CHAPTER .11 PROBABILITY DISTRIBUTION
(2 MARKS, 3 MARKS)

2 - MARKS

EXERCISE 11.1(1)

Suppose X is the number of tails occurred when three fair coins are tossed once simultaneously. Find the values of the random variable X and number of points in its inverse images.

Solution: Number of coins = 3 $n(s) = 2^3 = 8$

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

Let X be the discrete random variable denoting no of tails

X = { 0, 1, 2 , 3}

Values of X	0	1	2	3	total
No. of elements in inverse images	1	3	3	1	8

EXERCISE 11.2 -

1. Three fair coins are tossed simultaneously. Find the probability mass function for number of heads occurred.

SOLUTION:

When three coins are tossed, the sample space is

S = { HHH, HHT, HTH, HTT, THH, THT, TTH, TTT }

' X ' is the random variable denotes the number of heads.

∴'X ' can taKe the values of 0,1,2 and 3

$P(X = 0) = P(\text{No heads}) = \frac{1}{8}$

$P(X = 1) = P(1 \text{ head}) = \frac{3}{8}$

$P(X = 2) = P(2 \text{ heads}) = \frac{3}{8}$

$P(X = 3) = P(3 \text{ heads}) = \frac{1}{8}$

The probability mass function is $f(x) = \begin{cases} 1/8 & \text{for } x = 0.3 \\ 3/8 & \text{for } x = 1,2 \end{cases}$

EXERCISE 11.3

1. The probability density function of X is given by $f(x) = \begin{cases} kxe^{-2x} & \text{for } x > 0 \\ 0 & \text{for } x < 0 \end{cases}$ **Find the value of k .**

SOLUTION: Since f(X) is a pdf $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_0^{\infty} k\,x\,e^{-2x}\,dx = 1$ (since x > 0)

$k\frac{1}{2^2} = 1 \Rightarrow k\,\frac{1}{4} = 1 \Rightarrow k = 4$

EXERCISE 11.4

5. A commuter train arrives punctually at a station every half hour. Each morning, a student leaves his house to the train station. Let X denote the amount of time, in minutes, that the student waits for the train from the time he reaches the train station. It is known that the pdf of X is $f(x) = \begin{cases} \frac{1}{30} & 0 < x < 30 \\ 0 & \text{elsewhere} \end{cases}$ **Obtain and interpret the expected value of the random variable X .**

SOLUTION: $f(x) = \begin{cases} \frac{1}{30} & 0 < x < 30 \\ 0 & \text{elsewhere} \end{cases}$

$E(x) = \int_{-\infty}^{\infty} xf(x)\,dx$

$E(X) = \int_0^{30} x\frac{1}{30}\,dx$

$= \frac{1}{30}\left[\frac{x^2}{2}\right]_0^{30} = \frac{1}{30}\left[\frac{30^2}{2} - 0\right] = \frac{1}{30}\left[\frac{900}{2}\right] = 15$

6. The time to failure in thousands of hours of an electronic equipment used in a manufactured computer has the density function $f(x) = \begin{cases} 3e^{-3x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$ **Find the expected life of this electronic equipment.**

SOLUTION: $f(x) = \begin{cases} 3e^{-3x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$

$E(x) = \int_{-\infty}^{\infty} xf(x)\,dx$ $\quad \left[\int_0^{\infty} x^n e^{-ax}dx = \frac{n!}{a^{n+1}}\right]$

$= 3\int_0^{\infty} xe^{-3x}dx = 3\frac{1!}{3^{1+1}} = 3\frac{1!}{3^2} = = \frac{1}{3}$ $\quad$ [n = 1, a = 3]

EXERCISE 11.5

Examples - 11.19 (each)

1. Compute P(X = k) for the binomial distribution, B(n , p) where

(i) $n = 6,\ p = \frac{1}{3},\ k = 3$

SOLUTION : $n = 6, p = \frac{1}{3}$; $k = 3$

$q = 1 - p = 1 - \frac{1}{3} = \frac{3-1}{3} = \frac{2}{3}$

$P(X = x) = nC_x\, p^x q^{n-x}$ $\quad$ [n = 4 , x = 3, n- x = 4-3 = 1]

$P(X = 3) = 4C_3 (\frac{1}{3})^3 (\frac{2}{3})^1 = 4(\frac{1}{27})(\frac{2}{3}) = \frac{8}{81}$

(ii) $n = 10,\ p = \frac{1}{5},\ k = 4$

$n = 10, p = \frac{1}{5}$; $k = 4$

$q = 1 - p = 1 - \frac{1}{5} = \frac{5-1}{5} = \frac{4}{5}$

$P(X = x) = nC_x\, p^x q^{n-x}$ $\quad$ [n = 10 , x = 4, n- x = 10-4 = 6]

$P(X = 4) = 10C_4 (\frac{1}{5})^4 (\frac{4}{5})^6$

(iii) $n = 9,\ p = \frac{1}{2},\ k = 7$

$n = 9, p = \frac{1}{2}$; $k = 7$

$q = 1 - p = 1 - \frac{1}{2} = \frac{2-1}{2} = \frac{1}{2}$

$P(X = x) = nC_x\, p^x q^{n-x}$ $\quad$ [n = 9 , x = 7, n- x = 9 -7 = 2]

$P(X = 7) = 9C_7 (\frac{1}{2})^7 (\frac{1}{2})^2$

3 Using binomial distribution find the mean and variance of X for the following experiments

(i) A fair coin is tossed 100 times, and X denote the number of heads.

SOLUTION: $n = 100$ $\quad p = \frac{1}{2}$ $\quad q = \frac{1}{2}$

Mean = n p = $100(\frac{1}{2}) = 50$

Variance = npq = $100(\frac{1}{2})(\frac{1}{2}) = 25$

(ii) A fair die is tossed 240 times, and X denote the number of times that four appeared.

S = { 1, 2, 3, 4, 5, 6 } $\quad n = 240$ $\quad p = \frac{1}{6}$ $\quad q = 1 - \frac{1}{6} = \frac{5}{6}$

Mean = n p = $240(\frac{1}{6}) = 40$

Variance = npq = $240(\frac{1}{6})(\frac{5}{6}) = \frac{100}{3}$

4.The probability that a certain kind of component will survive a electrical test is $\frac{3}{4}$**. Find the probability that exactly 3 of the 5 components tested survive.**

SOLUTION:

$n = 5$; $\quad p = \frac{3}{4}$; $q = 1 - \frac{3}{4} = \frac{4-3}{4} = \frac{1}{4}$; $x = 3$; $5C_3 = \frac{5.4.3}{3.2.1} = 10$

$P(X = x) = nC_x\, p^x q^{n-x}$ $\quad$ [n = 5 , x = 3, n- x = 5 - 3 = 2]

$P(X = 3) = 5C_3 (\frac{3}{4})^3 (\frac{1}{4})^2 = 10\frac{3^3}{4^5} = \frac{270}{1024}$

3 - MARKS

EXERCISE 11.1

(2) In a pack of 52 playing cards, two cards are drawn at random simultaneously. If the number of black cards drawn is a random variable, find the values of the random variable and number of points in its inverse images.

SOLUTION: No of cards = 52; No of cards drawn = 2;

Total number of points = $52C_2 = \frac{52 \times 51}{2 \times 1} = 1326$

X be the discrete random variable denoting number of black cards

x = { 0, 1, 2}

$X(0) = X(2 \text{ Red cards}) = 26C_2 = \frac{26 \times 25}{2 \times 1} = 325$

$X(1) = X(1 \text{ Red, } 1 \text{ Black}) = 26C_1 \times 26C_1 = 26 \times 26 = 676$

$X(2) = X(2 \text{ Black cards}) = 26C_2 = \frac{26 \times 25}{2 \times 1} = 325$

Values of X	0	1	2	total
No. of elements in inverse images	325	676	325	1326

EXERCISE 11.2

2. A six sided die is marked '1' on one face, '3' on two of its faces, and '5' on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find

(i) the probability mass function

(ii) the cumulative distribution function

(iii) $P(4 \le X < 10)$ (iv) $P(X \ge 6)$

Solution:

+	1	3	3	5	5	5
1	2	4	4	6	6	6
3	4	6	6	8	8	8
3	4	6	6	8	8	8
5	6	8	8	10	10	10
5	6	8	8	10	10	10
5	6	8	8	10	10	10

Given that die is marked ' 1 ' on one face, ' 3 ' on two of its faces and ' 5 ' on remaining three faces. i.e., {1, 3,3,5,5,5} in a single die.

$P(X = 2) = \frac{1}{36}$; $P(X = 4) = \frac{4}{36}$; $P(X = 6) = \frac{10}{36}$;

$P(X = 8) = \frac{12}{36}$; $P(X = 10) = \frac{9}{36}$

(i) Probability mass function:

x	2	4	6	8	10	Total
f(x)	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$	1

(ii) The Cumulative distribution function:

$$F(x) = \begin{cases} 0 & \text{for } x < 2 \\ 1/36 & \text{for } x \le 2 \\ 5/36 & \text{for } x \le 4 \\ 15/36 & \text{for } x \le 6 \\ 27/36 & \text{for } x \le 8 \\ 1 & \text{for } x \le 10 \end{cases}$$

(iii)$P(4 \le X < 10) = P(X = 4) + P(X = 6) + P(X = 8)$

$= \frac{4}{36} + \frac{10}{36} + \frac{12}{36} = \frac{26}{36} = \frac{13}{18}$

(iv) $P(X \ge 6) = P(X = 6) + P(X = 8) + P(X = 10)$

$= \frac{10}{36} + \frac{12}{36} + \frac{9}{36} = \frac{31}{36}$

Exercise 11. 4

Question 1.

For the random vaniable X with the given probability mass function as below, find the mean and variance

(i) $\mathbf{f(x) = \begin{cases} \frac{1}{10} & x = 2, 5 \\ \frac{1}{5} & x = 0, 1, 3, 4 \end{cases}}$

Solution:

(i) Given probability mass function

$$f(x) = \begin{cases} \frac{1}{10} & x = 2,5 \\ \frac{1}{5} & x = 0,1,3,4 \end{cases}$$

x	0	1	2	3	4	5
f(x)	1/5	1/5	1/10	1/5	1/5	1/10

Mean $E(X) = \Sigma x f(x) = 0 + \frac{1}{5} + \frac{1}{5} + \frac{3}{5} + \frac{4}{5} + \frac{1}{2}$

$= \frac{9}{5} + \frac{1}{2} = \frac{18+5}{10} = \frac{23}{10} = 2.3$

$E(X^2) = \Sigma x^2 f(x)$

$= 0 + 1^2(\frac{1}{5}) + 2^2(\frac{1}{10}) + 3^2(\frac{1}{5}) + 4^2(\frac{1}{5}) + 5^2(\frac{1}{10})$

$= 0 + \frac{1}{5} + \frac{4}{10} + \frac{9}{5} + \frac{16}{5} + \frac{25}{10} = \frac{1}{5} + \frac{2}{5} + \frac{9}{5} + \frac{16}{5} + \frac{5}{2} = \frac{28}{5} + \frac{5}{2}$

$= \frac{56+25}{10} = \frac{81}{10}$

Variance $Var(X) = E(X^2) - [E(X)]^2$

$= \frac{81}{10} - \frac{529}{100} = \frac{810-529}{100} = \frac{281}{100} = 2.81$

(ii) Given probability mass function

$\mathbf{f(x) = \begin{cases} \frac{4-x}{6}, x = 1, 2, 3 \end{cases}}$

x	1	2	3
f(x)	1/2	1/3	1/6

Mean $E(X) = \Sigma x f(x) = \frac{1}{2} + \frac{2}{3} + \frac{1}{2} = 1 + \frac{2}{3} = \frac{5}{3} = 1.67$

$E(X^2) = \Sigma x^2 f(x) = 1^2(\frac{1}{2}) + 2^2(\frac{1}{3}) + 3^2(\frac{1}{6})$

$= \frac{1}{2} + \frac{4}{3} + \frac{9}{6} = \frac{3+8+9}{6} = \frac{20}{6} = \frac{10}{3}$

Variance $Var(X) = E(X^2) - [E(X)]^2$

$= \frac{10}{3} - \left(\frac{5}{3}\right)^2 = \frac{10}{3} - \frac{25}{9} = \frac{30-25}{9} = \frac{5}{9} = 0.56$

(iii) Given probability mass function

$\mathbf{f(x) = \begin{cases} 2(x-1), & 1 < x < 2 \\ 0, & \text{otherwise} \end{cases}}$

Here ' X ' is a continuous random variable

Mean $E(X) = \int_{-\infty}^{\infty} x f(x) dx$

$= 2\int_1^2 x(x-1)dx = 2\int_1^2 (x^2 - x)dx = 2\left[\frac{x^3}{3} - \frac{x^2}{2}\right]_1^2$

$= 2\left[\left(\frac{8}{3} - 2\right) - \left(\frac{1}{3} - \frac{1}{2}\right)\right] = 2\left[\frac{2}{3} + \frac{1}{6}\right] = 2\left(\frac{5}{6}\right) = \frac{5}{3}$

$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x) dx$

$= 2\int_1^2 x^2(x-1)dx = 2\int_1^2 (x^3 - x^2)dx$

$= 2\left[\frac{x^4}{4} - \frac{x^3}{3}\right]^2 = \frac{17}{6}$

Variance $Var(X) = E(X^2) - [E(X)]^2$

$= \frac{17}{6} - \left(\frac{5}{3}\right)^2 = \frac{17}{6} - \frac{25}{9} = \frac{51-50}{18} = \frac{1}{18}$

(iv) $f(x) = \begin{cases} \frac{1}{2}e^{-\frac{x}{2}} & \text{for } x>0 \\ 0 & \text{otherwise} \end{cases}$

Since ' X ' is a continuous random variable

Mean $E(X) = \int_{-\infty}^{\infty} xf(x)dx$ $\quad [\int_0^{\infty} x^n e^{-ax}dx = \frac{n!}{a^{n+1}}]$

$= \frac{1}{2}\int_0^{\infty} xe^{-\frac{x}{2}}dx = \frac{1}{2}\frac{1!}{(\frac{1}{2})^{1+1}}$ $\quad [n = 1, a = \frac{1}{2}]$

$= \frac{1}{2}(\frac{1}{\frac{1}{4}}) = \frac{4}{2} = 2$

$E(X^2) = \int_{-\infty}^{\infty} x^2 \cdot f(x)dx$

$= \frac{1}{2}\int_0^{\infty} x^2 \cdot e^{-x/2}dx$ $\quad [n = 2, a = \frac{1}{2}]$

$= \frac{1}{2}\frac{2!}{(\frac{1}{2})^{2+1}} = \frac{1}{2}(\frac{2}{\frac{1}{8}})$

$= \frac{16}{2} = 8$

Variance $Var(X) = E(X^2) - [E(X)]^2 = 8 - 4 = 4$

2.Two balls are drawn in succession without replacement from an urn containing four red balls and three black balls. Let X be the possible outcomes drawing red balls. Find the probability mass function and mean for X.

Solution:

Number of Red balls = 4; Number of Black balls = 3

Total number of balls = 7

Two balls are drawn without replacement

$n(s) = 7C_2$

X denote number of red balls $\quad X = \{0, 1, 2\}$

$P(X = 0) = P(0R, 2B) = \frac{3C_2}{7C_2} = \frac{\frac{3 \times 2}{2 \times 1}}{\frac{7 \times 6}{2 \times 1}} = \frac{3 \times 2}{7 \times 6} = \frac{1}{7}$

$P(X = 1) = P(1R, 1B) = \frac{4C_1\, 3C_1}{7C_2} = \frac{4 \times 3}{\frac{7 \times 6}{2 \times 1}} = \frac{4 \times 3 \times 2}{7 \times 6} = \frac{4}{7}$

$P(X = 2) = P(2R, 0B) = \frac{4C_2}{7C_2} = \frac{\frac{4 \times 3}{2 \times 1}}{\frac{7 \times 6}{2 \times 1}} = \frac{4 \times 3}{7 \times 6} = \frac{2}{7}$

∴ Probability mass function

x	0	1	2
f(x)	1/7	4/7	2/7

$E(X) = \Sigma xf(x) = 0(\frac{1}{7}) + 1(\frac{4}{7}) + 2(\frac{2}{7}) = \frac{8}{7} = 0 + \frac{4}{7} + \frac{4}{7} = \frac{8}{7}$

Question 3.

If μ and σ^2 are the mean and variance of the discrete random variable X, and $E(X + 3) = 10$ and $E(X + 3)^2 = 116$, find μ and σ^2.

Solution:

Mean = μ, $\quad$ Vaniance = σ^2

Given $E(X + 3) = 10$ and $E(X + 3)^2 = 116$

$E(X) + 3 = 10$ $\quad E(X^2 + 6X + 9) = 116$

$E(X) = 10 - 3$ $\quad E(X^2) + 6E(X) + 9 = 116$

$E(X) = 7$ $\quad E(X^2) + 6(7) + 9 = 116$

∴ Mean $\mu = E(X) = 7$ $\quad E(X^2) + 51 = 116$

$E(X^2) = 116 - 51 = 65$

Variance $Var(X) = E(X^2) - [E(X)]^2$

$65 - 49 = 16 = \sigma^2$ $\quad$ ∴ $\mu = 7$ and $\sigma^2 = 16$

4. Four fair coins are tossed once. Find the probability mass function, mean and variance for number of heads occurred.

SOLUTION:

n = 4, X - random variable denoting no.of heads

X= { 0, 1, 2, 3, 4 }

$P(x= 0) = 4C_0 (\frac{1}{2})^4 = 1.\frac{1}{16} = \frac{1}{16}$

$P(x= 1) = 4C_1 (\frac{1}{2})^4 = 4.\frac{1}{16} = \frac{4}{16}$

$P(x= 2) = 4C_2 (\frac{1}{2})^4 = 6.\frac{1}{16} = \frac{6}{16}$

$P(x= 3) = 4C_3 (\frac{1}{2})^4 = 4.\frac{1}{16} = \frac{4}{16}$

$P(x= 4) = 4C_4 (\frac{1}{2})^4 = 1.\frac{1}{16} = \frac{1}{16}$

Probabaility mass function

X	0	1	2	3	4
P(X=x)	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

$P = \frac{1}{2}$ and $q = \frac{1}{2}$

Mean = np = $4(\frac{1}{2}) = 2$ & Variance = npq = $4(\frac{1}{2})(\frac{1}{2}) = 1$

Question 7.

The probability density function of the random variable X is given by $f(x) = \begin{cases} 16xe^{-4x} & \text{for } x > 0 \\ 0 & \text{for } x \le 0 \end{cases}$ Find the mean and vaniance of X

Solution:

Given p.d.f. is $f(x) = \begin{cases} 16xe^{-4x} & \text{for } x>0 \\ 0 & \text{for } x\le 0 \end{cases}$

Mean $E(X) = \int_{-\infty}^{\infty} xf(x)dx$

$= 16\int_0^{-\infty} x^2e^{-4x}dx$ $\quad [\int_0^{\infty} x^n e^{-ax}dx = \frac{n!}{a^{n+1}}]$

$= 16 \times \frac{2!}{4^{2+1}} = 16 \times \frac{2 \times 1}{4^3} = 16 \times \frac{2}{64} = \frac{1}{2}$ $\quad [n = 2 ; a = 4]$

$E(X^2) = \int_{-\infty}^{\infty} x^2f(x)dx = 16\int_0^{\infty} x^3e^{-4x}dx$ $\quad [n = 3; a = 4]$

$= 16 \times \frac{3!}{4^{3+1}} = 16 \times \frac{3 \times 2 \times 1}{4^4} = 16 \times \frac{6}{256} = \frac{6}{16} = \frac{3}{8}$

Variance $Var(X) = E(X^2) - [E(X)]^2 = \frac{3}{8} - \frac{1}{4} = \frac{3-2}{8} = \frac{1}{8}$

Question 8.

A lottery with 600 tickets gives one prize of ₹200, four prizes of ₹100, and six prizes of ₹50. If the ticket costs is ₹2, find the expected winning amount of a ticket.

Solution: Given, total number of tickets = 600

One prize of Rs. 200; Four prizes of R. 100

Six prizes of Rs. 50

Let ' X ' be the random variable "denotes the winning amount" and it can take the values 200,100 and 50 .

$p(X= 200) = \frac{1}{600}$; $P(X = 100) = \frac{4}{600}$; $P(x = 50) = \frac{6}{600}$

∴ Probability mass function is

x	200	100	50
f(x)	1/600	4/600	6/600

∴ $E(X) = \Sigma xf(x) = \frac{200}{600} + \frac{400}{600} + \frac{300}{600} = \frac{900}{600} = 1.5$

Expected winning amount = Amount won − Cost of lottery

= 1.50 − 2.00 =− 0.50

ie., Loss of Rs. 0.50

EXERCISE 11.5: **2.**

The probability that Mr. Q hits a target at any trial is $\frac{1}{4}$.

Suppose he tries at the target 10 times. Find the probability that he hits the target

(i) exactly 4 times (ii) at least one time.

Solution:

Let ' p ' be the probability of hitting the trial

i.e., $p = \frac{1}{4}$, $\therefore q = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$

number of trials = $n = 10$

$P(X = x) = nC_x p^x q^{n-x}, x = 0,1,2, \ldots\ldots, n$

(i) exactly 4 times is

$P(X = 4) = 10C_4\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^{10-4}$

$= 10C_4\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^6$

(ii) atleast one time

$P(X \geq 1) = 1 - P(X < 1)$

$= 1 - P(X = 0)$

$= 1 - 10C_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^{10}$

$= 1 - \left(\frac{3}{4}\right)^{10}$

Question 5.

A retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 5% . The inspector of the retailer randomly picks 10 items from a shipment. What is the Probability that there will be

(i) at least one defective item (ii) exactly two defective items.

Solution:

Given $n = 10$

Probability of a defective item $= p = 5\% = \frac{5}{100}$

$\therefore q = 1 - p = 1 - \frac{5}{100} = \frac{95}{100}$

Let ' X ' be the random variable denotes the number of defective items.

$P(X = x) = nC_x p^x q^{n-x}, x = 0,1,2, \ldots\ldots, n$

(i) Probability that atleast one defective item will be there

$P(X \geq 1) = 1 - P(X < 1) = 1 - [P(X = 0)]$

$= 1 - \left[10C_0\left(\frac{5}{100}\right)^0\left(\frac{95}{100}\right)^{10-0}\right] = 1 - \left(\frac{95}{100}\right)^{10} = 1 - (0.95)^{10}$

(ii) Probability that exactly two defective item will be there

$P(X = 2) = 10C_2\left(\frac{5}{100}\right)^2\left(\frac{95}{100}\right)^8$

$= 10C_2(0.05)^2(0.95)^8$

Question 8.

If $X \sim B(n, p)$ such that $4P(X = 4) = P(x = 2)$ and $n = 6$. Find the distribution, mean and standard deviation.

Solution:

$n = 6.$

$P(X = x) = nC_x p^x q^{n-x}, x = 0,1,2, \ldots\ldots, n$

$4P(X = 4) = P(X = 2)$

$4(6C_4 p^4 q^2) = 6C_2 p^4 q^4 \quad [\therefore 6C_4 = 6C_2]$

$4p^2 = q^2$

$\Rightarrow 4p^2 = (1 - p)^2$

$4p^2 - p^2 + 2p - 1 = 0$

$3p^2 + 2p - 1 = 0$

$(3p - 1)(p + 1) = 0$

$\therefore p = \frac{1}{3}$; $p = -1$ is not possible.

If $p = \frac{1}{3}$ then $q = 1 - \frac{1}{3} = \frac{2}{3}$

Binomial Distribution is $B\left(6, \frac{1}{3}\right)$

Mean $np = 6 \times \frac{1}{3} = 2$

Standard deviation $= \sqrt{npq} = \sqrt{6 \times \frac{1}{3} \times \frac{2}{3}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

Question 9.

In a binomial distribution consisting of 5 independent trials, the probability of 1 and 2 successes are 0. 4096 and 0. 2048 respectively. Find the mean and variance of the distribution.

solution:

Number of trials $n = 5$

$P(X = x) = nC_x p^x q^{n-x}, x = 0,1,2, \ldots\ldots, n$

Given $P(X = 1) = 0.4096$ and $P(X = 2) = 0.2048$

$P(X = 1) = 0.4096 \Rightarrow 5C_1 p^1 q^4 = 5pq^4 = 0.4096$ ---(1)

$P(X = 2) = 0.2048 \Rightarrow 5C_2 p^2 q^3 = 10p^2 q^3 = 0.2048$ ---(2)

Dividing (1) by (2)

$\Rightarrow \frac{5pq^4}{10p^2q^3} = \frac{0.4096}{0.2048}$

$\frac{q}{2p} = 2 \Rightarrow q = 4p$

$\Rightarrow 1 - p = 4p \quad [\because q = 1 - p]$

$\Rightarrow 5p = 1 \Rightarrow p = \frac{1}{5}$

& $q = 1 - \frac{1}{5} = \frac{5-1}{5} = \frac{4}{5}$

Mean $= np = 5 \times \frac{1}{5} = 1$

Variance $= npq = 5 \times \frac{1}{5} \times \frac{4}{5} = \frac{4}{5}$

CHAPTER 12 - DISCRETE MATHEMATICS
(2 MARKS, 3 MARKS, 5 MARKS)

2 - MARKS

EXERCISE 12.1(1)

Determine whether ∗is a binary operation on the sets given below.

Solution:

(i) $a * b = a.|b|$ **on R**

Let a, b ∈ R, then |b| ∈ R, a.|b| ∈ R

⇒ ∗ is a binary operator on R

(ii) $a*b = \min(a,b)$ **on A = {1,2,3, 4,5}**

Let a, b ∈ A

$a*b = \min(a,b) = \begin{cases} a & \text{if } a \le b \\ b & \text{if } b \le a \end{cases}$

in either case a∗b ∈ A

⇒ ∗ is a binary operator on A

(iii) $(a*b) = a\sqrt{b}$ **is binary on R .**

Let a, b ∈ R; $\sqrt{b} \notin$ R

∴ $a\sqrt{b} \notin$ R

⇒ a∗b ∉ R

⇒ ∗ is not a binary operator on R

EXERCISE 12.1 (2).

On Z , define ⊗ by $(m \otimes n) = m^n + n^m$: **∀ m, n ∈ Z.**

Is ⊗ binary on Z ?

SOLUTION:

Let m,n ∈ Z, m > 0 and n < 0 then $m^n \notin Z$

⇒ $(m \otimes n) = m^n + n^m \notin Z$, ∴ ⊗ is not a binary on Z

EXERCISE 12.1 (3).

Let ∗ be defined on R by (a∗b) = a + b + ab − 7 .

Is ∗binary on R ? If so, find $3 * \left(\frac{-7}{15}\right)$

SOLUTION:

Let a, b ∈ R, then ab ∈ R

∴ a + b + ab – 7 ∈ R ⇒ (a∗b) = a + b + ab – 7 ∈ R

⇒ ∗ is a binary operator on R

$3 * \left(\frac{-7}{15}\right) = 3 + \left(\frac{-7}{15}\right) + 3\left(\frac{-7}{15}\right) - 7 = 3 - \frac{7}{15} - \frac{21}{15} - 7 = \frac{45-7-21-105}{15} = \frac{-88}{15}$

4. Let $A = \{a + \sqrt{5}\,b : a,b \in Z\}$ **. Check whether the usual multiplication is a binary operation on A .**

SOLUTION:

Let $a + \sqrt{5}b,\ c + \sqrt{5}d \in A$; a,b,c,d ∈ Z

$(a + \sqrt{5}b)(c + \sqrt{5}d) = ac + \sqrt{5}\,ad + \sqrt{5}\,bc + 5bd)$

$= (ac + 5bd) + \sqrt{5}(ad+bc) \in A$

[ac, bd, ad, bc ∈ Z and ac + 5bd, ad + bc ∈ Z]

∴ usual multiplication is a binary operator on A.

6. Fill in the following table so that the binary operation ∗on A = {a,b,c} is commutative.

∗	a	b	c
a	b		
b	c	b	a
c	a		c

SOLUTION:

a ∗ b = b ∗ a = c

a ∗ c = c ∗ a = a

c ∗ b = b ∗ c = a

∗	a	b	c
a	b	c	a
b	c	b	a
c	a	a	c

7. Consider the binary operation ∗ defined on the set A = {a,b,c,d} by the following table:

∗	a	b	c	d
A	a	c	b	d
B	d	a	b	c
C	c	d	a	a
D	d	b	a	c

Is it commutative and associative?

Solution:

∗	a	b	c	d
a	a	c	b	d
b	d	a	b	c
c	c	d	a	a
d	d	b	a	c

Commutative Propertty:

a ∗ b = c & b ∗ a = d

a ∗ b ≠ b ∗ a

Commutative property not satisfied

Associative property : a∗(b ∗ c) = (a ∗ b) ∗ c

L.H.S: a∗(b ∗ c) = a ∗ b = c

R.H.S: (a ∗ b) ∗ c = c ∗ c = a

L.H.S ≠ R.H.S. a∗(b ∗ c) ≠ (a ∗ b) ∗ c

ASSOCIATIVE PROPERTY NOT SATISFIED.

EXERCISE 12.2 (1)

Let p : Jupiter is a planet and q : India is an island be any two simple statements. Give verbal sentence describing each of the following statements.

(i) ¬p (ii) p ∧ ¬q (iii) ¬p ∨q (iv) p→¬q (v) p↔q

Solution:

(i) ¬p : Jupiter is not a planet

(ii) p ∧ ¬q: Jupiter is a planet and India is not a island

(iii) ¬p ∨q: Jupiter is not a planet or India is a land

(iv) p→¬q : If Jupiter is a planet then India is not a island

(v) p↔q : Jupiter is not a planet if and only if India is a island

Exercise 12.2(2)

Write each of the following sentences in symbolic form using statement variables p and q .

Solution:

p: 19 is a prime number and

q: All angles of a triangle are equal

(i) 19 is not a prime number and all the angles of a triangle are equal. Ans : ¬p ∧ q

(ii) 19 is a prime number or all the angles of a triangle are not equal. Ans : p v q

(iii) 19 is a prime number and all the angles of a triangle are equal. Ans: p ∧ q

(iv) 19 is not a prime number. Ans : ¬p

Exercise 12.2 q.no (3), (4) objectives

3 - MARKS

EXERCISE 12.1

7. Let $A = \begin{pmatrix} 1010 \\ 0101 \\ 1001 \end{pmatrix}$, $B = \begin{pmatrix} 0101 \\ 1010 \\ 1001 \end{pmatrix}$, $C = \begin{pmatrix} 1101 \\ 0110 \\ 1111 \end{pmatrix}$ **be any three boolean matricesof the same type. Find (i) A∨B (ii) A∧B (iii) (A∨B) ∧C (iv) (A∧B) ∨C .**

SOLUTION:

$$A \vee B = \begin{pmatrix} 1010 \\ 0101 \\ 1001 \end{pmatrix} \vee \begin{pmatrix} 0101 \\ 1010 \\ 1001 \end{pmatrix} = \begin{pmatrix} 1V0 & 0V1 & 1V0 & 0V1 \\ 0V1 & 1V0 & 0V1 & 1V0 \\ 1V1 & 0V0 & 0V0 & 1V1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

$$A \wedge B = \begin{pmatrix} 1010 \\ 0101 \\ 1001 \end{pmatrix} \wedge \begin{pmatrix} 0101 \\ 1010 \\ 1001 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \wedge 0 & 0 \wedge 1 & 1 \wedge 0 & 0 \wedge 1 \\ 0 \wedge 1 & 1 \wedge 0 & 0 \wedge 1 & 1 \wedge 0 \\ 1 \wedge 1 & 0 \wedge 0 & 0 \wedge 0 & 1 \wedge 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

$$(A \vee B) \wedge C = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix} \wedge \begin{pmatrix} 1101 \\ 0110 \\ 1111 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \wedge 1 & 1 \wedge 1 & 1 \wedge 0 & 1 \wedge 1 \\ 1 \wedge 0 & 1 \wedge 1 & 1 \wedge 1 & 1 \wedge 0 \\ 1 \wedge 1 & 0 \wedge 1 & 0 \wedge 1 & 1 \wedge 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

$$(A \wedge B) \vee C = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \vee \begin{pmatrix} 1101 \\ 0110 \\ 1111 \end{pmatrix}$$

$$= \begin{pmatrix} 0V1 & 0V1 & 0V0 & 0V1 \\ 0V0 & 0V1 & 0V1 & 0V0 \\ 1V1 & 0V1 & 0V1 & 1V1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

EXERCISE 12.2

Exercise 12.2(5)(i).

Write the converse, inverse, and contrapositive of each of the following implication.

(i) If x and y are numbers such that x = y , then $x^2 = y^2$

Solution:

(i) Conditional statement: p → q

If x and y are numbers such that x = y , then $x^2 = y^2$

(ii) Converse statement: q → p .

If x and y are numbers such that $x^2 = y^2$ then x = y

(iii) Inverse Statement: ¬p → ¬q

If x and y are numbers such that x ≠ y , then $x^2 \neq y^2$

(iv) Contrapositive statement: ¬q → ¬p

If x and y are numbers such that $x^2 \neq y^2$ then x ≠ y

Exercise 12.2(5)(ii).

Write the converse, inverse, and contrapositive of each of the following implication.

(ii) If a quadrilateral is a square then it is a rectangle

Solution:

(i) Conditional statement: p → q

If a quadrilateral is a square then it is a rectangle

(ii) Converse statement: q → p .

If a quadrilateral is a rectangle then it is a square

(iii) Inverse Statement: ¬p → ¬q

If a quadrilateral is not a square then it is not a rectangle

(iv) Contrapositive statement: ¬q → ¬p

If a quadrilateral is a not a rectangle then it is a not a square

5. (i) Define an operation ∗ on Q, $a * b = \left(\frac{a+b}{2}\right)$; a, b ∈ Q . Examine the closure, commutative, and associative properties satisfied by ∗on ℚ.

(ii) Define an operation ∗ on Q, $a * b = \left(\frac{a+b}{2}\right)$; a, b ∈ Q . Examine the existence of identity and the existence of inverse for the operation ∗on ℚ.

Solution:

Closure property: Let a, b ∈ Q then $\frac{a+b}{2} \in \mathbb{Q}$

⇒ a ∗ b ∈ ℚ

∴ closure property satisfied

Commutative property:

Let a, b ∈ Q, to verify a ∗ b = b ∗ a

L.H.S: $a * b = \frac{a+b}{2}$ & R.HS: $b * a = \frac{b+a}{2} = \frac{a+b}{2}$ = L.H.S

∴ Commutative property satisfied

Associative property

Let a, b, c ∈ Q, to verify a∗ (b ∗c) = (a∗ b) ∗ c

L.H.S: $a * (b * c) = a * \left(\frac{b+c}{2}\right) = \frac{a + \frac{b+c}{2}}{2} = \frac{2a+b+c}{4}$

R.H.S: $(a * b) * c = \left(\frac{a+b}{2}\right) * c = \frac{\frac{a+b}{2} + C}{2} = \frac{a+b+2c}{4}$

L.H.S ≠ R.H.S. Associative property not satisfied

(ii) Identity property:

Let e be the identity element such that

$a * e = a \Rightarrow \frac{a+e}{2} = a \Rightarrow a + e = 2a$

⇒ e = 2a - a = a since e = a which is not unique

So identity property not satisfied

Since identity property not satisfied inverse also not satisfied

Exercise 12.2 (6)

Construct the truth table for the following statements.

(i) ¬p∧¬q

Solution:

No of simple statements = 2; No. of rows = $2^2 = 4$

p	q	¬p	¬q	¬p∧¬q
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Last column corresponding to ¬p∧¬q

Exercise 12.2 (6):

Construct the truth table for the following statements.

(ii) ¬(p∧¬q)

Solution:

No of simple statements = 2; No. of rows = $2^2 = 4$

p	q	¬q	p ∧ ¬q	¬(p∧¬q)
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

Last column corresponding to **¬(p∧¬q)**

Exercise 12.2 (6): Construct the truth table for

(iii) (p ∨q) ∨ ¬q

Solution: No of simple statements = 2; No. of rows = $2^2 = 4$

p	q	¬q	p v q	(p ∨ q) ∨ ¬q
T	T	F	T	T
T	F	T	T	T
F	T	F	T	T
F	F	T	F	T

Last column corresponding to **(p ∨q) ∨ ¬q**

Exercise 12.2 (6):

Construct the truth table for the following statements.

(iv) $(\neg p\to r)\wedge(p\leftrightarrow q)$

Solution:

No of simple statements = 3; No. of rows = $2^3 = 8$

p	q	r	$\neg p$	T→F F $\neg p\to r$	T → F F F ← T F $p\leftrightarrow q$	$(\neg p\to r)\wedge(p\leftrightarrow q)$
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	T	F	F
T	F	F	F	T	F	F
F	T	T	T	T	F	F
F	T	F	T	F	F	F
F	F	T	T	T	T	T
F	F	F	T	F	T	F

Last column corresponding $(\neg p\to r)\wedge(p\leftrightarrow q)$

Exercise 12.2: (7)

Verify whether the following compound propositions are tautologies or contradictions or contingency:

(i) $(p\wedge q)\wedge\neg(p\vee q)$

Solution:

No of simple statements = 2; No. of rows = $2^2 = 4$

p	q	$p\wedge q$	$p\vee q$	$\neg(p\vee q)$	$(p\wedge q)\wedge\neg(p\vee q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

Since last column contains ONLY F So it is contradiction

Exercise 12.2: (7)

Verify whether the following compound propositions are tautologies or contradictions or contingency:

(ii) $((p\vee q)\wedge\neg p)\to q$

Solution:

No of simple statements = 2; No. of rows = $2^2 = 4$

p	q	$p\vee q$	$\neg p$	$(p\vee q)\wedge\neg p$	T → F F $((p\vee q)\wedge\neg p)\to q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

Since last column contains ONLY T so it is tautology

Exercise 12.2: (7)

Verify whether the following compound propositions are tautologies or contradictions or contingency:(iii) $(p\to q)\leftrightarrow(\neg p\to q)$

Solution:

No of simple statements = 2; No. of rows = $2^2 = 4$

p	q	T→F F $p\to q$	$\neg p$	T → F F $\neg p\to q$	T → F F F ← T F $(p\to q)\leftrightarrow(\neg p\to q)$
T	T	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	F	F

Since last column contains both T and F it is contigencey

Exercise 12.2: (7)

Verify whether the following compound propositions are tautologies or contradictions or contingency:

(iv) $((p\to q)\wedge(q\to r))\to(p\to r)$

Solution:

No of simple statements = 3; No. of rows = $2^3 = 8$

p	q	r	T →F F $p\to q$	T →F F $q\to r$	$(p\to q)\wedge(q\to r)$	T →F F $p\to r$	$((p\to q)\wedge(q\to r))\to(p\to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Since last column contains ONLY T so it is tautology

Exercise 12.2: (8): Show that (i) $\neg(p\wedge q)\equiv\neg p\vee\neg q$

Solution: No of simple statements = 2; No. of rows = $2^2 = 4$

p	q	$p\wedge q$	L.H.S $\neg(p\wedge q)$	$\neg p$	$\neg q$	R.H.S $\neg p\vee\neg q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Since column corresponding to L.H.S and R.H.S are identical,
Hence $\neg(p\wedge q)\equiv\neg p\vee\neg q$

Exercise 12.2: (8): Show that (ii) $\neg(p\to q)\equiv p\wedge\neg q$

Solution: No of simple statements = 2; No. of rows = $2^2 = 4$

p	q	T→F F $p\to q$	L.H.S $\neg(p\to q)$	$\neg q$	R.H.S $p\wedge\neg q$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	F	T	F

Since column corresponding to L.H.S and R.H.S are identical,
Hence $\neg(p\to q)\equiv p\wedge\neg q$

Exercise 12.2: (9): Prove that $q\to p\equiv\neg p\to\neg q$

Solution: No of simple statements = 2; No. of rows = $2^2 = 4$

p	q	L.H.S T → F F $q\to p$	$\neg p$	$\neg q$	R.H.S T → F F $\neg p\to\neg q$
T	T	T	F	F	T
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	T	T

Since column corresponding to L.H.S and R.H.S are identical,
Hence $q\to p\equiv\neg p\to\neg q$

Exercise 12.2: (10):

Show that $p\to q$ and $q\to p$ are not equivalent

Solution:

No of simple statements = 2; No. of rows = $2^2 = 4$

p	q	T→F F $p\to q$	T → F F $q\to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Since column corresponding $p\to q$ AND $q\to p$ are NOT identical . $p\to q$ and $q\to p$ are not equivalent

Exercise 12.2: (11):

Show that ¬(p↔q) ≡ p ↔ ¬q

Solution:

No of simple statements = 2; No. of rows = $2^2 = 4$

p	q	p ↔ q	¬(p↔q)	¬q	p ↔ ¬q
T	T	T	F	F	F
T	F	F	T	T	T
F	T	F	T	F	T
F	F	T	F	T	F

Since column corresponding p → q AND q → p are NOT identical

p→q and q→p are not equivalent

Exercise 12.2: (13):

Using truth table check whether the statements ¬(p∨q)∨(¬p∧q) and ¬p are logically equivalent.

Solution:

No of simple statements = 2; No. of rows = $2^2 = 4$

p	q	p∨q	¬(p∨q)	¬p	¬p∧q	¬(p∨q)∨(¬p∧q)
T	T	T	F	F	F	F
T	F	T	F	F	F	F
F	T	T	F	T	T	T
F	F	F	T	T	F	T

Since column corresponding ¬(p∨q)∨(¬p∧q) and ¬p are identical

Hence ¬(p∨q)∨(¬p∧q) and ¬p are logically equivalent

5 MARKS

Example 12.9

Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation $+_5$ on Z_5 using table corresponding to addition modulo 5.

SOLUTION: Z_5= { [0], [1], [2], [3], [4] } = { 0, 1, 2, 3, 4 }

$+_5$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

CLOSURE PROPERTY:

All the elements in the table are form the set only

Closure property is verified

Commutative property :

Table is symmetric about main diagonal

Commutative property is verified

Associative property:

$+_5$ is alwys associative, Associative property is verified

Identity property:

0 ∈ Z_5 is the identity element, identity property is verified.

Inverse property:

ELEMENT	0	1	2	3	4
INVERSE	0	4	3	2	1

Inverse property is verified

Example 12.10

Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation x_{11} on a subset A = {1,3,4,5,9} of the set of remainders {0,1,2,3,4,5,6,7,8,9,10}.

SOLUTION:

A= {1,3,4,5,9}

x_{11}	1	3	4	5	9
1	1	3	4	5	9
3	3	9	1	4	5
4	4	1	5	9	3
5	5	4	9	3	1
9	9	5	3	1	4

CLOSURE PROPERTY:

All the elements in the table are form the set only

Closure property is verified

Commutative property :

Table is symmetric about main diagonal

Commutative property is verified

Associative property:

x_{11} is alwys associative, Associative property is verified

Identity property:

1 ∈ A is the identity element, identity property is verified.

Inverse property:

ELEMENT	1	3	4	5	9
INVERSE	1	4	3	9	5

Inverse Property satisfied

10. (i) Let A be Q\{1}. Define ∗on A by $x * y = x + y - xy$. Is ∗ binary on A ? If so, examine the commutative and associative properties satisfied by ∗on A .
(ii) Let A be Q\{1}. Define ∗on A by $x * y = x + y - xy$. Is ∗binary on A ? If so, examine the existence of identity, existence of inverse properties for the operation ∗on A .

SOLUTION:

Given $A = Q\backslash\{1\}$ & $x * y = x + y - xy$

Closure property : Let a,b ∈ A then a ≠1 and b ≠1

Then a + b, ab ∈ A also a + b - ab ∈ A ⇒ a∗b ∈ A

To verify closure property we must prove a ∗ b ≠1

Let a ∗ b = 1 ⇒ a + b - ab = 1 ⇒ b (1 - a) = 1 - a

$\Rightarrow b = \frac{1-a}{1-a} = 1$ which is a contradiction b ≠1

a∗b = a + b - ab ∈ A Closure property verified

Commutative property :

Let a, b ∈ A. To verify a∗b = b ∗a

L.H.S. : a∗b = a + b - ab & R.H.S.: b ∗a = b + a - ba

L.H.S = R.H.S Commutative property satisfied.

Associative Property:

Let a, b, c ∈ A. To verify a∗(b ∗ c) = (a ∗ b) ∗ c

L.H.S.:

a∗(b ∗ c) = a ∗ (b + c - bc) = a +(b + c - bc)- a(b + c - bc)

= a + b + c - ab - bc - ac + abc

R.H.S:

(a ∗ b) ∗ c = (a + b - ab)∗c = a + b - ab + c - (a + b - ab)c

= a + b + c - ab - bc- ac + abc

L.H.S = R.H.S Associative property satisfied

(iii) Identity property:

Let e be the identity element such that

a ∗ e = a ⇒ a + e - ae = a ⇒ e(1-a) = a - a = 0

$\Rightarrow e = \frac{0}{1-a} = 0 \in A$

Identity property satisified

(v) Inverse property:

Let a ∈ A and let a' ∈ A be the inverse of a such that

a ∗ a' = o ⇒ a + a' - aa' = 0 ⇒ a' - aa' = -a ⇒a' (1-a) = -a

$\Rightarrow a' = \frac{-a}{1-a} \in A$ inverse property not satisfied

Example 12.19: Using the equivalence property

Show that p↔q ≡ (p ∧ q) v (¬p ∧ ¬q)

Solution:

p↔q ≡ (p → q) ∧ (q → p)
≡ (¬p v q) ∧ (¬q v p)
≡ [(¬p v q) ∧ ¬q] v [(¬p v q) ∧ p]
≡ [¬q ∧ (¬p v q)] v [p ∧ (¬p v q)]
≡ [(¬q ∧ ¬p) v (¬q ∧ q)] v [(p ∧ ¬p) v (p ∧ q)]
≡ [(¬q ∧ ¬p) v 𝔽] v [𝔽 v (p ∧ q)]
≡ (¬q ∧ ¬p) v(p ∧ q)
≡ (p ∧ q) v (¬q ∧ ¬p) Hence proved

5 (i) Define an operation ∗ on Q, $a * b = \left(\frac{a+b}{2}\right)$; a, b ∈ Q . Examine the closure, commutative, and associative properties satisfied by ∗on ℚ.

(ii) Define an operation ∗ on Q, $a * b = \left(\frac{a+b}{2}\right)$; a, b ∈ Q . Examine the existence of identity and the existence of inverse for the operation ∗on ℚ.

Solution:

Closure property: Let a, b ∈ Q then $\frac{a+b}{2} \in \mathbb{Q}$ ⇒ a ∗ b ∈ ℚ

∴ closure property satisfied

Commutative property:

Let a, b ∈ Q, to verify a ∗ b = b ∗ a

L.H.S: $a * b = \frac{a+b}{2}$ & R.HS: $b * a = \frac{b+a}{2} = \frac{a+b}{2}$ = L.H.S

∴ Commutative property satisfied

Associative property

Let a, b, c ∈ Q, to verify a∗ (b ∗c) = (a∗ b) ∗ c

L.H.S: $a * (b * c) = a * \left(\frac{b+c}{2}\right) = \frac{a+\frac{b+c}{2}}{2} = \frac{2a+b+c}{4}$

R.H.S: $(a * b) * c = \left(\frac{a+b}{2}\right) * c = \frac{\frac{a+b}{2}+C}{2} = \frac{a+b+2c}{4}$

L.H.S ≠ R.H.S. Associative property not satisfied

(ii) Identity property:

Let e be the identity element such that

$a * e = a \Rightarrow \frac{a+e}{2} = a \Rightarrow a + e = 2a$

⇒ e = 2a - a = a since e = a which is not unique

So identity property not satisfied

Since identity property not satisfied inverse also not satisfied

Example 12.7

Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for following operation on the given set. $\mathbf{m * n = m + n - mn}$; $\mathbf{n \in Z}$

solution

Closure property : Let a,b ∈ Z

Then a + b, ab ∈ Z also a + b - ab ∈ Z ⇒ a∗b ∈ Z

Closure property verified

Commutative property :

Let a, b ∈ Z. To verify a∗b= b ∗a

L.H.S. : a∗b = a + b - ab & R.H.S.: b ∗a = b + a - ba

L.H.S = R.H.S Commutative property satisfied.

Associative Property:

Let a, b, c ∈ Z. To verify a∗(b ∗ c) = (a ∗ b) ∗ c

L.H.S.:

a∗(b ∗ c) = a ∗ (b + c - bc)

= a +(b + c - bc)- a(b + c - bc)

= a + b + c - ab - bc - ac + abc

R. H.S:

a ∗ b) ∗ c = (a + b - ab)∗c = a + b - ab + c - (a + b - ab)c

= a + b + c - ab - bc- ac + abc

(iv) Identity property:Let e be the identity element such that

a ∗ e = a ⇒ a + e - ae = a ⇒ e(1-a) = a - a = 0

⇒ e = $\frac{0}{1-a}$ = 0∈ Z; Identity property verified

(v) Inverse property:

Let a ∈ Z and let a’ ∈ Z be the inverse of a such that

a ∗ a’ = o ⇒ a + a’ - aa’ = 0 ⇒ a’ - aa’ = -a ⇒a’ (1-a) = -a

⇒ a’ = $\frac{-a}{1-a}$ ∉ Z inverse property not verified

Exercise 12.1

9.(i) Let $\mathbf{M} = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in \mathbf{R} - \{0\}\right\}$ and Let * be the matrix multiplication. Determine whether M is closed under *. If so, examine the commutative and associative properties satisfied by * on M.

(ii) Let $\mathbf{M} = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in \mathbf{R} - \{0\}\right\}$ and let * be the matrix multiplication. Determine whether M is closed under *. If so, examine the existence of an identity, the existence of inverse properties for the operation * on M.

Solution:

$M = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\}\right\}$; ∗ be the matrix multiplication

Closure property :

Let $A = \begin{pmatrix} a & a \\ a & a \end{pmatrix}$ and a ≠ 0; Let $B = \begin{pmatrix} b & b \\ b & b \end{pmatrix}$ and b ≠ 0

$A * B = \begin{pmatrix} a & a \\ a & a \end{pmatrix}\begin{pmatrix} b & b \\ b & b \end{pmatrix} = \begin{pmatrix} ab+ab & ab+ab \\ ab+ab & ab+ab \end{pmatrix}$

$= \begin{pmatrix} 2ab & 2ab \\ 2ab & 2ab \end{pmatrix} \in M$ (2ab ≠ 0 as a ≠ 0 and b ≠ 0)

Commutative property:

Let A, B ∈ M To verify A ∗ B = B ∗ A

L.H.S. $A * B = \begin{pmatrix} a & a \\ a & a \end{pmatrix}\begin{pmatrix} b & b \\ b & b \end{pmatrix} = \begin{pmatrix} ab+ab & ab+ab \\ ab+ab & ab+ab \end{pmatrix}$

$= \begin{pmatrix} 2ab & 2ab \\ 2ab & 2ab \end{pmatrix}$

R.H.S. $B * A = \begin{pmatrix} b & b \\ b & b \end{pmatrix}\begin{pmatrix} a & a \\ a & a \end{pmatrix} = \begin{pmatrix} ab+ab & ab+ab \\ ab+ab & ab+ab \end{pmatrix}$

$= \begin{pmatrix} 2ab & 2ab \\ 2ab & 2ab \end{pmatrix}$

L.H.S. = R.H.S. ∴ commutative property satisfied

Associative property:

Matrix multiplication is always associative

∴ Associative property is verified.

Identity property:

Let $E = \begin{pmatrix} e & e \\ e & e \end{pmatrix}$ be the identity element such that A∗E = A

To prove: E ∈ M

$A*E = A \Rightarrow \begin{pmatrix} a & a \\ a & a \end{pmatrix}\begin{pmatrix} e & e \\ e & e \end{pmatrix} = \begin{pmatrix} a & a \\ a & a \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 2ae & 2ae \\ 2ae & 2ae \end{pmatrix} = \begin{pmatrix} a & a \\ a & a \end{pmatrix} \Rightarrow 2ae = a \Rightarrow e = \frac{1}{2} \neq 0$

cha $\therefore E = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \in M$ is the identity element

We can prove E∗A = A

INVERSE PROPERTY:

Let $A' = \begin{pmatrix} a' & a' \\ a' & a' \end{pmatrix}$ be the inverse of A ∈M, such that A∗A’ = E

To prove A’ ∈ M

$A*A' = E \Rightarrow \begin{pmatrix} a & a \\ a & a \end{pmatrix}\begin{pmatrix} a' & a' \\ a' & a' \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 2aa' & 2aa' \\ 2aa' & 2aa' \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \Rightarrow 2aa' = \frac{1}{2} \Rightarrow a' = \frac{1}{4a} \neq 0$ (a≠0)

$\Rightarrow A' = \begin{pmatrix} \frac{1}{4a} & \frac{1}{4a} \\ \frac{1}{4a} & \frac{1}{4a} \end{pmatrix} \in M$ is the inverse of A

We can prove A’∗ A = E

CHAPTER 1 - MATRICES AND DETERMINANTS

5 MARKS

Example 1.1

If $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$, verify that $A(\text{adj } A)=(\text{adj } A)=|A|I_3$.

Example 1.10

If $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, find x and y such that $A^2 + xA + yI_2 = O_2$. Hence, find A^{-1}.

Example 1.12

If $A=\frac{1}{7}\begin{bmatrix} 6 & -3 & a \\ b & -2 & 6 \\ 2 & c & 3 \end{bmatrix}$ is orthogonal, find a , b ,and c , and hence A^{-1}.

EXERCISE 1.1

3. If $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$, show that $[F(\alpha)]^{-1} = F(-\alpha)$.

4. If $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$, show that $A^2 - 3A - 7I_2 = O_2$. Hence find A^{-1}.

Example 1.19

Show that the matrix $\begin{bmatrix} 3 & 1 & 4 \\ 2 & 0 & -1 \\ 5 & 2 & 1 \end{bmatrix}$ is non-singular and reduce it to the identity matrix by elementary row transformations.

Example 1.21 Find the inverse of $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by Gauss-Jordan method.

EXERCISE 1.2

3. Find the inverse of each of the following by Gauss-Jordan method:

(ii) $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$

Example 1.23

Solve the following system of equations, using matrix inversion method: $2x_1 + 3x_2 + 3x_3 = 5$,

$x_1 - 2x_2 + x_3 = -4$, $3x_1 - x_2 - 2x_3 = 3$.

Example 1.24

If $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the system of equations $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$.

EXERCISE 1.3

1. Solve the following system of linear equations by matrix inversion method:

(iii) $2x + 3y - z = 9$, $x + y + z = 9$, $3x - y - z = 1$

(iv) $x + y + z - 2 = 0$, $6x - 4y + 5z - 31 = 0$, $5x + 2y + 2z = 13$

2. If $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the system of equations

$x + y + 2z = 1$, $3x + 2y + z = 7$, $2x + y + 3z = 2$.

4. The prices of three commodities A , B and C are ₹ x , y , and z per units respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C . Person Q purchases 2 units of C and sells 3 units of A and one unit of B . Person R purchases one unit of A and sells 3 unit of B and one unit of C . In the process, P , Q ,and R earn ₹15,000, ₹1,000 and ₹4,000 respectively. Find the prices per unit of A , B ,and C . (Use matrix inversion method to solve the problem.)

Example 1.25 Solve, by Cramer's rule, the system of equations $x_1 - x_2 = 3$, $2x_1 + 3x_2 + 4x_3 = 17$, $x_2 + 2x_3 = 7$

Example 1.26

In a T20 match, Chennai Super Kings needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball traversed along a path in a vertical plane and the equation of the path is $y = ax^2 + bx + c$ with respect to a xy - coordinate system in the vertical plane and the ball traversed through the points (10,8) , (20,16) , (30,18) , can you conclude that Chennai Super Kings won the match? Justify your answer. (All distances are measured in metres and the meeting point of the plane of the path with the farthest boundary line is (70,0) .)

EXERCISE 1.4

5. A family of 3 people went out for dinner in a restaurant. The cost of two dosai, three idlies and two vadais is ₹150. The cost of the two dosai, two idlies and four vadais is ₹200. The cost of five dosai, four idlies and two vadais is ₹250. The family has ₹350 in hand and they ate 3 dosai and six idlies and six vadais. Will they be able to manage to pay the bill within the amount they had ?

Example 1.27

Solve the following system of linear equations, by Gaussian elimination method :

$4x + 3y + 6z = 25$, $x + 5y + 7z = 13$, $2x + 9y + z = 1$

Example 1.28

The upward speed $v(t)$ of a rocket at time t is approximated by $v(t) = at^2 + bt + c$, $0 \le t \le 100$ where a , b , and c are constants. It has been found that the speed at times

$t = 3$, $t = 6$, and $t = 9$ seconds are respectively, 64, 133, and 208 miles per second respectively. Find the speed at time t =15 seconds. (Use Gaussian elimination method.)

EXERCISE 1.5

1. Solve the following systems of linear equations by Gaussian elimination method:

(i) $2x - 2y + 3z = 2$, $x + 2y - z = 3$, $3x - y + 2z = 1$

(ii) $2x + 4y + 6z = 22$, $3x + 8y + 5z = 27$, $-x + y + 2z = 2$

2. If $ax^2 + bx + c$ is divided by $x + 3$, $x - 5$, and $x - 1$, the remainders are 21 , 61 and 9 respectively.Find a , b , and c .

(Use Gaussian elimination method.)

3. An amount of ₹65,000 is invested in three bonds at the rates of 6%, 8% and 10% per annum respectively. The total annual income is ₹4,800. The income from the third bond is ₹600 more than that from the second bond. Determine the price of each bond. (Use Gaussian elimination method.)

4. A boy is walking along the path $y = ax^2 + bx + c$ through the points $(-6, 8)$, $(-2, -12)$, and $(3, 8)$. He wants to meet his friend at P(7,60). Will he meet his friend?

(Use Gaussian elimination method.)

Example 1.29

Test for consistency of the following system of linear equations and if possible solve : $x + 2y - z = 3$,

$3x - y + 2z = 1$, $x - 2y + 3z = 3$, $x - y + z + 1 = 0$.

Example 1.30

Test for consistency of the following system of linear equations and if possible solve:

$4x - 2y + 6z = 8$, $x + y - 3z = -1$, $15x - 3y + 9z = 21$.

Example 1.31

Test for consistency of the following system of linear equations and if possible solve:

$x - y + z = -9$, $2x - 2y + 2z = -18$, $3x - 3y + 3z + 27 = 0$

Example 1.32

Test the consistency of the following system of linear equations

$x - y + z = -9$, $2x - y + z = 4$,

$3x - y + z = 6$, $4x - y + 2z = 7$

Example 1.33

Find the condition on a , b ,and c so that the following system of linear equations has one parameter family of solutions: $x + y + z = a$, $x + 2y + 3z = b$, $3x + 5y + 7z = c$.

Example 1.34

Investigate for what values of λ and μ the system of linear equations $x + 2y + z = 7$, $x + y + \lambda z = \mu$, $x + 3y - 5z = 5$ has (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

EXERCISE 1.6

1. Test for consistency and if possible, solve the following systems of equations by rank method.

(i) $x - y + 2z = 2$, $2x + y + 4z = 7$, $4x - y + z = 4$

(ii) $3x + y + z = 2$, $x - 3y + 2z = 1$, $7x - y + 4z = 5$

(iv) $2x - y + z = 2$, $6x - 3y + 3z = 6$, $4x - 2y + 2z = 4$

2. Find the value of k for which the equations

$kx - 2y + z = 1$, $x - 2ky + z = -2$, $x - 2y + kz = 1$

have (i) no solution (ii) unique solution

(iii) infinitely many solution

3. Investigate the values of λ and μ the system of linear equations $2x + 3y + 5z = 9$, $7x + 3y - 5z = 8$,

$2x + 3y + \lambda z = \mu$, have (I)no solution (ii) a unique solution (iii) an infinite number of solutions.

Example 1.36

Solve the system: $x + 3y - 2z = 0$, $2x - y + 4z = 0$,

$x - 11y + 14z = 0$.

Example 1.37

Solve the system: $x + y - 2z = 0$, $2x - 3y + z = 0$,

$3x - 7y + 10z = 0$, $6x - 9y + 10z = 0$.

Example 1.38

Determine the values of λ for which the following system of equation$(3\lambda - 8)x + 3y + 3z = 0$, $3x + (3\lambda - 8)y + 3z = 0$

$3x + 3y + (3\lambda - 8)z = 0$ has a non-trivial solution

Example 1.39

By using Gaussian elimination method, balance the chemical reaction equation: $C_5H_8 + O_2 \rightarrow CO_2 + H_2O$

Example 1.40

If the system of equations

$px + by + cz = 0$, $ax + qy + cz = 0$, $ax + by + rz = 0$

has a non-trivial solution and $p \ne a$, $q \ne b$, $r \ne c$,

prove that $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$

CHAPTER 4 - INVERSE TROGNOMETRIC EQUATION

EXAMPLE 4.4 :

Find the domain of $\sin^{-1}(2-3x^2)$

Exercise 4.1 - 6(i) :

Find the domain of $f(x)=\sin^{-1}\left(\frac{x^2+1}{2x}\right)$

Example 4.7

Find the domain of $\cos^{-1}\left(\frac{2+\sin x}{3}\right)$.

Exercise 4.2 - 6(i)

Find the domain of $f(x)=\sin^{-1}\left(\frac{|x|-2}{3}\right)+\cos^{-1}\left(\frac{1-|x|}{4}\right)$

Exercise 4.3-4(ii)

Find the value of $\sin\left(\tan^{-1}\left(\frac{1}{2}\right)-\cos^{-1}\left(\frac{4}{5}\right)\right)$

Exercise 4.3-4(iii)

Find the value of $\cos\left(\sin^{-1}\left(\frac{4}{5}\right)-\tan^{-1}\left(\frac{3}{4}\right)\right)$

Example 4.20

Evaluate $\sin\left[\sin^{-1}\left(\frac{3}{5}\right)+\sec^{-1}\left(\frac{5}{4}\right)\right]$

Example 4.22

If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$ and $0<x,y,z<1$, show that $x^2+y^2+z^2+2xyz=1$

Example 4.23

If $a_1, a_2, a_3, \ldots a_n$ is an arithmetic progression with common difference d, prove that

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\ldots+\tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right]=\frac{a_n-a_1}{1+a_1a_n}$$

Example 4.27:

Solve $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$, if $6x^2<1$

Example 4.28

Solve $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$

Example 4.29

Solve $\cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right)=\sin\left\{\cot^{-1}\left(\frac{3}{4}\right)\right\}$

Exercise 4.5 3(ii)

Find the value of $\cot\left(\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{4}{5}\right)$

Exercise 4.5 3(iii)

Find the value of $\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$.

CHAPTER 10 - APPLICATION PROBLEMS

Example 10.27

The growth of a population is proportional to the number present. If the population of a colony doubles in 50 years, in how many years will the population become triple?

Example 10.28

A radioactive isotope has an initial mass 200mg , which two years later is 50mg . Find the expression for the amount of the isotope remaining at any time. What is its half-life? (half-life mean the time taken for the radioactivity of a specified isotope to fall to half its original value).

Example 10.29

In a murder investigation, a corpse was found by a detective at exactly 8 p.m. Being alert, the detective also measured the body temperature and found it to be 70°F. Two hours later, the detective measured the body temperature again and found it to be 60°F. If the room temperature is 50°F, and assuming that the body temperature of the person before death was 98.6°F, at what time did the murder occur?

[log(2.43)=0.88789 ; log(0.5)=-0.69315]

Example 10.30

A tank contains 1000 litres of water in which 100 grams of salt is dissolved. Brine (Brine is a high-concentration solution of salt (usually sodium chloride) in water) runs in a rate of 10 litres per minute, and each litre contains 5 grams of dissolved salt. The mixture of the tank is kept uniform by stirring. Brine runs out at 10 litres per minute. Find the amount of salt at any time t .

EXERCISE 10.8

1. The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present.

Given that the number triples in 5 hours, find how many bacteria will be present after 10 hours?

2. Find the population of a city at any time t, given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3,00,000 to 4,00,000.

3. The equation of electromotive force for an electric circuit containing resistance and self-inductance is $\mathbf{E} = \mathbf{Ri} + \mathbf{L}\frac{\mathbf{di}}{\mathbf{dt}}$, where E is the electromotive force is given to the circuit, R the resistance and L, the coefficient of induction. Find the current i at time t when $E = 0$.

4. The engine of a motor boat moving at 10 m / s is shut off. Given that the retardation at any subsequent time (after shutting off the engine) equal to the velocity at that time. Find the velocity after 2 seconds of switching off the engine.

5. Suppose a person deposits ₹10,000 in a bank account at the rate of 5% per annum compounded continuously. How much money will be in his bank account 18 months later?

6. Assume that the rate at which radioactive nuclei decay is proportional to the number of such nuclei that are present in a given sample. In a certain sample 10% of the original number of radioactive nuclei have undergone disintegration in a period of 100 years. What percentage of the original radioactive nuclei will remain after 1000 years?

7. Water at temperature 100℃ cools in 10 minutes to 80℃ in a room temperature of 25℃ . Find (i) The temperature of water after 20 minutes (ii) The time when the temperature is 40℃

$\left[\log_e \frac{11}{15} = -0.3101 \ ; \ \log_e 5 = 1.6094\right]$

8. At 10.00 A.M. a woman took a cup of hot instant coffee from her microwave oven and placed it on a nearby Kitchen counter to cool . At this instant the temperature of the coffee was 180°F, and 10 minutes later it was 160°F . Assume that constant temperature of the kitchen was 70°F .

(i)What was the temperature of the coffee at 10.15A.M.?

(ii)The woman likes to drink coffee when its temperature is between 130°F and 140°F between what times should she have drunk the coffee?

9.A pot of boiling water at 100℃ is removed from a stove at time $t = 0$ and left to cool in the kitchen. After 5 minutes, the water temperature has decreased to 80℃ , and another 5 minutes later it has dropped to 65 ℃ . Determine the temperature of the kitchen.

10. A tank initially contains 50 litres of pure water. Starting at time $t = 0$ a brine containing with 2 grams of dissolved salt per litre flows into the tank at the rate of 3 litres per minute. The mixture is kept uniform by stirring and the well-stirred mixture simultaneously flows out of the tank at the same rate. Find the amount of salt present in the tank at any time $t > 0$.