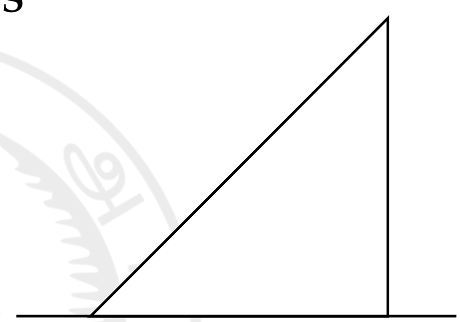
DIRECTORATE OF SCHOOL EDUCATION TAMILNADU

12JPCM10 (2023-24)	JEE PRACTICE QUESTIONS (TEST-10)	Class : XII Time : 1.15 hrs Total Marks : 180

Answer key

12th – MATHS



31. Ans : B

Here α = β = γ

$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

$$\cos\alpha = \frac{1}{\sqrt{3}}$$

PM = projection of AP and PQ

$$= |(x_2 - x_1)l + (y_2 - y_1)m + (z_2 - z_1)n|$$

$$= \left|(-2+3)\frac{1}{\sqrt{3}} + (3-5)\frac{1}{\sqrt{3}} + (1-2)\frac{1}{\sqrt{3}}\right|$$

$$= \frac{2}{\sqrt{3}}$$

$$AP = \sqrt{6}$$

$$AM = \sqrt{(AP)^2 - (PM)^2} = \sqrt{6 - \frac{4}{3}} = \sqrt{\frac{14}{3}}$$

32. Ans : A

$l + m + n = 0 \quad \rightarrow \quad (1)$

$l^2 + m^2 - n^2 = 0$

$\therefore l^2 + m^2 - (-l - m)^2 = 0$ by (1)

$lm = 0$

$l = 0$ (or) $m = 0$

case I : If $l = 0$

then $m + n = 0$

$$\frac{l}{o}=\frac{m}{-1}=\frac{n}{1}=\frac{1}{\sqrt{2}}$$

Case II : m = 0

$l + n = 0$

$$\frac{l}{-1}=\frac{m}{0}=\frac{n}{1}=\frac{1}{\sqrt{2}}$$

Angle between the lines is given by

Cos θ = 0 (-1) × (-1) + (-1) (0) + (1) (1)

= 1

$$\theta=\frac{\pi}{2}$$

33. Ans : D

Let P divides the line paining (3, 2, -1) and (6, 2, 2) in the ratio λ : 1

$$x=\frac{6\lambda+3}{\lambda+1},\quad y=\frac{2\lambda+2}{\lambda+1},\quad z=\frac{2\lambda-1}{\lambda+1}$$

$$x=\frac{6\lambda+3}{\lambda+1} \qquad y=\frac{2(2)+2}{2+1} \qquad z=\frac{2(2)-1}{2+1} \quad =\frac{3}{3}=1$$

$$5=\frac{6\lambda+3}{\lambda+1} \qquad =\frac{6}{3} \qquad z = 1$$

5λ + 5 = 6λ + 3

λ = 2 y = 2

$$\therefore P=(5,2,1)$$

Direction cosines of OP = $\left(\frac{5}{\sqrt{30}},\frac{2}{\sqrt{30}},\frac{1}{\sqrt{30}}\right)$

34. Ans: B

$$L:\frac{x-2}{3}=\frac{y-(-1)}{-2}=\frac{z-2}{0}$$

$$M:\frac{x-1}{1}=\frac{y-\left(\frac{-3}{2}\right)}{\frac{\alpha}{2}}=\frac{z-(-5)}{2}$$

L and M are perpendicular

$$3(1)-2\left(\frac{\alpha}{2}\right)+0\,(2)=0$$

3 – α = 0

α = 3

$$M:\frac{x-1}{1}=\frac{y-\left(\frac{-3}{2}\right)}{\frac{3}{2}}=\frac{z-(-5)}{2},\ N=\frac{x-1}{-3}=\frac{y-\frac{1}{2}}{-2}=\frac{z-0}{4}$$

Let θ be the angle between M and N

$$\cos\theta=\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$

$$\vec{a}=\vec{\imath}+|\frac{3}{2}\vec{\jmath}+2\vec{k},\vec{b}=-3\hat{\imath}-2\hat{\jmath}+4\hat{k}$$

$$\cos\theta=\frac{2}{\sqrt{29}\sqrt{29}}=\frac{4}{29}\Rightarrow\sec\theta=\frac{29}{4}$$

$$\theta=\sec^{-1}\left(\frac{29}{4}\right)$$

35. Ans : C

We have z = 0 for the point where the line intersects the curve

$$\frac{x-2}{3}=\frac{y+1}{2}=\frac{0-1}{-1}$$

$$\frac{x-2}{3}=1,\ \frac{y+1}{2}=1$$

$x = 5$ $y = 1$

$xy = c^2$

$xy = c^2$

$(5)\ (1) = c^2$

$c = \pm\sqrt{5}$

36. Ans: B

The direction cosines *l, m, n* of the line are given by

$$\frac{l}{6}=\frac{m}{2}=\frac{n}{3}=\frac{\sqrt{l^2+m^2+m^2}}{\sqrt{6^2+2^2+3^2}}=\frac{1}{\sqrt{49}}=\frac{1}{7}$$

$$(l,m,n)=\left(\frac{6}{7},\frac{2}{7},\frac{3}{7}\right)$$

The required length of projection is given by

$$|l(x_2-x_1)+m(y_2-y_1)+n(z_2-z_1)|$$

$$=\left|\frac{6}{7}(2-(-1))+\frac{2}{7}(5-0)+\frac{3}{7}(1-3)\right|$$

$$=\left|\frac{18}{7}+\frac{10}{7}-\frac{6}{7}\right|=\frac{22}{7}$$

37. Ans: B

Centre of a tetrahedron = (1, 2, -1)

$$\left(\frac{1}{4}\sum x, \frac{1}{4}\sum y, \frac{1}{4}\sum z\right)=(1,2,-1)$$

$$\frac{a+1+2+o}{4}=1,\ \frac{2+b+1+0}{4}=2,\ \frac{3+2+c+0}{4}=-1$$

a = 1, b = 5, c = -9

p (a, b, c) = P (1, 5, -9)

The distance from p to ox, oy, oz are

$$\sqrt{b^2+c^2}, \sqrt{c^2+a^2}, \sqrt{a^2+b^2} == \sqrt{5^2+(-9)^2}, \sqrt{(-9)^2+1^2}, \sqrt{1^2+5^2}$$

$$= \sqrt{106}, \sqrt{82}, \sqrt{26}$$

38. Ans : B

$$\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}=\lambda,\ \frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}=\mu$$

Given lines are intersecting

(2λ + 1, 3λ - 1, 4λ + 1) = (μ + 3, 2μ + k, μ)

2λ +1 = μ + 3 → (1)

3λ – 1 = 2μ + k → (2)

μ = 4 λ + 1 → (3)

Solving (1), (3) and putting the value of λ and μ in (2) we get K = $\frac{9}{2}$

39. Ans : B)

$$\frac{x-2}{1}=\frac{y+3}{-2}=\frac{z+5}{-2}$$

Direction cosines of this line is $\frac{1}{3}, \frac{-2}{3}, \frac{-2}{3}$

Hence, the equation of line can be put in the form $\frac{x-2}{1/3}=\frac{y+3}{-2/3}=\frac{z+5}{-2/3}=r$

Any point on its is $\left(2+\frac{r}{3}, -3-\frac{2r}{3} -5-\frac{2r}{3}\right)$

Given r = ± 6

Points are (4, -7,-9) and (0, 1, -1)

40. Ans : A)

$(\hat{i}+j+k)x+(3\hat{i}-3j+k)y+(-4\hat{i}+5j)=\lambda(x\hat{i}-yj+2k)$

comparing co-efficients

$$(1-\lambda)x+3y-4z=0$$
$$x-(3+\lambda)y+5z=0$$
$$3x+y-\lambda z=0$$
$$\begin{vmatrix}1-\lambda & 3 & -4\\ 1 & -(3+\lambda) & 5\\ 3 & 1 & -\lambda\end{vmatrix}=0$$
$$\lambda(\lambda+1^2)=0$$

λ= 0, - 1

41. Ans : B

Let the given position vectors be of points A, B and C respectively then

$$|\overrightarrow{AB}|=\sqrt{(\beta-\alpha)^2+(\gamma-\beta)^2+(\alpha-\gamma)^2}$$
$$|\overrightarrow{BC}|=\sqrt{(\gamma-\beta)^2+(\alpha-\gamma)^2+(\beta-\alpha)^2}$$
$$|\overrightarrow{CA}|=\sqrt{(\alpha-\gamma)^2+(\beta-\alpha)^2+(\gamma-\beta)^2}$$
$$|\overrightarrow{AB}|=|\overrightarrow{BC}|=|\overrightarrow{CA}|$$

$\therefore \Delta$ABC is an equilateral triangle

42. Ans : D

Given $\vec{a}=\hat{i}+j+k \ \ \vec{b}=4\hat{i}+3j+4k \text{ and } \vec{c}=\hat{i}+\alpha j+\beta k$ are linearly independent

$$\vec{a}=\hat{i}+j+k \ \ \vec{b}=4\hat{i}+3j+4k \text{ and } \vec{c}=\hat{i}+\alpha j+\beta k$$
$$\begin{vmatrix}1 & 1 & 1\\ 4 & 3 & 4\\ 1 & \alpha & \beta\end{vmatrix}=0$$

1- β = 0

β = 1

$$|\vec{c}|=\sqrt{3}\Rightarrow 1+\alpha^2+\beta^2=3$$
$$a^2=1 \quad \alpha=\pm 1$$

43. Ans : B

$$\left|(\vec{a}\times\vec{b})\times\vec{c})\right| = \left|\vec{a}\times\vec{b}\right|\left|\vec{c}\right| \sin 30°$$

$$= \frac{1}{2}\left|\vec{a}\times\vec{b}\right|\left|\vec{c}\right|$$

$$\vec{a} = 2\hat{i} + j - 2k, \quad \vec{b} = \hat{i} + j$$

$$\vec{a}\times\vec{b} = 2\hat{i} - 2j + k$$

$$\left|\vec{a}\times\vec{b}\right| = \sqrt{9} = 3$$

$$\left|\vec{c}-\vec{a}\right| = 2\sqrt{2}$$

$$\left|\vec{c}-\vec{a}\right|^2 = 8$$

$$\left|\vec{c}\right|^2 + \left|\vec{a}\right|^2 - 2\vec{a}.\vec{c} = 8$$

$$\vec{a}.\vec{c} = |\vec{c}|, \quad |\vec{a}| = 3$$

$$\therefore \left|\vec{c}\right|^2 - 2\left|\vec{c}\right| + 1 = 0$$

$$\left(\left|\vec{c}\right| - 1\right)^2 = 0$$

$$\left|\vec{c}\right| = 1$$

$$\therefore \left|(\vec{a}\times\vec{b})\times\vec{c}\right| = \frac{1}{2}(3)\ (1) = \frac{3}{2}$$

44. Ans : C

Any Vector coplanar is $\vec{a}$ and $\vec{b}$ is

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

$$= \hat{i} - j + k + \lambda(2\hat{i} + j + k)$$

$$= (1+2\lambda)\hat{i} + (-1+\lambda)j + (1+\lambda)k$$

$\vec{r}$ is orthogonal to $5\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$

$$5(1+2\lambda) + 2(-1+\lambda) + 6(1+\lambda) = 0$$

$$\lambda = -\frac{1}{2}$$

$$\therefore \vec{r} = 3j - k$$

Received unit vector = $\hat{r} = \dfrac{3j-k}{\sqrt{10}}$

45. Ans : B

Given $|\vec{a}| = |\vec{b}| = 1$

$$(\vec{a} + 2\vec{b}) . (5\vec{a} - 4\vec{b}) = 0$$

$$5|\vec{a}|^2 - 8|\vec{b}|^2 - 4\vec{a}.\vec{b} + 10\vec{b}.\vec{a} = 0$$

$$5 - 8 + 6\vec{a}.\vec{b} = 0$$

$$6|\vec{a}|\,|\vec{b}|\cos\theta = 3$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = 60^\circ$$

DIRECTORATE OF SCHOOL EDUCATION TAMILNADU

11JPCM10 (2023-24)	JEE PRACTICE QUESTIONS (TEST-10)	Class : XI Time : 1.15 hrs Total Marks : 180

Answer key

11th – MATHS

31. Ans : A)

$$(\hat{i}+j+k)x+(3\hat{i}-3j+k)y+(-4\hat{i}+5j)=\lambda(x\hat{i}-yj+2k)$$

comparing co-efficients

$$(1-\lambda)x+3y-4z=0$$
$$x-(3+\lambda)y+5z=0$$
$$3x+y-\lambda z=0$$
$$\begin{vmatrix}1-\lambda & 3 & -4\\ 1 & -(3+\lambda) & 5\\ 3 & 1 & -\lambda\end{vmatrix}=0$$
$$\lambda(\lambda+1^2)=0$$

$\lambda = 0, -1$

32. Ans : B

Let the given position vectors be of points A, B and C respectively then

$$|\overrightarrow{AB}|=\sqrt{(\beta-\alpha)^2+(\gamma-\beta)^2+(\alpha-\gamma)^2}$$
$$|\overrightarrow{BC}|=\sqrt{(\gamma-\beta)^2+(\alpha-\gamma)^2+(\beta-\alpha)^2}$$
$$|\overrightarrow{CA}|=\sqrt{(\alpha-\gamma)^2+(\beta-\alpha)^2+(\gamma-\beta)^2}$$
$$|\overrightarrow{AB}|=|\overrightarrow{BC}|=|\overrightarrow{CA}|$$

$\therefore \Delta ABC$ is an equilateral triangle

33. Ans : D

Given $\vec{a}=\hat{i}+j+k \quad \vec{b}=4\hat{i}+3j+4k \text{ and } \vec{c}=\hat{i}+\alpha j+\beta k$ are linearly independent

$$\vec{a}=\hat{i}+j+k \quad \vec{b}=4\hat{i}+3j+4k \text{ and } \vec{c}=\hat{i}+\alpha j+\beta k$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \alpha & \beta \end{vmatrix}=0$$

$1-\beta=0$

$\beta=1$

$|\vec{c}|=\sqrt{3} \Rightarrow 1+\alpha^2+\beta^2=3$

$a^2=1 \quad \alpha=\pm 1$

34. Ans : B

$\left|(\vec{a}\times\vec{b})\times\vec{c})\right|=\left|\vec{a}\times\vec{b}\right|\left|\vec{c}\right|\sin 30^\circ$

$=\frac{1}{2}\left|\vec{a}\times\vec{b}\right|\left|\vec{c}\right|$

$\vec{a}=2\hat{i}+j-2k, \quad \vec{b}=\hat{i}+j$

$\vec{a}\times\vec{b}=2\hat{i}-2j+k$

$\left|\vec{a}\times\vec{b}\right|=\sqrt{9}=3$

$\left|\vec{c}-\vec{a}\right|=2\sqrt{2}$

$\left|\vec{c}-\vec{a}\right|^2=8$

$\left|\vec{c}\right|^2+\left|\vec{a}\right|^2-2\vec{a}.\vec{c}=8$

$\vec{a}.\vec{c}=|\vec{c}|, \quad |\vec{a}|=3$

$\therefore \left|\vec{c}\right|^2-2\left|\vec{c}\right|+1=0$

$\left(\left|\vec{c}\right|-1\right)^2=0$

$\left|\vec{c}\right|=1$

$\therefore \left|(\vec{a}\times\vec{b})\times\vec{c}\right|=\frac{1}{2}(3)\ (1)=\frac{3}{2}$

35. Ans : C

Any Vector coplanar is $\vec{a}$ and $\vec{b}$ is

$$\vec{r}=\vec{a}+\lambda\vec{b}$$

$$=\hat{i}-j+k+\lambda(2\hat{i}+j+k)$$

$$= (1+2\lambda)\hat{i} + (-1+\lambda)\,j + (1+\lambda)k$$

$\vec{r}$ is orthogonal to $5\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$

$$5(1+2\lambda) + 2(-1+\lambda) + 6(1+\lambda) = 0$$

$$\lambda = -\frac{1}{2}$$

$$\therefore \vec{r} = 3j - k$$

Received unit vector = $\hat{r} = \dfrac{3j-k}{\sqrt{10}}$

36. Ans : B

Given $|\vec{a}| = |\vec{b}| = 1$

$$\left(\vec{a} + 2\vec{b}\right).\,(5\vec{a} - 4\vec{b}) = 0$$

$$5|\vec{a}|^2 - 8|\vec{b}|^2 - 4\vec{a}.\vec{b} + 10\vec{b}.\vec{a} = 0$$

$$5 - 8 + 6\vec{a}.\vec{b} = 0$$

$$6|\vec{a}|\,|\vec{b}|\cos\theta = 3$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = 60^\circ$$

37. Ans : A

$\vec{a}.\,\vec{b} = 0$

$\vec{a}$ and $\vec{b}$ are mutually perpendicular

$$|\vec{a}| = 2, \quad |\vec{b}| = 3$$

Let $\vec{a} = 2\hat{i}, \vec{b} = 3j$ such that $\vec{a} = \hat{i},\ \vec{b} = j$

$$(\vec{a} \times (\vec{a} \times (\vec{a} \times (\vec{a} \times \vec{b})))) = 2\hat{i} \times (2i \times (2i \times (2\vec{j} \times 3\vec{j}))))$$

$$= 48\,j = 48\,b$$

38. Ans : B)

Angular velocity $\vec{w} = 4\left(\dfrac{\hat{i} + 2j - 2k}{\sqrt{1+4+4}}\right) = \dfrac{4}{3}\left(\hat{i} + 2j - 2k\right)$

$$\vec{r} = \overrightarrow{OP} - \overrightarrow{OA}$$

$$= \left(4\hat{i} + j + k\right) - (3\hat{i} - 2j - k)$$

$\hat{i}+3j+2k$

$\vec{v}=\vec{w}\times\vec{r}=\frac{4}{3}\left(\hat{i}+2j-2k\right)\times(\hat{i}+3j+2k)$

$=\frac{4}{3}(10\hat{i}-4j+k)$

39. Ans : D

Let $\vec{a}=2x^2\hat{i}+4xj+k,\ \vec{b}=7\hat{i}-2j+xk.$ The angle between $\vec{a}$ and $\vec{b}$ is obtuse.

$$\vec{a}.\vec{b}<0$$

$$14x^2 - 8x + x < 0$$

$$x\ (2x - 1) < 0$$

$$x\in(0,\frac{1}{2}) \quad \rightarrow (1)$$

Also Given that $\vec{b}.\vec{k}=x$ and

$$\frac{\vec{b}.\vec{k}}{|\vec{b}|}<\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$$

$$2x>\sqrt{3}\ \sqrt{53+x^2}$$

$$x^2>159 \quad \rightarrow (2)$$

From (1), (2) there is no common values for x

40. Ans: D

$$(3\vec{a}+\vec{b}).(\vec{a}-4\vec{b})=3|\vec{a}|^2-11\vec{a}.\vec{b}-4|\vec{b}|^2$$

$$= 3(36) - 11\ (6.8.\ \cos\ \pi) - 4(64) > 0$$

$\therefore$ Angle between $\vec{a}$ and $\vec{b}$ is a cute

$\therefore$ The longer diagonal is given by

$$\vec{\alpha}=(3\vec{a}+\vec{b})+(\vec{a}-4\vec{b})=4\vec{a}-3\vec{b}$$

$$\overrightarrow{|\alpha|}^2=|4\vec{a}-3\vec{b}|^2=16|\vec{a}|^2+9|\vec{b}|^2=4\vec{a}.\vec{b}$$

$$= 16(36) + 9(64) - 24\ (6)\ (8)\ \cos\pi$$

$$= 16 \times 144$$

$$|4\vec{a}-3\vec{b}|=48$$

41. Ans : C

Let x be the projection

$$\vec{a} = x\frac{(\hat{i}+j)}{\sqrt{2}} + x\frac{(-\hat{i}+j)}{\sqrt{2}} + xk$$

$$\vec{a} = \frac{2xj}{\sqrt{2}} + xk$$

$$= \frac{\sqrt{2}}{\sqrt{3}}j + \frac{k}{\sqrt{3}}$$

42. Ans : B)

$$\overrightarrow{DA} = \vec{a}, \overrightarrow{AB} = \vec{b} \text{ and } \overrightarrow{CB} = k\vec{a}$$

Given X and Y are the mid points of DB and AC then

$$\overrightarrow{OX} = \frac{\overrightarrow{OB}+\overrightarrow{OD}}{2}, \overrightarrow{OY} = \frac{\overrightarrow{OA}+\overrightarrow{OC}}{2}$$

$$\overrightarrow{XY} = \overrightarrow{OY} - \overrightarrow{OX} = \frac{\overrightarrow{DA}+\overrightarrow{BC}}{2} = \frac{\vec{a}-k\vec{a}}{2} = \frac{(1-k)\vec{a}}{2}$$

$$\left|\overrightarrow{xy}\right| = \pm\left(\frac{1-k}{2}\right)a = 4 \qquad \text{(Given)}$$

Taking +ve sign

$$\left(\frac{1-k}{2}\right)17 = 4 \; [|\vec{a}| = 17]$$

$$1-k = \frac{8}{17}$$

$$k = \frac{9}{17}$$

Taking – ve sign

$$-\left(\frac{1-k}{2}\right)17 = 4$$

$$k = \frac{25}{17}$$

43. Ans : A)

$$\vec{r} = \vec{b} + t\vec{c}$$

$$\vec{r} = (1+t)\vec{i} + (2+t)\vec{j} - (1+2t)\vec{k}$$

Projection of $\vec{r}$ on $\vec{a}$ is $\sqrt{\frac{2}{3}}$

$$\frac{\vec{r}.\vec{a}}{|\vec{a}|} = \sqrt{\frac{2}{3}}$$

$$\frac{2(1+t)-(2+t)-(1+2t)}{\sqrt{6}} = \pm\sqrt{\frac{2}{3}}$$

-t – 1 = ± 2

t = -3, 1

$$\therefore\ \vec{r} = -2\hat{i} - j + 5k$$

$$\vec{r} = 2\hat{\imath} + 3\hat{\jmath} - 3\hat{k}$$

44. Ans : A

$$\vec{a} = 7\hat{i} - 4j - 4k,\ \vec{b} = -2\hat{i} - j + 2k$$

Angel bisector of A divides BC in the ration $|\overrightarrow{AB}| : |\overrightarrow{AC}|$

$$\overrightarrow{AD} = \left[\frac{9(-2\hat{i} - j + 2k) + 3(7\hat{i} - 4j - 4k)}{9+3}\right] = \frac{\hat{i} - 7j + 2k}{4}$$

$$\vec{C} = \left(\frac{\overrightarrow{AD}}{|\overrightarrow{AD}|}\right) 5\sqrt{6} = \frac{5}{3}\left(\hat{i} - 7j + 2k\right)$$

45. Ans: B)

Let *xo* be the first term and x be the common ratio of G.P.

$$a = x_o x^{p-1}$$

$$b = x_o x^{q-1}$$

$$c = x_o x^{r-1}$$

$\log a = \log x_o + (p-1)\log x,\ \log b = \log x_o + (q-1)\log x,\ \log c = \log x_o + (r-1)\log x$

Now

$$(\log a^2\hat{i} + \log b^2 j + \log c^2 k).\ ((q-r)\hat{i} + (r-p)j + 1 + (p-q)k).$$

$$= \sum 2\log a(q-r) = 2\sum \log x_o + (p-1)\log x][q-r] = 0$$

$\therefore$ Angle between them is $\frac{\pi}{2}$