DIRECTORATE OF SCHOOL EDUCATION
TAMILNADU

12JPCM12 (2023-24)	JEE PRACTICE QUESTIONS (TEST-12)	Class : XII Time : 1.15 hrs Total Marks : 180

Answer key

12th – MATHS

31. Ans: B) $2|x|$

$$f(x)=\begin{cases}-x^2, & x<0\\ x^2, & x\geq 0\end{cases}$$

$$f^1(x)=\begin{cases}-2x, & x<0\\ 2x, & x\geq 0\end{cases}$$

$$=2|x|$$

32. Ans: A) 0

$$f(x)=\sqrt{x-1}+\sqrt{25+(x-1)-10\sqrt{x-1}}$$

$$=\sqrt{x-1}+\sqrt{(5-\sqrt{x-1})^2}$$

$$=\sqrt{x-1}+(5-\sqrt{x-1})$$

$$f(x)=5\Rightarrow f^1(x)=0$$

33. Ans: C) 1

$$\frac{dy}{dx}=\begin{vmatrix}\cos x & -\sin x & \cos x\\ \cos x & -\sin x & \cos x\\ x & 1 & 1\end{vmatrix}+\begin{vmatrix}\sin x & \cos x & \sin x\\ -\sin x & -\cos x & -\sin x\\ x & 1 & 1\end{vmatrix}+\begin{vmatrix}\sin x & \cos x & \sin x\\ \cos x & -\sin x & \cos x\\ 1 & 0 & 0\end{vmatrix}$$

$= 0 - 0 + 1\ (\cos^2 x + \sin^2 x)$

$= 1$

34. Ans: B) 1

$$y=(1+x)(1+x^2)(1+x^4).....(1+x^{2^n})$$

$$\frac{(1-x)(1+x)(1+x^2)(1+x^4)....(1+x^{2^n})}{(1-x)}$$

$$y = \frac{1-x^{2^{n+1}}}{1-x}$$

$$\frac{dy}{dx} = \frac{(1-x)\times -2^{n+1}x^{2^{n+1}-1} + (1-x^{2^{n+1}})}{(1-x)^2}$$

$$\left(\frac{dy}{dx}\right)_{x=1} = 1$$

35. Ans: A) A.P

Let $f(x) = ax^2+bx+c$

$f(1) = a + b + c$

$f(-1) = a - b + c$

$a + b + c = a - b + c$

$b = 0$

Also $2b = a + c$

$f^1(x) = 2ax + b = 2ax$

$f^1(a) = 2a^2$

$f^1(b) = 2ab$

$f^1(c) = 2ac$

=> *They are in A.P*

36. Ans: C) $\frac{1}{e\log 5}$

$$f(x) = \log_5(\log_3 x) = \log_5\left(\frac{\log x}{\log 3}\right)$$

$$= \log_5(\log x) - \log_5(\log 3)$$

$$f(x) = \frac{\log(\log x)}{\log 5} - \log_5(\log 3)$$

$$f^1(x) = \frac{1}{\log 5}\times\frac{1}{\log x}\times\frac{1}{x} - 0$$

$$f^1(e) = \frac{1}{\log 5}\times\frac{1}{\log e}\times\frac{1}{e}$$

$$f^1(e) = \frac{1}{e\log 5}$$

37. Ans : C) $1+[g(x)]^5$

$$f(g(x)) = x$$
$$f^1(g(x)) \times g^1(x) = 1$$
$$g^1(x) = \frac{1}{f^1(g(x)}$$
$$g^1(x) = 1+[g(x)]^5$$

38. Ans: C) -y

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-1/2}\left[\frac{(1+x)(-1)-(1-x)(1)}{(1+x)^2}\right]$$
$$= \frac{1}{2}\sqrt{\frac{1+x}{1-x}}\left[\frac{-2}{(1+x)^2}\right]$$
$$\frac{dy}{dx} = -\sqrt{\frac{1+x}{1-x}} \times \frac{1}{(1+x)^2}$$
$$(1-x^2)\frac{dy}{dx} = -(1-x^2)\sqrt{\frac{1+x}{1-x}} \times \frac{1}{(1+x)^2}$$
$$= -(1+x)(1-x)\sqrt{\frac{1+x}{1-x}} \times \frac{1}{(1+x)^2}$$
$$= -\sqrt{\frac{1-x}{1+x}}$$
$$(1-x^2)\frac{dy}{dx} = -y$$

39. Ans: C) $\frac{-3}{\sqrt{9-x^2}}$

$$\frac{d}{dx}[\cos^{-1}(4\left(\frac{x}{3}\right)^3 - 3\left(\frac{x}{3}\right)]$$
$$\frac{d}{dx}[3\cos^{-1}\left(\frac{x}{3}\right)]$$
$$= 3 \times \frac{-1}{\sqrt{1-(x/3)^2}} \times \frac{1}{3}$$

$= \frac{-3}{\sqrt{9-x^2}}$

40. Ans: C) $\frac{1}{\sqrt{2}}$

$\frac{dy}{dx} = \sec(\tan^{-1}x)\tan(\tan^{-1}x) \times \frac{1}{1+x^2}$

$\left(\frac{dy}{dx}\right)_{x=1} = \sqrt{2} \times 1 \times \frac{1}{2}$

$\frac{1}{\sqrt{2}}$

41. Ans: C) 5

$\lim_{h\to 0} \frac{1}{h} f(1+h) = 5$

$\lim_{h\to 0} \frac{f^1(1+h)(0+1)}{1} = 5$

$f^1(1) = 5$

42. Ans: B) 3

$y = \frac{1}{x} \Rightarrow \frac{dy}{dx} = \frac{-1}{x^2} \Rightarrow x^2 dy + dx = 0$

$\frac{x^2 xy}{\sqrt{1+x^4}} + \frac{dx}{\sqrt{1+x^4}} = 0$

$\frac{dy}{\sqrt{\frac{1}{x^4}+1}} + \frac{dx}{\sqrt{1+x^4}} = 0 \Rightarrow \frac{dy}{\sqrt{1+y^4}} + \frac{dx}{\sqrt{1+x^4}} = 0$

Req.value = 0 + 3 = 3

43. Ans: A) $\frac{1}{4}e^{x/2}\cot e^{x/2}$

$\frac{d}{dx}[\log\sqrt{\sin\sqrt{e^x}}] = \frac{d}{dx}[\frac{1}{2}\log\sin\sqrt{e^x}]$

$\frac{1}{2}\cot\sqrt{e^x} \times \frac{1}{2\sqrt{e^x}} e^x$

$= \frac{1}{4}\cot\sqrt{e^x}e^{x/2}$

$= \frac{1}{4}e^{x/2}\cot e^{x/2}$

44. Ans: B) $\frac{1}{1+(1+x)^2}$

$$y=\sum_{r=1}^{x}\tan^{-1}\frac{1}{1+r+r^2}=\sum_{r=1}^{x}\tan^{-1}\left(\frac{(r+1)-r}{1+(r+1)r}\right)$$

$$\sum_{r=1}^{x}[\tan^{-1}(r+1)-\tan^{-1}r]$$

$= (tan^{-1}(2) - tan^{-1}(1)) + (tan^{-1}(3) - tan^{-1})2) + \ldots + (tan^{-1}(x+1) - tan^{-1}(x)$

$y = tan^{-1}(x+1) - tan^{-1}(1)$

$$\frac{dy}{dx}=\frac{1}{1+(x+1)^2}$$

45. Ans: C) $\frac{-1}{14}$

$8f(x) + 6f(\frac{1}{x}) = x + 5$ - (1)

$8f(\frac{1}{x}) + 6f(x) = \frac{1}{x} + 5$ - (2)

$$(1)\times 8 \Rightarrow\ 64f(x)+48f(\frac{1}{x})=8x+40$$

$$(2)\times 6 \Rightarrow\ 36f(x)+48f(\frac{1}{x})=\frac{6}{x}+30$$

$$28f(x)=8x-\frac{6}{x}+10$$

$$f(x)=\frac{1}{28}\left(8x-\frac{6}{x}+10\right)$$

$y=x^2 f(x)$

$= x^2 \times \frac{1}{28}\left(8x - \frac{6}{x} + 10\right)$

$y = \frac{1}{28}(8x^3 - 6x + 10x^2)$

$\frac{dy}{dx} = \frac{1}{28}(24x^2 - 6 + 20x)$

At x = 1, $\frac{dy}{dx} = \frac{-1}{14}$

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11JPCM12 (2023-24)	JEE PRACTICE QUESTIONS (TEST-12)	Class : XI Time : 1.15 hrs Total Marks : 180

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$$y = \frac{1-x^{2^{n+1}}}{1-x}$$

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