DIRECTORATE OF SCHOOL EDUCATION
TAMILNADU

12JPCM03 (2023-24)	JEE PRACTICE QUESTIONS (TEST-3)	Class : XII Time : 1.15 hrs Total Marks : 180

Answer key

12th – MATHS

31. Ans (B)

$-1 \le 3x+5 \le 1$

$-6 \le 3x \le -4$

$-2 \le x \le \frac{-4}{3}$

$x \in [-2, \frac{-4}{3}]$

32. Ans (A)

$f(x) = \cot^{-1}(2x - x^2)$

$\theta = \cot^{-1}[1-(x-1)^2]$

$\cot\theta = 1-(x-1)^2$

$\cot\theta \le 1, \theta \in (0,\pi)$

$\frac{\pi}{4} \le f(x) < \pi$

So, $R_f = (\pi/4, \pi)$

33. Ans (C)

$4\sin^{-1}(x-1) + \cos^{-1}(x-1) = \pi$

$3\sin^{-1}(x-1) + \sin^{-1}(x-1) + \cos^{-1}(x-1) = \pi$

$3\sin^{-1}(x-1) + \pi/2 = \pi$

$3\sin^{-1}(x-1) = \pi/2$

$\sin^{-1}(x-1) = \pi/6$

$$x-1=\frac{1}{2}$$

$$x=1/2+1$$

$$x=3/2$$

34. Ans A

Let $\frac{1}{2}\cot^{-1}\left(\frac{3}{5}\right)=\theta$

$$\cos^{-1}\left(\frac{3}{5}\right)=2\theta$$

$$\cos 2\theta=\frac{3}{5}$$

$$2\cos^2\theta-1=\frac{3}{5}$$

$$2\cos^2\theta=\frac{8}{5}$$

$$\cos^2\theta=\frac{4}{5}$$

$$\cos\theta=\frac{2}{\sqrt{5}}$$

35. D) $\frac{2x}{5x+3}\le -1 \quad (or) \quad \frac{2x}{5x+3}\ge 1$

$$\frac{2x}{5x+3}+1\le 0 \ (or) \quad \frac{2x}{5x+3}-1\ge 0$$

$$\frac{7x+3}{5x+3}\le 0 \quad (or) \ \frac{-3x-3}{5x+3}\ge 0 \text{ b}$$

$$x=\frac{-3}{7}, x=\frac{-3}{5} \quad (or) \quad \frac{3x+3}{5x+3}\le 0$$

$$x=-1 \ x=\frac{-3}{5}$$

$$\therefore \alpha=-1, \beta=\frac{-3}{5}, \gamma=-3/5, \delta=\frac{-3}{7}$$

$$x\in\left[-1,\frac{-3}{5}\right]\cup\left(\frac{-3}{5},\frac{-3}{7}\right)$$

$$|3\alpha+10(\beta+\gamma)+21\delta|$$

$$=\left|3(-1)+10\left(\frac{-3}{5}-\frac{3}{5}\right)+21\left(\frac{-3}{7}\right)\right|$$

$= |-3-12-9|$

$= |-24|$

$= 24$

36. Ans A)

$\sin(\sin^{-1}\frac{1}{3}+\sin^{-1}\frac{1}{2})$

$=\sin(\sin^{-1}\frac{1}{3})\cos(\sin^{-1}\frac{1}{2})+\cos(\sin^{-1}\frac{1}{3})\sin(\sin^{-1}\frac{1}{2})$

$=\frac{1}{3}\sqrt{1-\frac{1}{4}}+\frac{1}{2}\sqrt{1-\frac{1}{9}}$

$=\frac{1}{3}\frac{\sqrt{3}}{2}+\frac{1}{2}\frac{\sqrt{8}}{3}$

$=\frac{\sqrt{3}+\sqrt{8}}{\sqrt{36}}$

a = 3, b = 8, C = 36

a + b = 11

37. Ans (b)

$\tan^{-1}\left(\frac{1-x}{1+x}\right),\ 0\le x\le 1$

$\tan^{-1} 1 - \tan^{-1} x$

$=\pi/4-\tan^{-1}x$

max. value = $\pi/4$ at $x = 0$

min value = 0 at $x = 1$

38. Ans (B) 2n

$$\sum_{i=1}^{2n}\sin^{-1}x_i = n\pi$$

$\sum_{i=1}^{2n}\sin^{-1}x_i = 2n\left(\frac{\pi}{2}\right)$

$\sin^{-1}(x_1)+\sin^{-1}(x_2)+.....+\sin^{-1}(x_{2n})=2n\left(\frac{\pi}{2}\right)$

$\sin^{-1}x_1=\frac{\pi}{2},\ \sin^{-1}x_2=\frac{\pi}{2},.....\sin^{-1}(x_{2n})=\pi/2$

$\therefore x_1=1,\ x_2=1,.....x_{2n}=1$

Thus $\sum_{i=1}^{2n} X_i$

$= x_1 + x_2 + \ldots\ldots\ldots + x_{2n}$

$= 1 + 1 + \ldots\ldots\ldots\ldots + 1$

$= 2n$

39. Solution Ans : C)

$\text{Sin } 15^o = \frac{\sqrt{3}-1}{2\sqrt{2}}$, $\cos 15^o = \frac{\sqrt{3}+1}{2\sqrt{2}}$,

$\sin 15^o \text{ con} 15^o = \frac{1}{2}\sin 30^o \ \frac{1}{4}$

$\sin 15 \cos 75^o = \sin^2 15^o = \frac{4-2\sqrt{3}}{8}$

40. Ans : A) 0

$4\cos 18^o - \frac{3}{\cos 18} - 2\frac{\sin 18}{\cos 18}$

$= \frac{4\cos^2 18 - 3 - 2\sin 18}{\cos 18}$

$= \frac{4\cos^3 18 - 3\cos 18 - 2\sin 18\cos 18}{\cos^2 18}$

$= \frac{\cos(3\times 18°) - \sin(2\times 18°)}{\cos^2 18}$

$= \frac{\cos 54 - \sin 36}{\cos^2 18}$

$= \frac{0}{\cos^2 18}$

$= 0$

41. Ans C) 11

$\Delta = \frac{1}{2}.\ 3.4 = 6$

$s = \frac{1}{2}(3+4+5) = 6$

$r_1 = \frac{\Delta}{5-a} = \frac{6}{6-3} = 2$

$r_2 = \frac{\Delta}{5-b} = \frac{6}{6-4} = 3$

$$r_3 = \frac{\Delta}{5-c} = \frac{6}{6-5} = 6$$

$$\therefore r_1 + r_2 + r_3 = 2 + 3 + 6 = 11$$

42. Ans (A)

$$\frac{1}{x} + \frac{1}{y}$$

$$y = \cot B - \cot A = \frac{1}{\tan B} - \frac{1}{\tan A} = \frac{x}{\tan A \tan B}$$

$$\tan A \ \tan B = \frac{x}{y}$$

Now cot (A – B) = $\frac{1}{\tan(A-B)}$

$$= \frac{1+\tan A \tan B}{\tan A - \tan B}$$

$$= \frac{1 + x/y}{x}$$

$$= \frac{1}{x} + \frac{1}{y}$$

43. Ans C $\sqrt{3}$

$$\sin 20^\circ \left(4 + \frac{1}{\cos 20^\circ}\right) = \frac{\sin 20^\circ}{\cos 20^\circ}(4\cos 20^\circ + 1)$$

$$= \frac{1}{\cos 20^\circ}(4\sin 20^\circ \cos 20^\circ + \sin 20^\circ)$$

$$= \frac{1}{\cos 20^\circ}(2\sin 40^\circ + \sin 20^\circ)$$

$$= \frac{1}{\cos 20^\circ}(\sin 40^\circ + \sin 40^\circ + \sin 20^\circ)$$

$$= \frac{1}{\cos 20^\circ}(\sin 40^\circ + 2\sin 30 \cos 10^\circ)$$

$$= \frac{1}{\cos 20^\circ}(\sin 40^\circ + \cos 10^\circ)$$

$$= \frac{1}{\cos 20^\circ}(\sin 40^\circ + \sin 80^\circ)$$

$$\frac{2\sin 60^\circ \cos 20^\circ}{\cos 20}$$

$$= \sqrt{3}$$

44. *And D)*

Sin 8x + 7 sin 6x + 18 sin 4x + 12 sin 2x

= sin 8x + sin 6x + 6[sin6x + sin 4x] + 12 [sin 4m + sin 2x]

= 2sin 7x cos x + 6.2 sin 5x cos x + 12.2 sin 3x cos x

= 2cos x [sin 7x + 6 sin 5x + 12 sin 3x]

The given fraction is 2 cos x

45. Ans (C)

$m = \cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$n = \sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$\therefore \frac{n}{m} = \tan\left(\frac{\alpha+\beta}{2}\right)$

Now,

$$\sin(\alpha+\beta) = \frac{2\tan\left(\frac{\alpha+\beta}{2}\right)}{1+\tan^2\left(\frac{\alpha+\beta}{2}\right)} = \frac{2n/m}{1+\frac{n^2}{m^2}} = \frac{2mn}{m^2+n^2}$$

DIRECTORATE OF SCHOOL EDUCATION
TAMILNADU

11JPCM03 (2023-24)	JEE PRACTICE QUESTIONS (TEST-3)	Class : XI Time : 1.15 hrs Total Marks : 180

Answer key

11th – MATHS

31. Solution Ans : C)

$\sin 15^o = \frac{\sqrt{3}-1}{2\sqrt{2}}$, $\cos 15^o = \frac{\sqrt{3}+1}{2\sqrt{2}}$,

$\sin 15^o \cos 15^o = \frac{1}{2}\sin 30^o \ \frac{1}{4}$

$\sin 15 \cos 75^o = \sin^2 15^o = \frac{4-2\sqrt{3}}{8}$

32. Ans : A) 0

$4\cos 18^o - \frac{3}{\cos 18} - 2\frac{\sin 18}{\cos 18}$

$= \frac{4\cos^2 18 - 3 - 2\sin 18}{\cos 18}$

$= \frac{4\cos^3 18 - 3\cos 18 - 2\sin 18\cos 18}{\cos^2 18}$

$= \frac{\cos(3\times 18°) - \sin(2\times 18°)}{\cos^2 18}$

$= \frac{\cos 54 - \sin 36}{\cos^2 18}$

$= \frac{0}{\cos^2 18}$

$= 0$

33. Ans C) 11

$\Delta = \frac{1}{2}.\ 3.4 = 6$

$$s = \frac{1}{2}(3+4+5) = 6$$

$$r_1 = \frac{\Delta}{5-a} = \frac{6}{6-3} = 2$$

$$r_2 = \frac{\Delta}{5-b} = \frac{6}{6-4} = 3$$

$$r_3 = \frac{\Delta}{5-c} = \frac{6}{6-5} = 6$$

$$\therefore r_1 + r_2 + r_3 = 2+3+6 = 11$$

34. Ans (A)

$$\frac{1}{x} + \frac{1}{y}$$

$$y = \cot B - \cot A = \frac{1}{\tan B} - \frac{1}{\tan A} = \frac{x}{\tan A \tan B}$$

$$\tan A \ \tan B = \frac{x}{y}$$

Now cot (A – B) = $\frac{1}{\tan(A-B)}$

$$= \frac{1+\tan A \tan B}{\tan A - \tan B}$$

$$= \frac{1+ x/y}{x}$$

$$= \frac{1}{x} + \frac{1}{y}$$

35. Ans C $\sqrt{3}$

$$\sin 20^\circ \left(4 + \frac{1}{\cos 20^\circ}\right) = \frac{\sin 20^\circ}{\cos 20^\circ}(4\cos 20^\circ + 1)$$

$$= \frac{1}{\cos 20^\circ}(4\sin 20^\circ \cos 20^\circ + \sin 20^\circ)$$

$$= \frac{1}{\cos 20^\circ}(2\sin 40^\circ + \sin 20^\circ)$$

$$= \frac{1}{\cos 20^\circ}(\sin 40^\circ + \sin 40^\circ + \sin 20^\circ)$$

$$= \frac{1}{\cos 20^\circ}(\sin 40^\circ + 2\sin 30 \cos 10^\circ)$$

$$= \frac{1}{\cos 20^\circ}(\sin 40^\circ + \cos 10^\circ)$$

$= \frac{1}{\cos 20°}(\sin 40° + \sin 80°)$

$\frac{2\sin 60° \cos 20°}{\cos 20}$

$= \sqrt{3}$

36. *And D)*

Sin 8x + 7 sin 6x + 18 sin 4x + 12 sin 2x

= sin 8x + sin 6x + 6[sin6x + sin 4x] + 12 [sin 4m + sin 2x]

= 2sin 7x cos x + 6.2 sin 5x cos x + 12.2 sin 3x cos x

= 2cos x [sin 7x + 6 sin 5x + 12 sin 3x]

The given fraction is 2 cos x

37. Ans (C)

$m = \cos \alpha + \cos \beta = 2 \cos \left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$n = \sin \alpha + \sin \beta = 2\sin \left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$\therefore \frac{n}{m} = \tan\left(\frac{\alpha+\beta}{2}\right)$

Now,

$\sin(\alpha+\beta) = \frac{2\tan\left(\frac{\alpha+\beta}{2}\right)}{1+\tan^2\left(\frac{\alpha+\beta}{2}\right)} = \frac{2n/m}{1+\frac{n^2}{m^2}} = \frac{2mn}{m^2+n^2}$

38. Ans : C

$5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$

Let u =tan²x, we have

$5(u - \frac{1}{1+u}) = 2\left(\frac{1-u}{1+u}\right) + 9$

5(u² + u-1) = 2 – 2u + 9 + 9u

5u² – 2u – 16 = 0 ⇒ (5u + 8) (u – 2) = 0

But u is positive u = 2 tan² x = 2

Cos2x = $\frac{1-\tan^2 x}{1+\tan^2 x} = \frac{1-2}{1+2} = \frac{-1}{3}$

$\Rightarrow \cos 4x = 2\cos^2 2x - 1 = 2\left(\frac{-1}{3}\right)^2 - 1 = \frac{-7}{9}$

39. Ans : A)

$\cos 60° = \frac{4+25-c^2}{2.2.5} \Rightarrow \frac{1}{2} = \frac{29-c^2}{20}$

$\Rightarrow 10 = 29 - c^2$

$c^2 = 19 \Rightarrow c = \sqrt{19}$

$\cos 120 = \frac{a^2+b^2-c^2}{2ab}$

$-\frac{1}{2} = \frac{a^2+b^2-19}{2ab}$

$a^2+b^2 - 19 = -ab$

$a^2 + b^2 + ab = 19$

Area of quadrilateral = $= \frac{1}{2} \times 2 \times 5 \times \sin 60° + \frac{1}{2} ab \sin 120° = 4\sqrt{3}$

$\Rightarrow \frac{5\sqrt{3}}{2} + \frac{ab\sqrt{3}}{4} = 4\sqrt{3}$

$\frac{ab}{4} = 4 - \frac{5}{2} = \frac{8-5}{2} = \frac{3}{2}$

$ab = 6$

$a^2 + b^2 = 13$

$a = 2, b = 3$

$\therefore$ perimeter of quadrilateral = 2 + 5 + 2 + 3 = 12

40. Ans : (C) 7

$\cos x + \cos 2x + \cos 3x + \cos 4x = 0$

$(\cos x + \cos 3x) + (\cos 2x + \cos 4x) = 0$

$2 \cos x \cos 2x + 2 \cos x \cos 3x = 0$

$2 \cos x [\cos 2x + \cos 3x] = 0$

$2 \cos x \ \ 2 \cos \frac{5x}{2} \cos \frac{x}{2} = 0$

When $x = (0, 2\pi)$

$\therefore x = \pi/2, \frac{3\pi}{2}, \pi, \pi/5, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{9\pi}{5}$

Thus we have 7 solutions.

41. Ans (A) P

$\frac{\cos\theta}{p} = \frac{\sin\theta}{q} \Rightarrow \tan\theta = q/p$

$$\frac{p}{\sec 2\theta}+\frac{q}{\operatorname{cosec} 2\theta}=p\cos 2\theta+q\sin 2\theta$$

$$= p\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)+q\frac{2\tan\theta}{1+\tan^2\theta}$$

$$= p\frac{\left(1-q^2/p^2\right)}{\left(1+q^2/p^2\right)}+q\frac{2\left(q/p\right)}{\left(1+q^2/p^2\right)}$$

$$= p\frac{p^2-q^2}{p^2+q^2}+\frac{2pq^2}{p^2+q^2}=P\frac{p^2-q^2+2q^2}{p^2+q^2}$$

$$= p\frac{p^2+q^2}{p^2+q^2}=p$$

42. Ans A & C

$$a^2=b^2+c^2-2bc\cos A$$

$$=b^2+8-2b\sqrt{8}\frac{1}{\sqrt{2}}$$

$a^2 = b^2 + 8 - 4b$

$6 = b^2 + 8 - 4b$

$b^2 - 4b + 2 = 0$

$$\therefore b=2\pm\sqrt{2}$$

$$\Delta=\frac{1}{2}bc\sin A$$

$$=\frac{1}{2}(2\pm\sqrt{2})\sqrt{8}.\frac{1}{\sqrt{2}}$$

$$\Delta=(2\pm\sqrt{2})$$

43. Ans D cot 6^o

tan 42 tan 66^o tan 78^o

= tan(60 – 18) tan (60 +18) tan 66^o

$$= \frac{\tan 3\times 18}{\tan 18}\tan 66$$

$$= \frac{\tan 54\ \tan 66}{\tan 18}$$

$$= \frac{\tan(60-6)\tan(60+6)}{\tan 18} = \frac{\tan(3\times 6)}{\tan 6^\circ}.\frac{1}{\tan 18}$$

$$= \frac{1}{\tan 6^\circ}$$

$= \cot 6^{o}$

44. Ans D $\theta = 2n\pi + \pi/6$

$\tan\theta + \sec\theta = \sqrt{3}$

Taking reciprocal

$\sec\theta - \tan\theta = \frac{1}{\sqrt{3}}$

solving, we get

$\sec\theta = \frac{2}{\sqrt{3}}$, $\tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \pi/6$

$\therefore \theta = 2n\pi + \frac{\pi}{6}$

45. Ans (C) $7/5$

$\cos\alpha + \cos\beta = \frac{3}{2}$

$2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{3}{2}$ → (1)

$\sin\alpha + \sin\beta = 1/2$

$2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = 1/2$ → (2)

Dividing (2) by (1) $\tan\left(\frac{\alpha+\beta}{2}\right) = 1/3$

θ is the arithmetic mean of α and β

$\therefore \theta = \frac{\alpha+\beta}{2}$

$\tan\theta = 1/3$

$\cos\theta = 3/\sqrt{10}$

Now

$\sin 2\theta + \cos 2\theta$

$= 2\sin\theta + \cos\theta + 2\cos^2 = \theta - 1$

$= 2\left(\frac{1}{\sqrt{10}}\right)\left(\frac{3}{\sqrt{10}}\right) + 2\left(\frac{9}{10}\right) - 1$

$= \frac{6}{10} + \frac{18}{10} - 1 = \frac{14}{10} = \frac{7}{5}$