



# DIRECTORATE OF SCHOOL EDUCATION TAMILNADU

12JPCM11 (2023-24)	JEE PRACTICE QUESTIONS (TEST-11)	Class : XII Time : 1.15 hrs Total Marks : 180
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## Answer key

### 12th - MATHS

31. Ans: C)

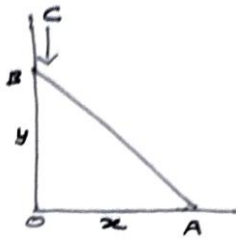
OC - wall

AB - Ladder

OA = x, OB = y, AB = 20ft

$$x^2 + y^2 = 20^2$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$



$$\frac{dy}{dt} = \frac{-x}{\sqrt{400 - x^2}} \frac{dx}{dt}$$
$$= \frac{-16}{\sqrt{400 - 16^2}} \frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{-4}{3} \frac{dx}{dt}$$

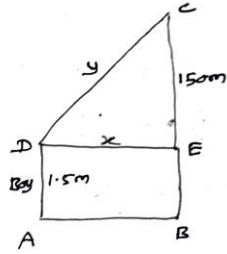
-ve sign indicates, y decreases.

32. Ans: B)

C - Kite position

BC = 151.5 m

CE = BC - BE = 150m



$$y^2 = x^2 + 150^2$$

$$\frac{dy}{dt} = \frac{x}{y} \frac{dx}{dt}$$

$$= \frac{x}{y} (10)$$

$$= \frac{10\sqrt{y^2 - 150^2}}{y}$$

$$= \frac{10\sqrt{(250)^2 - (150)^2}}{250} = 8 \text{ m/s}$$

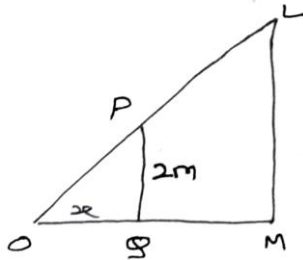
33. Ans: B)

L - Lamp

PQ - Man

OQ = x (shadow)

Let MQ = y



$$\frac{dy}{dt} = \text{speed of man} = 3 \text{ m/s}$$

$\Delta OPQ$  and  $\Delta OLM$  are similar

$$\frac{OM}{OQ} = \frac{LM}{PQ}$$

$$\frac{x+y}{x} = \frac{5}{2}$$

$$y = \frac{3}{2}x$$

$$\frac{dy}{dt} = \frac{3}{2} \frac{dx}{dt}$$

$$3 = \frac{3}{2} \frac{dx}{dt}$$

$$\frac{dx}{dt} = 2m/s$$

34. Ans: B)

$r$  = base radius

$l$  = slant height

$h$  = height

$$l^2 = r^2 + h^2, \quad h = \frac{l}{\sqrt{2}}, \quad r = \frac{l}{\sqrt{2}}$$

$$V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \left( \frac{l}{\sqrt{2}} \right)^2 \left( \frac{l}{\sqrt{2}} \right) = \frac{\pi}{6\sqrt{2}} l^3$$

$$\frac{dV}{dt} = \frac{3\pi}{6\sqrt{2}} (l^2) \frac{dl}{dt}$$

$$2 = \frac{3\pi}{6\sqrt{2}} (4)^2 \frac{dl}{dt}$$

$$\frac{dl}{dt} = \frac{\sqrt{2}}{4\pi} \text{ cm/s}$$

$$\lambda = \frac{\sqrt{2}}{4\pi}$$

$$40\pi\sqrt{2}\lambda = 40\pi\sqrt{2} \left( \frac{\sqrt{2}}{4\pi} \right) = 10 \times 2 = 20$$

35. Ans: B)

$$f^{-1}(x) = \left[ \frac{\sqrt{a+4}}{1-a} - 1 \right] (5x^4 - 3) < 0$$

$$\left[ \frac{\sqrt{a+4}}{1-a} - 1 \right] x^4 < \frac{3}{5}$$

$$\left[ \frac{\sqrt{a+4}}{1-a} - 1 \right] \leq 0$$

$$\frac{\sqrt{a+4}}{1-a} \leq 1 \quad [\because -4 \leq a < 1] \quad \rightarrow (1)$$

$$a + 4 \leq (1-a)^2$$

$$a^2 - 3a - 3 \geq 0$$

$$a \in \left(-\infty, \frac{3 - \sqrt{21}}{2}\right] \cup \left[\frac{3 + \sqrt{21}}{2}, \infty\right)$$

But  $-4 \leq a < 1$

$$a \in \left[-4, \frac{3 - \sqrt{21}}{2}\right] \text{ and } a > 1, (1) \text{ always true}$$

$$\text{Hence } a \in \left[-4, \frac{3 - \sqrt{21}}{4}\right] \cup [1, \infty)$$

36. Ans: B)

$$f'(x) = (ab - b^2 + 2) + \cos^4 x + \sin^4 x < 0$$

$$= ab - b^2 - 2 + (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x < 0$$

$$ab - b^2 - 1 < \frac{1}{2} (\sin^2 2x) < \frac{1}{2}$$

$$2ab - 2b^2 - 2 < 1$$

$$2b^2 - 2ab + 3 > 0$$

$$(-2a)^2 - 4 \cdot 2 \cdot 3 < 0$$

$$a^2 < 6$$

$$-\sqrt{6} < a < \sqrt{6}$$

37. Ans: B)

Since  $\sin^{-1}(\sin x + \cos x)^3$  and  $(\sin x + \cos x)^3$  are both increasing function,  $f(x)$  is increasing when  $\sin x + \cos x$  is increasing

$$\text{Let } g(x) = \sin x + \cos x$$

$$g'(x) = \cos x - \sin x = \sqrt{2} \cos\left(x + \frac{\pi}{4}\right) > 0$$

(or)

$$\cos\left(x + \frac{\pi}{4}\right) > 0$$

$$2n\pi - \frac{\pi}{2} < x + \frac{\pi}{4} < 2n\pi + \frac{\pi}{2}, n \in I$$

$$2n\pi - \frac{3\pi}{4} < x < 2n\pi + \frac{\pi}{4}$$

$$-\frac{3\pi}{4} < x < \frac{\pi}{4} \text{ and } \frac{5\pi}{4} < x < \frac{9\pi}{4}$$

But

$$x \in (0, 2\pi)$$

$$x \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right)$$

Then  $a = 0, b = 1, c = 5, d = 8$

$$\therefore a + 10b + 100c + 1000d = 8510$$

38. Ans: B)

$$f^1(x) = 3 \left[ 1 - \frac{\sqrt{21+4b-b^2}}{b+1} \right] x^2 + 5 > 0$$

$$1 - \frac{\sqrt{21+4b-b^2}}{b+1} \geq 0$$

$$\frac{\sqrt{21+4b-b^2}}{b+1} \leq 1 \quad \rightarrow (1)$$

$$21 - 4b - b^2 \geq 0 \Rightarrow -7 \leq b \leq 3 \quad \rightarrow (2)$$

$$\text{Case I : If } b + 1 < 0, b < -1 \quad \rightarrow (3)$$

Then Inequality (2) is always true, then from (2) and (3) we get

$$-7 \leq b < -1 \quad \rightarrow (4)$$

$$\text{Case II : If } b + 1 > 0, b > -1 \quad \rightarrow (5)$$

Form (1)

$$\sqrt{21-4b-b^2} \leq b+1$$

$$21 - 4b - b^2 \leq b^2 + 2b + 1$$

$$(b + 5)(b - 2) \geq 0$$

$$b \in (-\infty, -5) \cup (2, \infty) \quad \rightarrow (6)$$

$$\text{Form (5), (6) we get } b \geq 2 \quad \rightarrow (7)$$

From (4), (7) we get

$$b \in [-7, -1] \cup [2, \infty)$$

39. Ans: C)

$$\lim_{x \rightarrow \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2} = \lim_{x \rightarrow \alpha} \frac{1 - \cos[a(x - \alpha)](x - \beta)}{(x - \alpha)^2}$$

$$\lim_{x \rightarrow \alpha} \frac{2 \sin^2 \left( \frac{a(x - \alpha)(x - \beta)}{2} \right)}{(x - \alpha)^2}$$

$$\lim_{x \rightarrow \alpha} \frac{2 \sin^2 \left( \frac{a(x - \alpha)(x - \beta)}{2} \right)}{a^2(x - \alpha)^2(x - \beta)^2} \times \frac{a^2(x - \beta)^2}{4}$$

$$= \frac{a^2(\alpha - \beta)^2}{2}$$

40. Ans: C)

$$\lim_{x \rightarrow 0} \frac{x(1 + a \cos x) - b \sin x}{x^3} = 1$$

$$\lim_{x \rightarrow 0} \frac{(1 + a \cos x) - b \frac{\sin x}{x}}{x^2} = 1$$

$$\lim_{x \rightarrow 0} \frac{[1 + a(1 - \frac{x^2}{2!} + \frac{x^4}{4!} \dots)] \dots - b(1 - \frac{x^2}{3!} + \frac{x^4}{5!} \dots)}{x^2} = 1$$

$$\lim_{x \rightarrow 0} \frac{(1 + a - b) + \left(\frac{b}{3!} - \frac{a}{2!}\right)x^2 + \dots}{x^2} = 1$$

It is possible only when  $(1 + a - b) = 0$

$$\frac{b}{3!} - \frac{a}{2!} = 1$$

On solving we get  $a = \frac{-5}{2}, b = \frac{-3}{2}$

41. Ans: C)

$$\lim_{x \rightarrow \infty} \frac{729^x - 243^x - 81^x + 9^x + 3^x - 1}{x^3}$$

$$\lim_{x \rightarrow \infty} \frac{243^x(3^x - 1) - 9^x(9^x - 1) + (3^x - 1)}{x^3}$$

$$\lim_{x \rightarrow \infty} \frac{243^x(3^x - 1) - 9^x(3^x - 1) + (3^x + 1) + (3^x - 1)}{x^3}$$

$$\lim_{x \rightarrow \infty} \frac{[243^x - 9^x(3^x + 1) + 1](3^x - 1)}{x^3}$$

$$\lim_{x \rightarrow \infty} \left(\frac{27^x - 1}{x}\right) \left(\frac{9^x - 1}{x}\right) \left(\frac{3^x - 1}{x}\right)$$

$\log 27. \log 9. \log 3$

$$= 6[\log 3]^3$$

$$\therefore K = 6, M = 3, N = 3$$

$$\therefore KM + MN + NK = \frac{(K + M + N)^2 - (K^2 + M^2 + N^2)}{2} = 45$$

42. Ans: C)

$$\begin{aligned}
& \lim_{x \rightarrow 1} \frac{x + x^2 + \dots + x^n - n}{x - 1} \\
&= \lim_{x \rightarrow 1} \left[ \frac{x + x^2 + \dots + x^n - 1 - 1 - 1 \dots - 1}{x - 1} \right] \\
&= \lim_{x \rightarrow 1} \left[ \frac{(x - 1) + (x^2 - 1) + (x^3 - 1) + \dots + (x^n - 1)}{x - 1} \right] \\
&= \lim_{x \rightarrow 1} [1 + (x + 1) + (x^2 + x + 1) + \dots] \\
&= 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}
\end{aligned}$$

43. Ans: B)

For  $x > 0$

$\sin x < x$

$$\frac{\sin x}{x} < 1$$

$$\frac{99 \sin x}{x} > 99$$

$$\left[ \frac{99 \sin x}{x} \right] = 98$$

$$\therefore \lim_{x \rightarrow 0} \left[ \frac{100x}{\sin x} \right] + \left[ \frac{99 \sin x}{x} \right] = 100 + 98 = 198$$

$$\frac{x}{\sin x} > 1$$

$$\frac{100x}{\sin x} > 100$$

$$\left[ \frac{100x}{\sin x} \right] = 100$$

44. Ans: B)

$$y^2 - 4y + 11 = (y-2)^2 + 7$$

$\min(y^2 - 4y + 11) = 7$  (min is at  $y = 2$ )

$$\lim_{x \rightarrow 0} \left[ \min(y^2 - 4y + 11) \frac{\sin x}{x} \right]$$

$$\lim_{x \rightarrow 0} \left[ 7 \frac{\sin x}{x} \right]$$

For  $x > 0$

$\sin x < x$

$$\frac{\sin x}{x} < 1$$

$$\frac{7 \sin x}{x} < 7$$

For  $x < 0$

$\sin x > x$

$$\frac{\sin x}{x} < 1$$

$$\frac{7 \sin x}{x} < 7$$

$$\left[ \frac{7 \sin x}{x} \right] = 6$$

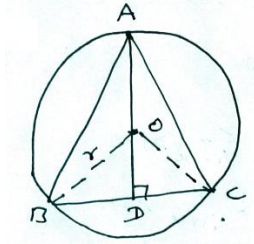
$$\therefore RHL = 6$$

$$LHL = RHL = 6$$

$$\left[ \frac{7 \sin x}{x} \right] = 6$$

$$LHL = 6$$

45. Ans: C)



$$AD = h$$

$$OD = h - r$$

$$BC = 2 DC = 2 \sqrt{r^2 - (h - r)^2} = 2\sqrt{2hr - h^2}$$

$$\text{Area } \Delta = \frac{1}{2} (BC)h = \frac{1}{2} (2\sqrt{2hr - h^2})h$$

$$h^{3/2} \sqrt{2r - h}$$

$$AB = AC = \sqrt{h^2 + (DC)^2} = \sqrt{h^2 + r^2 - (h - r)^2}$$

$$= \sqrt{2hr}$$

$$p = AB + BC + CA = \sqrt{2hr} + 2\sqrt{2hr - h^2}$$

$$= 2h^{1/2} (\sqrt{2r} + \sqrt{2r - h})$$

$$\therefore \lim_{h \rightarrow 0} \frac{\Delta}{p^3} = \lim_{h \rightarrow 0} \frac{h^{3/2} \sqrt{2r - h}}{8h^{3/2} (\sqrt{2r} + \sqrt{2r - h})^3} = \frac{\sqrt{2r}}{8(2\sqrt{2r})^3} = \frac{1}{128r}$$





# DIRECTORATE OF SCHOOL EDUCATION TAMILNADU

<b>11JPCM11 (2023-24)</b>	<b>JEE PRACTICE QUESTIONS (TEST-11)</b>	<b>Class : XI Time : 1.15 hrs Total Marks : 180</b>
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## Answer key

### 11<sup>th</sup> - MATHS

31. Ans: C)

$$\begin{aligned} \lim_{x \rightarrow \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2} &= \lim_{x \rightarrow \alpha} \frac{1 - \cos[a(x - \alpha)](x - \beta)}{(x - \alpha)^2} \\ \lim_{x \rightarrow \alpha} \frac{2 \sin^2\left(\frac{a(x - \alpha)(x - \beta)}{2}\right)}{(x - \alpha)^2} & \\ \lim_{x \rightarrow \alpha} \frac{2 \sin^2\left(\frac{a(x - \alpha)(x - \beta)}{2}\right)}{a^2(x - \alpha)^2(x - \beta)^2} \times \frac{a^2(x - \beta)^2}{4} & \\ = \frac{a^2(\alpha - \beta)^2}{2} & \end{aligned}$$

32. Ans: C)

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{x(1 + a \cos x) - b \sin x}{x^3} &= 1 \\ \lim_{x \rightarrow 0} \frac{(1 + a \cos x) - b \frac{\sin x}{x}}{x^2} &= 1 \\ \lim_{x \rightarrow 0} \frac{[1 + a(1 - \frac{x^2}{2!} + \frac{x^4}{4!} \dots)] - b(1 - \frac{x^2}{3!} + \frac{x^4}{5!} \dots)}{x^2} &= 1 \\ \lim_{x \rightarrow 0} \frac{(1 + a - b) + \left(\frac{b}{3!} - \frac{a}{2!}\right)x^2 + \dots}{x^2} &= 1 \end{aligned}$$

It is possible only when  $(1 + a - b) = 0$

$$\frac{b}{3!} - \frac{a}{2!} = 1$$

On solving we get  $a = \frac{-5}{2}$ ,  $b = \frac{-3}{2}$

33. Ans: C)

$$\lim_{x \rightarrow \infty} \frac{729^x - 243^x - 81^x + 9^x + 3^x - 1}{x^3}$$

$$\lim_{x \rightarrow \infty} \frac{243^x(3^x - 1) - 9^x(9^x - 1) + (3^x - 1)}{x^3}$$

$$\lim_{x \rightarrow \infty} \frac{243^x(3^x - 1) - 9^x(3^x - 1) + (3^x + 1) + (3^x - 1)}{x^3}$$

$$\lim_{x \rightarrow \infty} \frac{[243^x - 9^x(3^x + 1) + 1](3^x - 1)}{x^3}$$

$$\lim_{x \rightarrow \infty} \left( \frac{27^x - 1}{x} \right) \left( \frac{9^x - 1}{x} \right) \left( \frac{3^x - 1}{x} \right)$$

$$\log 27. \log 9. \log 3$$

$$= 6[\log 3]^3$$

$$\therefore K = 6, M = 3, N = 3$$

$$\therefore KM + MN + NK = \frac{(K + M + N)^2 - (K^2 + M^2 + N^2)}{2} = 45$$

34. Ans: C)

$$\lim_{x \rightarrow 1} \frac{x + x^2 + \dots + x^n - n}{x - 1}$$

$$= \lim_{x \rightarrow 1} \left[ \frac{x + x^2 + \dots + x^n - 1 - 1 - 1 \dots - 1}{x - 1} \right]$$

$$= \lim_{x \rightarrow 1} \left[ \frac{(x - 1) + (x^2 - 1) + (x^3 - 1) + \dots + (x^n - 1)}{x - 1} \right]$$

$$= \lim_{x \rightarrow 1} [1 + (x + 1) + (x^2 + x + 1) + \dots]$$

$$= 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

35. Ans: B)

For  $x > 0$

$$\sin x < x$$

$$\frac{\sin x}{x} < 1$$

$$\frac{99 \sin x}{x} > 99$$

$$\left[ \frac{99 \sin x}{x} \right] = 98$$

$$\therefore \lim_{x \rightarrow 0} \left[ \frac{100x}{\sin x} \right] + \left[ \frac{99 \sin x}{x} \right] = 100 + 98 = 198$$

$$\frac{x}{\sin x} > 1$$

$$\frac{100x}{\sin x} > 100$$

$$\left[ \frac{100x}{\sin x} \right] = 100$$

36. Ans: B)

$$y^2 - 4y + 11 = (y-2)^2 + 7$$

$$\min(y^2 - 4y + 11) = 7 \text{ (min is at } y = 2)$$

$$\lim_{x \rightarrow 0} \left[ \min(y^2 - 4y + 11) \frac{\sin x}{x} \right]$$

$$\lim_{x \rightarrow 0} \left[ 7 \frac{\sin x}{x} \right]$$

For  $x > 0$

$$\sin x < x$$

$$\frac{\sin x}{x} < 1$$

$$\frac{7 \sin x}{x} < 7$$

$$\left[ \frac{7 \sin x}{x} \right] = 6$$

$$\therefore RHL = 6$$

$$LHL = RHL = 6$$

For  $x < 0$

$$\sin x > x$$

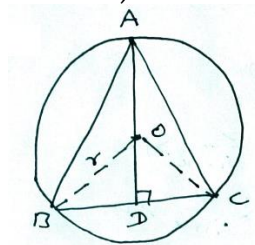
$$\frac{\sin x}{x} < 1$$

$$\frac{7 \sin x}{x} < 7$$

$$\left[ \frac{7 \sin x}{x} \right] = 6$$

$$LHL = 6$$

37. Ans: C)



$$AD = h$$

$$OD = h - r$$

$$BC = 2 DC = 2 \sqrt{r^2 - (r-h)^2} = 2\sqrt{2hr - h^2}$$

$$\text{Area } \Delta = \frac{1}{2} (BC)h = \frac{1}{2} (2\sqrt{2hr - h^2})h$$

$$h^{3/2}\sqrt{2r-h}$$

$$AB = AC = \sqrt{h^2 + (DC)^2} = \sqrt{h^2 + r^2 - (h-r)^2}$$

$$= \sqrt{2hr}$$

$$p = AB + BC + CA = \sqrt{2hr} + 2\sqrt{2hr - h^2}$$

$$= 2h^{1/2}(\sqrt{2r} + \sqrt{2r-h})$$

$$\therefore \lim_{h \rightarrow 0} \frac{\Delta}{p^3} = \lim_{h \rightarrow 0} \frac{h^{3/2}\sqrt{2r-h}}{8h^{3/2}(\sqrt{2r} + \sqrt{2r-h})^3} = \frac{\sqrt{2r}}{8(2\sqrt{2r})^3} = \frac{1}{128r}$$

38. Ans: A)

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{2\sqrt{2} \left[ 1 - \frac{1}{2\sqrt{2}} (\cos x + \sin x)^3 \right]}{1 - \sin 2x}$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{2\sqrt{2} \left[ 1 - \left( \frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x \right)^3 \right]}{1 - \sin 2x}$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{2\sqrt{2} \left[ 1 - \cos^3 \left( \frac{\pi}{4} - x \right) \right]}{1 - \sin 2x}$$

Put  $x = \frac{\pi}{4} + h$  then

$$= \lim_{h \rightarrow 0} \frac{2\sqrt{2} [1 - \cos^3 h]}{1 - \cos 2h} = \lim_{h \rightarrow 0} \frac{2\sqrt{2} [1 - \cos^3 h]}{2\sin^2 h}$$

$$= \lim_{h \rightarrow 0} \frac{2\sqrt{2} [1 - \cos^3 h][1 + \cosh + \cos^2 h]}{2(1 - \cosh)(1 + \cosh)}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{2}(1 + \cosh + \cos^2 h)}{1 + \cosh} = \frac{3\sqrt{2}}{2}$$

39. Ans: B)

Since  $f$  is continuity at  $x = 0$

$$\lim_{x \rightarrow 0} f(x) = f(0) = C$$

$$C = \lim_{x \rightarrow 0} \left( \frac{1 + a \cos 3x + b \cos 4x}{x^2 \sin^2 x} \right)$$

since it has a finite limit, so  $a + b + 1 = 0$

$\rightarrow (1)$

$$\begin{aligned} \therefore C &= \lim_{x \rightarrow 0} \left( \frac{-a - b + a \cos 2x + b \cos 4x}{x^4} \right) \\ &= \lim_{x \rightarrow 0} \left[ \frac{-a(1 - \cos 2x) - b(1 - \cos 4x)}{x^4} \right] \\ &= \lim_{x \rightarrow 0} \left[ \frac{-a \left( \frac{2 \sin^2 x}{x^2} \right) - b \left( \frac{2 \sin^2 2x}{x^2} \right)}{x^2} \right] \end{aligned}$$

Since it has a finite limit

$$2a + 8b = 0 \quad \rightarrow (2)$$

Solving (1), (2)  $a = \frac{-4}{3}$ ,  $b = \frac{1}{3}$

$$\begin{aligned} C &= \lim_{x \rightarrow 0} \left[ \frac{4(1 - \cos 2x) - (1 - \cos 4x)}{3x^4} \right] \\ &= \lim_{x \rightarrow 0} \left[ \frac{8 \sin^2 x - 8 \sin^2 x \cos^2 x}{3x^4} \right] \\ &= \lim_{x \rightarrow 0} \left[ \frac{8 \sin^2 x \times \sin^2 x}{(3)x^2 \times x^2} \right] \\ C &= \frac{8}{3} \end{aligned}$$

$$\therefore a + b + c + \frac{1}{3} = \frac{4}{3} + \frac{1}{3} + \frac{8}{3} + \frac{1}{3} = 2$$

40. Ans: D)

Since  $f$  is continuous at  $x = 0$

$$f(0) = \lim_{x \rightarrow 0} f(x), \quad \lim_{x \rightarrow 4} f(x) = f(4)$$

$$\begin{aligned} f(0) &= \frac{\left( \frac{4^x - 1}{x} \right)^3}{\sin \left( \frac{x}{p} \right) \log \left[ 1 + \left( \frac{x^2}{3} \right) \right]} \\ &= \frac{\left( \frac{x}{p} \times p \right) \left( \frac{x^2}{3} \times 3 \right)}{3p(\log 4)^3} \\ &= 3p(\log 4)^3 \\ &= 3p(\log 4)^3 = 12(\log 4)^3 \\ p &= 4 \end{aligned}$$

41. Ans: C)

$f$  is continuous at  $x = 0$

$$\lim_{x \rightarrow 0} f(x) = f(0)$$

$$\lim_{x \rightarrow 0} \left[ \frac{36^x - 9^x - 4^x + 1}{\sqrt{2} - \sqrt{1 + \cos x}} \right] = \lim_{x \rightarrow 0} \frac{(9^x - 1)(4^x - 1)(\sqrt{2} + \sqrt{1 + \cos x})}{1 - \cos x}$$

$$\lim_{x \rightarrow 0} \frac{\left(\frac{9^x - 1}{x}\right) \left(\frac{4^x - 1}{x}\right) (\sqrt{2} + \sqrt{1 + \cos x})}{\frac{2\sin^2\left(\frac{x}{2}\right)}{\frac{x^2}{4} \times 4}}$$

$$= \log 9 \cdot \log 4 \cdot 4\sqrt{2}$$

$$\therefore \log 9 \cdot \log 4 \cdot 4\sqrt{2} = K$$

$$16\sqrt{2} \log 3 \cdot \log 2 = K$$

42. Ans: D)

$$f(x) = [x \sin \pi x]$$

$$0 < x \sin \pi x \leq 1$$

$$f(x) = [x \sin \pi x] = 0$$

$\therefore f(x)$  is continuous and differentiable in  $(-1, 1)$

43. Ans: C)

$$f(0) = f(0^+) = \lim_{h \rightarrow 0} f(0+h)$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{a^2 - ah + h^2} - \sqrt{a^2 + ah + h^2}}{\sqrt{a+h} - \sqrt{a-h}}$$

$$= \lim_{h \rightarrow 0} \frac{(a^2 - ah + h^2)^{\frac{1}{2}} - (a^2 + ah + h^2)^{\frac{1}{2}}}{(a^2 - ah + h^2) - (a^2 + ah + h^2)} \times (-2ah) \bigg/ \frac{(a+h)^{3/2} - (a-h)^{1/2}}{(a+h) - (a-h)} \times 2h$$

$$= -ah \lim_{h \rightarrow 0} \frac{\frac{1}{2}(a^2 + ah + h^2)^{-\frac{1}{2}}}{\frac{1}{2}(a+h)^{-\frac{1}{2}}}$$

$$= -a \frac{(a^2)^{-\frac{1}{2}}}{(a)^{-\frac{1}{2}}} = -\sqrt{a}$$

44. Ans: D)

$f(o) = \text{RHL of } f(x) \text{ at } o$

$$= \lim_{x \rightarrow o^+} f(x)$$

$$= \lim_{h \rightarrow o^+} f(o+h)$$

$$= \lim_{h \rightarrow o} \frac{2 - (256 - 7h)^{\frac{1}{8}}}{(5h + 32)^{\frac{3}{5}} - 2}$$

$$\lim_{h \rightarrow o} \frac{\frac{(256)^{\frac{1}{8}} - (256 - 7h)^{\frac{1}{8}}}{256 - (256 - 7h)} \times 7h}{\frac{(5h + 32)^{\frac{3}{5}} - (32)^{\frac{3}{5}}}{(5h + 32) - 32} \times 5h}$$

$$= \frac{\frac{7\left(\frac{1}{8}\right)(256)^{-\frac{1}{8}}}{\frac{1}{5}(32)^{-\frac{4}{5}}} = \frac{7}{8}(2)^{-3} = \frac{7}{64}}$$

45. Ans: C)

$$\lim_{x \rightarrow \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right) = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\pi}{2} f(x)$$

$$h \rightarrow 0 f\left(\frac{\pi}{2} - h\right) = a = h \rightarrow 0 f\left(\frac{\pi}{2} + h\right)$$

$$h \rightarrow 0 \frac{1 - \cos^3 h}{3 \sin^2 h} = a = h \rightarrow 0 \frac{b(1 - \cosh)}{4h^2}$$

$$h \rightarrow 0 \frac{(1 - \cos h)(1 + \cosh + \cos^2 h)}{3(1 - \cos h)(1 + \cosh)} = a = h \rightarrow 0 \frac{b(1 - \cosh)}{4h^2}$$

$$\frac{3}{3(2)} = a = \frac{b}{8}$$

$$\frac{b}{a} = 8 \Rightarrow \left(\frac{b}{a}\right)^{5/3} = (8)^{5/3} = 32$$