DIRECTORATE OF SCHOOL EDUCATION TAMILNADU

12JPCM07 (2023-24)	JEE PRACTICE QUESTIONS (TEST-7)	Class : XII Time : 1.15 hrs Total Marks : 180

Answer key

12th – MATHS

31. Ans: D) 2

$$\int_{-\frac{\pi}{4}}^{\pi/4} \frac{x^9 - 3x^5 + 7x^3 - x + 1}{\cos^2 x} dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x(x^9 - 3x^5 + 7x^3 - x)dx + \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x dx$$

$$= 0 + 2\int_0^{\pi/4} \sec^2 x dx$$

$$= 2[\tan x]_0^{\pi/4}$$

$$= 2\left[\tan\frac{\pi}{4} - \tan 0\right] = 2(1-0) = 2$$

32. Ans : A) 20

$$\int_0^{10\pi} |\sin x| dx = 10\int_0^{\pi} |\sin x| dx$$

$(\because |\sin x|)$ is of period π)

$$= 10\int_0^{\pi} \sin x dx$$

$$= 10\int_0^{\pi} \sin x dx$$

$$= 10[-\cos x]_0^{\pi}$$

$$= 10[1+1] = 20$$

33. Ans: A) $2\sqrt{2}\pi$

$$\int_0^{4\pi} \frac{dx}{co^2x(2+\tan^2 x)} = \int_0^{4\pi} \frac{\sec^2 xdx}{2+\tan^2 x}$$

$$= 2\int_0^{2\pi} \frac{\sec^2 xdx}{2+\tan^2 x} = 4\int_0^{\pi} \frac{\sec^2 xdx}{2+\tan^2 x} = 8\int_0^{\pi/2} \frac{\sec^2 xdx}{2+\tan^2 x}$$

$$put \times \tan x = t \quad x=0 \quad t=0$$

$$\sec^2 xdx = dt \quad x = \pi/2 \quad t=\infty$$

$$= 8\int_0^{\infty} \frac{dt}{(\sqrt{2})^2 + t^2}$$

$$= 8\times\frac{1}{2}\left(\tan^{-1}\frac{t}{\sqrt{2}}\right)_0^{\infty}$$

$$= 4\sqrt{2}\,(\tan^{-1}(\infty) - \tan^{-1}(o))$$

$$= 4\sqrt{2}\left(\frac{\pi}{2}\right) = 2\sqrt{2}\pi$$

34. Ans: A) 2

$$I = \int_0^1 xf^{11} = (2x)dx$$

$$put \ t = 2x \Rightarrow dx = dt/2$$

$$= \ = \frac{1}{4}\int_0^2 tf^{11}(t)dt$$

$$= \frac{1}{4}\left[\left(tf^1(t)\right)_0^2 - \int_0^2 f^1(t)dt\right]$$

$$= \frac{1}{4}\left[\left(t \ \ f^1(t)\right)_0^2 - \right]\left(f(t)\right)_0^2$$

$$= \frac{1}{4}\left[\left(2f^1(2)\right) - f(2) + f(0)\right]$$

$$= \frac{1}{4}[10-3+1] = 2$$

35. Ans: D)

$-\sqrt{2}-\sqrt{3}+5$

$$\int_0^2 [x^2]dx = \int_0^1 [x^2]dx + \int_1^{\sqrt{2}} [x^2]dx + \int_{\sqrt{2}}^{\sqrt{3}} [x^2]dx + \int_{\sqrt{3}}^2 [x^2]dx$$

$$= \int_0^2 o\,dx + \int_1^{\sqrt{2}} 1\,dx + \int_{\sqrt{2}}^{\sqrt{3}} 2\,dx + \int_{\sqrt{3}}^{2} 3\,dx$$
$$= 0 + [x]_1^{\sqrt{2}} + [2x]_{\sqrt{2}}^{\sqrt{3}} + [3x]_{\sqrt{3}}^{2}$$
$$= \sqrt{2} - 1 + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3}$$
$$= 5 - \sqrt{3} - \sqrt{2}$$

36. Ans : B) $2\sqrt{2}$

$$\sin x + \cos x = 0$$
$$\tan x = -1 = \tan 3\pi/4$$
$$x = \frac{3\pi}{4}$$
$$\times\int_0^{\pi} |\sin x + \cos x|\,dx = 3\int_0^{\pi/4} |\sin x + \cos x|\,dx + \int_{3\pi/4}^{\pi} |\sin x + \cos x|\,dx$$
$$= \int_0^{3\pi/4} (\sin x + \cos x)dx - \int_{3\pi/4}^{\pi} (\sin x + \cos x)dx$$
$$= (-\cos x + \sin x)_0^{3\pi/4} - (-\cos x + \sin x)_{3\pi/4}^{\pi}$$
$$= \left[\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) + 1\right] - \left[1 - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right]$$
$$= 2\sqrt{2}$$

37. Ans : B) $\frac{3}{2}$

$$I = \int_3^6 \frac{\sqrt{x}}{\sqrt{9-x} + \sqrt{x}}dx \qquad \rightarrow (1)$$

Also $I = \int_3^6 \frac{\sqrt{9-x}}{\sqrt{x} + \sqrt{9-x}}\,dx \qquad \rightarrow (2)$

$(1) + (2) \Rightarrow 2I = \int_3^6 \frac{\sqrt{x} + \sqrt{9-x}}{\sqrt{9-x} + \sqrt{x}}dx$

$$= [x]_3^6$$
$$2I = 3 \Rightarrow I = \frac{3}{2}$$

38. Ans : B)

$\times 20+\frac{1}{\sqrt{2}}$

$$=10\int_{0}^{41\frac{\pi}{4}}|\cos x|dx=\int_{0}^{10\pi}|\cos x|dx+\int_{10\pi}^{\frac{4\pi}{4}}|\cos x|dx$$

$$=10\int_{0}^{\pi}|\cos x|dx+\int_{10\pi}^{10\pi+\frac{\pi}{4}}|\cos x|dx$$

($\because |\cos x|$ is a periodic function of period π)

$$=\left[\int_{0}^{\pi/2}\cos xdx-\int_{\pi/2}^{\pi}\cos xdx\right]+\int_{0}^{\pi/4}\cos xdx$$

$$=10[(\sin x)_{0}^{\pi/2}-(\sin x)_{\pi/2}^{\pi}]+(\sin x)_{0}^{\pi/4}$$

$$=10[(1-0)-()0-1)]+\frac{1}{\sqrt{2}}$$

$$=20+\frac{1}{\sqrt{2}}$$

39. *loga, (loga + logr), (loga + 2logr),…….*

It is an A.P

$$t_n=\log a+(n-1)\log r$$

$$Sn=\frac{n}{2}(\log a+\log a+(n-1)\log r)$$

$$=\frac{n}{2}(2\log a+(n-1)\log r)$$

$$=\frac{n}{2}\log a^{2(n-1)}_{r}$$

$$\Rightarrow\frac{n}{2}\log a^2r^{n-1}$$

40. Ans: B 3, 6

$$\frac{2ab}{a+b}=4$$

$$\frac{G^2}{A}=4\Rightarrow G^2=4A$$

Given $2A + G^2 = 27$

$$A=\frac{27}{6}\Rightarrow A=\frac{9}{2}$$

$$\Rightarrow a+b=9 \qquad (1)$$

$ab = 18$ (2)

$(a-b)^2 = 81-72$

$(a-b)^2 = 9$

$a-b=3$ (3)

(1) + (3) $\Rightarrow$ a = 6

b = 3

41. $2^{\frac{1}{4}}.4^{\frac{1}{8}}.8^{\frac{1}{16}}..........\infty = 2^{\frac{1}{4}+\frac{2}{8}+\frac{3}{16}}$

$= 2^{\frac{1}{4}\left(1+2\times(\frac{1}{2})+3\times(\frac{1}{2})^2+.........\right)}$

$= 2^{\frac{1}{4}(1-\frac{1}{2})^{-2}}$

$= 2^{\frac{1}{4}\left(\frac{2}{1}\right)^2}$

= 2

42. Ans: A) A.P

$\cos A = \dfrac{b^2+c^2-a^2}{2bc}$, $\sin A = ka$ k is constant

$\cot A = \dfrac{b^2+c^2-a^2}{2abck}$

$\cot B = \dfrac{c^2+a^2-b^2}{2abck}$

$\cot C = \dfrac{a^2+b^2-c^2}{2abck}$

Given $\dfrac{b^2+c^2-a^2}{2abck}, \dfrac{c^2+a^2-b^2}{2abck}, \dfrac{a^2+b^2-c^2}{2abck}$ are in A.P.

$\Rightarrow b^2+c^2-a^2, c^2+a^2-b^2, a^2+b^2-c^2$ are in A.P

$\Rightarrow -2a^2, -2b^2-2c^2$ are in A.P

$\Rightarrow a^2, b^2, c^2$ are in A.P

43. Ans: A) 231

$a_1^2+a_2^2+a_3^2 = 33033$

$a^2+a^2r^2+a^2r^4 = 11^2\times3\times7\times13$

$a^2(1+r^2+r^4) = 11^2\times273$

$a^2(1+r^2+r^4) = 11^2(1+16+256)$

$\Rightarrow a = 11$ *and* $r = 4$

$a_1 + a_2 + a_3 = a + ar + ar^2$
$= 11 + 44 + 176$
$= 231$

44. Ans : C) 4

$AM \geq GM$

$$\frac{4^{\sin^2 x} + 4^{\cos^2 x}}{2} \geq \sqrt{4^{\sin^2 x} \times 4^{\cos^2 x}}$$

$$4^{\sin^2 x} + 4^{\cos^2 x} \geq 2\sqrt{4}$$
$$= 4$$

So minimum value of $4^{\sin^2 x} + 4^{\cos^2 x}$ is 4

45. Ans: B)

$$\frac{n(2n^2 + 9n + 13)}{24}$$

$$T_r = \frac{1^3 + 2^3 + 3^3 + \ldots\ldots + r^3}{1 + 3 + 5 + \ldots\ldots + (2r - 1)}$$

$$\rightarrow \frac{\left[r\frac{(r+1)}{2}\right]^2}{\frac{r}{2}[1 + (2r - 1)]}$$

$$= \frac{\cancel{r^2}(r+1)^2}{4} \times \frac{2}{\cancel{r} \times \cancel{2r}}$$

$$T_r = \frac{(r+1)^2}{4}$$

$$S_n = \sum_{r=1}^{n} \frac{(r+1)^2}{4}$$

$$= \frac{1}{4}\left(2^2 + 3^2 + \ldots\ldots\ldots + (n+1)^2\right)$$

$$= \frac{1}{4}\left[\frac{(n+1)(n+2)(2n+3)}{6} - 1\right]$$

$$S_n = n\frac{(2n^2 + 9n + 13)}{24}$$

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11JPCM07 (2023-24)	JEE PRACTICE QUESTIONS (TEST-7)	Class : XI Time : 1.15 hrs Total Marks : 180

Answer key

11th – MATHS

46. *loga, (loga + logr), (loga + 2logr),…….*

It is an A.P

$t_n = \log a + (n-1)\log r$

$Sn = \frac{n}{2}(\log a + \log a + (n-1)\log r)$

$= \frac{n}{2}(2\log a + (n-1)\log r)$

$= \frac{n}{2}\log a^{2}_{r}{}^{(n-1)}$

$\Rightarrow \frac{n}{2}\log a^2 r^{n-1}$

47. Ans: B 3, 6

$\frac{2ab}{a+b} = 4$

$\frac{G^2}{A} = 4 \Rightarrow G^2 = 4A$

Given $2A + G^2 = 27$

$A = \frac{27}{6} \Rightarrow A = \frac{9}{2}$

$\Rightarrow a + b = 9$ (1)

$ab = 18$ (2)

$(a-b)^2 = 81 - 72$

$(a-b)^2 = 9$

$a - b = 3$ (3)

(1) + (3) $\Rightarrow$ a = 6

$b = 3$

48. $2^{\frac{1}{4}}.4^{\frac{1}{8}}.8^{\frac{1}{16}}..........\infty = 2^{\frac{1}{4}+\frac{2}{8}+\frac{3}{16}}$

$= 2^{\frac{1}{4}\left(1+2\times(\frac{1}{2})+3\times(\frac{1}{2})^2+.........\right)}$

$= 2^{\frac{1}{4}(1-\frac{1}{2})^{-2}}$

$= 2^{\frac{1}{4}\left(\frac{2}{1}\right)^2}$

$= 2$

49. Ans: A) A.P

$\cos A = \frac{b^2+c^2-a^2}{2bc}, \ \sin A = ka$ k is constant

$\cot A = \frac{b^2+c^2-a^2}{2abck}$

$\cot B = \frac{c^2+a^2-b^2}{2abck}$

$\cot C = \frac{a^2+b^2-c^2}{2abck}$

Given $\frac{b^2+c^2-a^2}{2abck}, \frac{c^2+a^2-b^2}{2abck}, \frac{a^2+b^2-c^2}{2abck}$ are in A.P.

$\Rightarrow b^2+c^2-a^2, c^2+a^2-b^2, a^2+b^2-c^2$ are in A.P

$\Rightarrow -2a^2, -2b^2-2c^2$ are in A.P

$\Rightarrow a^2, b^2, c^2$ are in A.P

50. Ans: A) 231

$a_1^2+a_2^2+a_3^2 = 33033$

$a^2+a^2r^2+a^2r^4 = 11^2\times 3\times 7\times 13$

$a^2(1+r^2+r^4) = 11^2\times 273$

$a^2(1+r^2+r^4) = 11^2(1+16+256)$

$\Rightarrow a = 11 \ and \ r = 4$

$a_1+a_2+a_3 = a+ar+ar^2$

$= 11 + 44 + 176$

$= 231$

51. Ans : C) 4

$AM \geq GM$

$$\frac{4^{\sin^2 x}+4^{\cos^2 x}}{2} \geq \sqrt{4^{\sin^2 x} \times 4^{\cos^2 x}}$$

$$4^{\sin^2 x}+4^{\cos^2 x} \geq 2\sqrt{4}$$

$$= 4$$

So minimum value of $4^{\sin^2 x}+4^{\cos^2 x}$ is 4

52. Ans: B)

$$\frac{n(2n^2+9n+13)}{24}$$

$$T_r = \frac{1^3+2^3+3^3+......+r^3}{1+3+5+......+(2r-1)}$$

$$\rightarrow \frac{\left[r\frac{(r+1)}{2}\right]^2}{\frac{r}{2}[1+(2r-1)]}$$

$$= \frac{r^2(r+1)^2}{4} \times \frac{2}{r \times 2r}$$

$$T_r = \frac{(r+1)^2}{4}$$

$$S_n = \sum_{r=1}^{n} \frac{(r+1)^2}{4}$$

$$= \frac{1}{4}\left(2^2+3^2+..........+(n+1)^2\right)$$

$$= \frac{1}{4}\left[\frac{(n+1)(n+2)(2n+3)}{6}-1\right]$$

$$S_n = n\frac{(2n^2+9n+13)}{24}$$

53. Ans: B: p^2, q^2, r^2 are in A.P.
p, q, r are in H.P.

$$q = \frac{2pr}{p+r}$$

$$q^2(p+r)^2 = 4p^2r^2 \qquad \rightarrow (1)$$

p,q, -2r are in G.P

$$q^2 = -2pr \qquad \rightarrow (2)$$

$$(1) \Rightarrow q^2(p+r)^2 = q^4$$

$q^2[(p+r)^2-q^2]=0$

$\Rightarrow p+r=q,\ as\ \ p+q+r\neq 0$

Also $q^2(p^2+2pr+r^2)=q^4$

$q^2(p^2+r^2-q^2)=q^4$ by (2)

$q^2(p^2+r^2-2q^2)=0$

$\Rightarrow p^2+r^2-2q^2=0$

$q^2=\dfrac{p^2+r^2}{2}$

$\Rightarrow p^2, q^2, r^2$ are in A.P

54. Ans: $\dfrac{\pi^2}{8}$

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots\ldots=\left(\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\ldots\ldots\right)-\left(\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}+\ldots\ldots\ldots\right)$$

$$=\frac{\pi^2}{6}-\frac{1}{4}\left(\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots\ldots\right)$$

$$=\frac{\pi^2}{6}\left(1-\frac{1}{4}\right)$$

$$=\frac{\pi^2}{6}\times\frac{3}{4}=\frac{\pi^2}{8}$$

55. Ans : A 7260

$1\geq r\leq 10$

$S_r=\dfrac{12}{2}\left[2r+(12-1)(2r-1)\right]$

$= \ =6[2r+22r-11]$

$S_r=6(24r-11)$

$$\sum_{r=1}^{10}S_r=6\sum_{r=1}^{10}(24r-11)$$

$$=6\times\left[24\sum_{r=1}^{10}r-10\times 11\right]$$

$$=6\left[\overset{12}{\cancel{24}}\times\frac{10\times 11}{\cancel{2}}-110\right]$$

= 6 [1320 – 110]

= 6 × 1210

= 7260

56. Ans : B 16

Let the two numbers be a and b

A = $\frac{a+b}{2} \Rightarrow 2A = a+b$

Now, a, g_1, g_2, b are in G.P

$$\frac{g_1}{a} = \frac{g_2}{g_1} = \frac{b}{g_2}$$

$$\frac{g_1}{a} = \frac{g_2}{g_1} \Rightarrow \frac{g_1^2}{g_2} = a$$

and $\frac{g_2}{g_1} = \frac{b}{g_2} \Rightarrow \frac{g_2^2}{g_1} = b$

$$\frac{g_1^2}{g_2} = \frac{g_2^2}{g_1} = a+b = 2A$$

57. Ans: B)

[3, ∞]

$$AM \geq GM$$

$$\frac{\frac{x}{y}+\frac{y}{z}+\frac{z}{x}}{3} \geq \left(\frac{x}{y}\times\frac{y}{z}\times\frac{z}{x}\right)^{1/3}$$

$$\frac{x}{y}\times\frac{y}{z}\times\frac{z}{x} \geq 3$$

58. Ans: A)

$$\frac{2+\sqrt{3}}{2-\sqrt{3}}$$

$$\sqrt{ab} = \frac{4ab}{a+b}$$

$$4\sqrt{ab} = a+b$$

$$4 = \sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}}$$

if $\frac{a}{b} = x^2 \Rightarrow x + \frac{1}{x} = 4$

$$x^2 - 4x + 1 = 0$$

$$\Rightarrow \quad x = 2 \pm \sqrt{3}$$

$$x^2 = \frac{\left(2+\sqrt{3}\right)^2}{4-3}$$

$$= \frac{\left(2+\sqrt{3}\right)^2}{2^2 - \sqrt{3}^2}$$

$$\frac{a}{b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$$

59. Ans : B)

$$\frac{a}{x} + \frac{c}{y} = 2$$

$$x = \frac{a+b}{2}, \ y = \frac{b+c}{2}$$

$$\frac{a}{x} + \frac{c}{y} = \frac{2a}{a+b} + \frac{2c}{b+c}$$

$$= 2\left[\frac{ab+ac+ac+bc}{ab+ac+b^2+bc}\right]$$

$$= 2\left[\frac{ab+2ac+bc}{ab+2ac+bc}\right]$$

$$\frac{a}{x} + \frac{c}{y} = 2$$

60. Ans A) $\frac{2}{1+\sqrt{5}}$

Let the sides be *a, ar, ar*² of which *ar*² is hypotenuse $\Rightarrow r > 1$

$$a^2 r^4 = a^2 + a^2 r^2$$

$$r^4 - r^2 - 1 = 0$$

image

$$r^2 = \frac{1 \pm \sqrt{5}}{2}$$

$$\Rightarrow r^2 = \frac{1+\sqrt{5}}{2} \text{ only}$$

C is the greater acute angle

$$\cos C = \frac{a}{ar^2} = \frac{1}{r^2} = \frac{2}{1+\sqrt{5}}$$