

$P(A)=\frac{n(A)}{n(s)} = \frac{1}{8}$

(ii) Let B be the event of getting entry fee back

B={ HHT,HTH,HTT,THH,THT,TTH}

n(B)=6

$P(B) = \frac{n(B)}{n(S)} = \frac{6}{8} = \frac{3}{4}$

(iii) Let C be the event of losing the entry fee

C ={TTT}

n(C)=1

$P(C) = \frac{n(C)}{n(S)} = \frac{1}{8}$

Examples & Exercise 8.4

Example 8.30:

A card is drawn from a pack of 52 cards Find the probability of getting a king or a heart or a red card.

Solution:

n(S) =52

Let A be the event of getting a King card

n(A) = 4

$P(A) = \frac{n(A)}{n(s)} = \frac{4}{52}$

Let B be the event of getting a heart card

n(B) = 13

$P(B) = \frac{n(B)}{n(s)} = \frac{13}{52}$

Let C be the event of getting a red card

n(C) = 26

$P(C) = \frac{n(C)}{n(s)} = \frac{26}{52}$

$P(A\cap B) = \frac{1}{52}$, $P(B\cap C) = \frac{13}{52}$

$P(A\cap C) = \frac{2}{52}$

$P(A\cap B\cap C) = \frac{1}{52}$

$$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) + P(A\cap B\cap C).$$

$$= \frac{4}{52} + \frac{13}{52} + \frac{26}{52} - \frac{1}{52} - \frac{13}{52} - \frac{2}{52} + \frac{1}{52}$$

$$= \frac{28}{52} = \frac{7}{13}$$

Practice Exercise : 8.4: 7

Example 8.31:

In a class of 50 students, 28 opted for NCC, 30 opted for NSS and 18 opted both NCC and NSS. One of the student is selected at random. Find the probability that

(i) The student opted for NCC but not NSS

(ii) The student opted for NSS but not NCC

(iii) The student opted for exactly one of them.

Solution:

n(S)=50

Let A and B be the events opted for NCC and NSS

Respectively.

n(A)=28, n(B)=30, $n(A\cap B) = 18$

$P(A) = \frac{n(A)}{n(s)} = \frac{28}{50}$

$P(B) = \frac{n(B)}{n(s)} = \frac{30}{50}$

$P(A\cap B) = \frac{n(A\cap B)}{n(s)} = \frac{18}{50}$

(i) $P(A\cap \bar{B}) = P(A) - P(A\cap B)$

$= \frac{28}{50} - \frac{18}{50} = \frac{10}{50} = \frac{1}{5}$

(ii) $P(\bar{A}\cap B) = P(B) - P(A\cap B)$

$= \frac{30}{50} - \frac{18}{50} = \frac{12}{50} = \frac{6}{25}$

(iii) $P[(A\cap \bar{B})\cup(\bar{A}\cap B)]$

$= P(A\cap \bar{B}) + P(\bar{A}\cap B)]$

$= \frac{1}{5} + \frac{6}{25} = \frac{5+6}{25} = \frac{11}{25}$

Example 8.32: A and B are two candidates seeking admission to IIT. The probability that A getting selected is 0.5 and the probability that both A and B getting selected is 0.3. Prove that the probability of B being selected is atmost 0.8

Solution:

$P(A)=0.5$, $P(A\cap B)=0.3$

$P(A\cup B) \leq 1$

$P(A)+P(B)-P(A\cap B) \leq 1$

$0.5+P(B)-0.3 \leq 1$

$0.2 + P(B) \leq 1$

$P(B) \leq 1 - 0.2$

$P(B) \leq 0.8$

Probability of B getting selected is atmost 0.8

Exercise 8.4:

Two dice are rolled once. Find the probability of getting an even number on the first die or a total of face sum 8.

Solution:

S={(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),
(3,1),(3,2),(,3,3),(3,4),(3,5),(3,6),
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

n(S)=36

Let A be the event of getting an even number on the first die.

A ={ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6),
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

n (A) = 18

$P(A) = \frac{n(A)}{n(s)} = \frac{18}{36} = \frac{1}{2}$

Let B be the event of getting a total of face sum 8.

B={(2,6),(3,5),(4,4),(5,3),(6,2)}

n (B)= 5

$P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$

(A∩B)= { (2,6), (4,4), (6,2)}

n(A∩B)=3

$P(A\cap B) = \frac{n(A\cap B)}{n(S)} = \frac{3}{36}$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= \frac{18}{36} + \frac{5}{36} - \frac{3}{36} = \frac{20}{36} = \frac{5}{9}$

2. A box contains cards numbered 3,5,7,9,…. 35,37. A card is drawn at random from the box. Find the probability that the drawn cards have either multiples of 7 or a prime number.

Solution:

S={3,5,7,9,11,13,15,17,19,21, 23, 25, 27,29,31,33,35,37}

n(S) = 18

Let A be the event of selecting a number which is multiple of 7.

A={7,21,35}

n(A)=3

$P(A)=\frac{n(A)}{n(s)}=\frac{3}{18}$

Let B be the event of selecting a prime number

B={3,5,7,11,13,17,19,23,29,31,37}

n(B)=11

$P(B)=\frac{n(B)}{n(s)}=\frac{11}{18}$

A∩B ={7} $n(A\cap B) = 1$

$P(A\cap B)=\frac{n(A\cap B)}{n(s)}=\frac{1}{18}$

P(A∪B)= P(A)+P(B)-P(A∩B)

$=\frac{3}{18}+\frac{11}{18}-\frac{1}{18}$

$=\frac{3+11-1}{18}=\frac{13}{18}$

---

3. Three unbiased coins are tossed once. Find the probability of getting atmost 2 tails or atleast 2 heads

Solution:

S = {HHH, HHT, HTT, TTT, TTH, THH ,HTH, THT}

n(S)=8

Let A be the event of getting atmost 2 tails.

A={HHH,HHT,HTT,TTH,THH, HTH,THT}

n (A)=7

$P(A)=\frac{n(A)}{n(s)}=\frac{7}{8}$

Let B be the event of getting atleast 2 heads.

B={HHH,HHT,THH,HTH}

n (B)=4

$P(B)=\frac{n(B)}{n(s)}=\frac{4}{8}$

A∩B = { HHH,HHT,THH,HTH}

$n(A\cap B) = 4$

$P(A\cap B)=\frac{n(A\cap B)}{n(s)}=\frac{4}{8}$

P(A∪B)= P(A)+P(B)-P(A∩B)

$=\frac{7}{8}+\frac{4}{8}-\frac{4}{8}=\frac{7}{8}$

---

4. The probability that a person will get an electrification contract is $\frac{3}{5}$ and the probability that he will not get plumbing contract is $\frac{5}{8}$. The probability of getting atleast one contract is $\frac{5}{7}$. What is the probability that he will get both ?

Sol:-Let A be the event of getting an electrification contract.

$P(A)=\frac{3}{5}$

Let B be the event of getting plumbing contract.

$P(\bar{B})=\frac{5}{8}$

$P(B)=1-P(\bar{B})$

$1-\frac{5}{8}=\frac{8-5}{8}$

$P(B)=\frac{3}{8}P(A\cup B)=\frac{5}{7}$

P(A∪B)= P(A)+P(B)-P(A∩B)

$\frac{5}{7}=\frac{3}{5}+\frac{3}{8}-P(A\cap B)$

$P(A\cap B)=\frac{3}{5}+\frac{3}{8}-\frac{5}{7}=\frac{168+105-200}{280}=\frac{73}{280}$

---

5. In a town of 8000 people 1300 are over 50 years and 3000 are females It is known that 30% of the females are over 50 years. What is the probability that a chosen individual from the town is either a female or over 50 years?

Solution: n (S) =8000

Let A be the event of selecting an individual over 50 years.

n(A)=1300

$P(A)=\frac{n(A)}{n(s)}=\frac{1300}{8000}=\frac{13}{80}$

Let B be the event of selecting a female

n(B) = 3000

$P(B)=\frac{n(B)}{n(s)}=\frac{3000}{8000}=\frac{3}{8}$

n (A∩B)=30% of 3000

$=\frac{30}{100}\times 3000 = 900$

$P(A\cap B)=\frac{n(A\cap B)}{n(s)}=\frac{900}{8000}=\frac{9}{80}$

P(A∪B)= P(A)+P(B)-P(A∩B)

$=\frac{13}{80}+\frac{30}{80}-\frac{9}{80}$

$=\frac{13+30-9}{80}=\frac{43-9}{80}=\frac{34}{80}=\frac{17}{40}$

---

6. A coin is tossed thrice. Find the probability of getting exactly two heads or atleast one tail or two consecutive heads.

Solution: S ={HHH, HHT, HTT, TTT,
TTH, THH, THT, HTH}

n (S)=8

Let A be the event of getting exactly two heads

A={HHT,HTH,THH}

n (A) =3

$P(A)=\frac{n(A)}{n(s)}=\frac{3}{8}$

Let B be the event of getting atleast one tail

B={HHT,HTT,TTT,TTH,THH,THT,HTH}

n(B)= 7

$P(B)=\frac{n(B)}{n(s)}=\frac{7}{8}$

Let C be the event of getting two consecutive heads.

C={ HHT,THH,HHH} n (C)= 3

$P(C) = \frac{n(C)}{n(s)} = \frac{3}{8}$

$A \cap B = \{ HHT, HTH, THH\}$

$n(A \cap B) = 3$

$P(A \cap B) = \frac{n(A \cap B)}{n(s)} = \frac{3}{8}$

$B \cap C = \{ HHT, THH\}$

$n(B \cap C) = 2$

$P(B \cap C) = \frac{n(B \cap C)}{n(s)} = \frac{2}{8}$

$A \cap C = \{ HHT, THH\}$

$n(A \cap C) = 2$

$P(A \cap C) = \frac{n(A \cap C)}{n(s)} = \frac{2}{8}$

$A \cap B \cap C = \{ HHT, THH\}$

$n(A \cap B \cap C) = 2$

$P(A \cap B \cap C) = \frac{n(A \cap B \cap C)}{n(s)} = \frac{2}{8}$

$P(A \cup B \cup C) = P(A)+P(B)+P(C) - P(A \cap B) - P(B \cap C)$
$- P(A \cap C) + P(A \cap B \cap C)$

$= \frac{3}{8} + \frac{7}{8} + \frac{3}{8} - \frac{3}{8} - \frac{2}{8} - \frac{2}{8} + \frac{2}{8}$

$= \frac{3+7+3-3-2-2+2}{8} = \frac{8}{8} = 1$

---

7. If A,B,C are any three events such that probability of B is twice as that of probability of A and probability of C is thrice as that of probability of A and if $P(A \cap B) = \frac{1}{6}$, $P(B \cap C) = \frac{1}{4}$, $P(A \cap C) = \frac{1}{8}$, $P(A \cup B \cup C) = \frac{9}{10}$, $P(A \cap B \cap C) = \frac{1}{15}$, then find P(A), P(B) and P(C)?

Solution:

$P(A \cap B) = \frac{1}{6}$ $P(B \cap C) = \frac{1}{4}$

$P(A \cap C) = \frac{1}{8}$, $P(A \cup B \cup C) = \frac{9}{10}$

$P(A \cap B \cap C) = \frac{1}{15}$

$P(B) = 2\,P(A)$

$P(C) = 3\,P(A)$

$P(A \cup B \cup C) = P(A)+P(B)+P(C) - P(A \cap B) - P(B \cap C)$
$- P(A \cap C) + P(A \cap B \cap C)$

$\frac{9}{10} = P(A)+2P(A)+P(A)$

$= \frac{1}{6} - \frac{1}{4} - \frac{1}{8} + \frac{1}{15}$

$\frac{9}{10} = 6P(A) - \frac{1}{6} - \frac{1}{4} - \frac{1}{8} + \frac{1}{15}$

$6P(A) = \frac{9}{10} + \frac{1}{6} + \frac{1}{4} + \frac{1}{8} - \frac{1}{15}$

$6P(A) = \frac{108+20+30+15-8}{120}$

$6P(A) = \frac{165}{120}$

$P(A) = \frac{165}{120 \times 6} = \frac{11}{48}$

$P(B) = 2 \times \frac{11}{48} = = \frac{22}{48} = \frac{11}{24}$

$P(C) = 3 \times \frac{11}{48} = \frac{33}{48} = \frac{11}{16}$

Practice Exercise 8.4 : 14.