DIRECTORATE OF SCHOOL EDUCATION TAMILNADU

12JPCM13 (2023-24)	JEE PRACTICE QUESTIONS (TEST-13)	Class : XII Time : 1.15 hrs Total Marks : 180

Answer key

12th – MATHS

31. Ans: C)

$F(x) = 2x^3 - 9ax^2 + 12a^2x + 1$

$f1(x) = 6x^2 - 18ax + 12a^2$

$f1(x) = 6(x^2 - 39x + 2a^2)$

$f1(x) = 6(x^2 - 39x + 2a^2)$

$f1(x) = 6(x - a)(x - 2a)$

For maximum of minimum $F1(x) = 0$

$6(x - a)(x - 2a) = 0$

$x = a \qquad x = 2a$

Thus p = point of maximum = a

Q = point of minimum = 2a

Now $p^2 = q \Rightarrow a^2 = 2a$

$a = 2$

32. Ans: C

$f(x) = 2^{(x^2-3)^3+27}$

Let $g(x) = (x^2 - 3)^2 + 27$

$g^1(x) = 3(x^2 - 3)^2 (2x)$

For maximum of minimum $g^1(x) = 0$

$3(x^2 - 3)^2 (2x) 50$

$x = 0 \quad x = \pm\sqrt{3}$

Thus the point of local minimum is x = 0

Hence the minimum value of

$f(x) = 2^{(0-3)^3+27}$

$= 2^0$

$= 1$

33. Ans: A)

$f(x) = \sin 2x - x$

$f(x) = 2\cos 2x - 1$

For max or min $f(x) = 0 \Rightarrow 2\cos 2x - 1 = 0$

$\cos 2x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{-\pi}{6}$

$f\left(+\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{5}{6} = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$

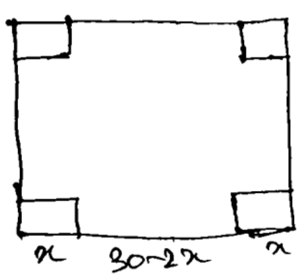
$f\left(-\pi/6\right) = \sin\left(-5/6\right) + \pi/6 = \pi/2 = \frac{-\sqrt{3}}{2} + \pi/6$

$f\left(\pi/2\right) = \sin(\pi) - \pi/2 = -\pi/2$

$f\left(-\pi/2\right) = \sin(-\pi) + \pi/2 = \pi/2$

Difference = $\left(-\pi/2 - \left(\pi/2\right)\right) = \pi$

34. Ans : C



$V = (30 - 2x)^2 x$

$\frac{dv}{dx} = (30 - 2x)(30 - 6x)$

$T.S.A = (30 - 2x)2 + 4(30 = 2x)$

$= 400 + 400$

$= 800\ cm^2$

35. Ans: C)

$$f(x) = ax^2 + \frac{b}{x} \Rightarrow f^1(x) = 2ax - \frac{b}{x^2}$$

So $f^1(x) = 0 \Rightarrow 2ax = \frac{b}{x2} \Rightarrow x = \left(\frac{b}{2a}\right)^{1/3}$

$$f^{11}(x) = 2a + \frac{2b}{x^3} \Rightarrow$$

$$= 2a + \frac{2b}{b/2a}$$

= 2a + 4a

$$f^{11}(x) = 6a > 0$$

So, $f(x)$ is minimum at $x = \left(\frac{b}{2a}\right)^{1/3}$

$$\therefore \min f = a\left(\frac{b}{2a}\right)^{2/3} + b.\left(\frac{2a}{b}\right)^{1/3}$$

$$= \left(\frac{27ab^2}{4}\right)^{1/3} \geq C$$

$$\Rightarrow \frac{ab^2}{C^3} \geq \frac{4}{27}$$

36. Ans: C) 3

Let $P(x) = ax^3 + bx^2 + cx + d$

$P^1(x) = 3ax^2 + 2bx + c$

$P^{11}(x) = 6ax + 2b$

$P1(x)$ has minimum at $x = 1$

$P^{11}(1) = 6a + 2b \quad \Rightarrow \quad b = -3a$

$P^1\ (x)\ 3ax^2 - 6ax + c$

$P(x)$ has maximum at $x = -1 \Rightarrow P^1\ (-1) = 0$

$3a + 6a + c = 0\ c = -9a$

$P^1(x) = 3ax^2 - 6ax - 9a$

$= 3a\ (x^2 - 2x - 3)$

$= 3a\ (x+1)\ (x-3)$

$P(x)$ has minimum at $x = 3$

37. Ans: D)

$f^1(x) = 3 \sin^2 x \cos x + 2\lambda \sin x \cos x$

$= \sin x \cos x\, (3 \sin x + 2\lambda)$

$f^1(x) =$ 0 has two roots in $\left(-\pi/2, \pi/2\right)$

$x = 0,$ $\qquad x = \sin^{-1}\left(\frac{-2\lambda}{3}\right)$

$\Rightarrow -1 < \frac{-2\lambda}{3} < 1$

$-3 < -2\lambda < 3$

$\frac{-3}{2} < -\lambda < \frac{3}{2}$

λ = 0 gives only one root

$\therefore \lambda = \left(\frac{-3}{2}, 0\right) \cup \left(0, 3/2\right)$

38. Ans: A) 6, 0

$f(x) = |(x-2)\,(x-3)|$

$= \begin{cases} x^2 - 5x + 6, & 0 \le x \le 2 \\ -x^2 + 5x - 6, & 2 < x \le 5/2 \end{cases}$

$f^1(x)$ $2x-5 < 0$ *for* $0 \le x \le 2,$ Decrease from 6 to 0 in (0, 2)

$f1(x) = 5 - 2x > 0$ *for* $2 \le x \le \frac{5}{2}$

$f(x)$ decease from o to $\frac{1}{4}$ is $\left(2, 5/2\right)$

∴ Greatest value = 6, least value = 0

39. *Ans : C)*

Let $x = a \cos^2 \theta + b \sin^2 \theta$

$\therefore a - x = (a-b)\sin^2\theta, x - b = (a-b)\cos^2\theta$

$\therefore y = (a-b)\sin\theta\cos\theta - (a-b)\theta$

$$= \frac{(a-b)\sin 2\theta}{2} - (a-b)\theta$$

$$\Rightarrow \frac{y}{\theta} = (a-b)2\cos 2\theta - (a-b)$$

$= -(a-b)2\sin^2\theta = -2(a-b)\sin^2\theta$

and $\frac{dx}{d\theta} = (b-a)\sin 2\theta$

$$\therefore \ \frac{dy}{dx} = \frac{2(a-b)\sin^2\theta}{(b-a)\sin 2\theta} = \tan\theta = \sqrt{\left(\frac{a-x}{x-b}\right)}$$

40. Ans : A)

$$y = \tan^{-1}\sqrt{\left(\frac{1-\cos\left(\frac{\pi}{2}+x\right)}{1+\cos\left(\frac{\pi}{2}+x\right)}\right)} = \tan^{-1}\left|tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right| \qquad \text{.... (i}$$

Now, $\frac{\pi}{2} < x < \pi$

$\therefore \quad \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$

or $\quad \frac{\pi}{2} < \frac{\pi}{4} + \frac{x}{2} < \frac{3\pi}{4}$

$\therefore \quad \left|\tan\left(\frac{\pi}{2}+\frac{x}{2}\right)\right| = -\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$ ($\because$ in II quadrant)

$$= \tan\left\{\pi - \left(\frac{\pi}{4}+\frac{x}{2}\right)\right\}$$

From Eq, (i)

$$y = \tan^{-1}\tan\left\{\pi - \left(\frac{\pi}{4}+\frac{x}{2}\right)\right\}$$

$$= \pi - \left(\frac{\pi}{4}+\frac{x}{2}\right)$$

$$= \frac{3\pi}{4} - \frac{x}{2}$$

($\because$ Principle value of $\tan^{-1}$ tan x in - $\frac{\pi}{2} to \frac{x}{2}$)

$$\therefore \frac{dy}{dx} = -\frac{1}{2}$$

41. Ans : C)

Let $y = f(x) \Rightarrow x = f^{-1}(y)$

then $\quad f(x) = x + \tan x$

$\Rightarrow \quad y = f^{-1}(y) + \tan(f^{-1}(y)$

$\Rightarrow \quad y = g(y) + \tan(g(y) \ or\ x = g(x) + \tan(g(x))$ .... (i)

Differentiating both sides, then we get

$$1 = g^1(x) + sec^2\, g(x).\, g^1(x)$$

$$g^1(x)=\frac{1}{1+\sec^2(g(x))}=\frac{1}{1+1+\tan^2(g(x))}$$

$$=\frac{1}{2+(x-g(x))^2}$$ [from Eq. (i)]

$$=\frac{1}{2+(g(x)-x)^2}$$

42. Ans : C)

$$x^2+y^2=t-\frac{1}{t}, x^4+y^4=t^2+\frac{1}{t^2}$$

$$=\left(t-\frac{1}{t}\right)^2+2$$

$$=X^4+y^4+2x^2y^2+2$$

$\therefore$ $x^2y^2 = -1$

$\Rightarrow$ $x^2.2y\frac{dy}{dx}+y^2.2x=0$

$\Rightarrow$ $x^3y\frac{dy}{dx}=-x^2y^2=1$

43. Ans : B)

Let $u=\sin^{-1}\left(\frac{2x}{1+x^2}\right)=2\tan^{-1}x$

$\therefore$ $\frac{du}{dx}=\frac{2}{1+x^2}$

and $v=\tan^{-1}\left(\frac{2x}{1-x^2}\right)=2\tan^{-1}x$

Differential coefficient

$\therefore$ $\frac{dv}{dx}=\frac{2}{1+x^2}$

$\therefore$ $\frac{du}{dv}=\frac{\left(\frac{du}{dx}\right)}{\left(\frac{dv}{dx}\right)}=1$

44. Ans : C)

Since, $y = \sin x^\circ$

$$=\sin\left(\frac{\pi x}{180}\right)$$

$$\therefore \frac{dy}{dx} = \frac{\pi}{180}\cos\left(\frac{\pi x}{180}\right)$$

$$= \frac{\pi}{180} = \cos x^2$$

and = u = $cos\ x$

$$\therefore \frac{du}{dx} = \sin x$$

Then $\frac{dy}{du} = \frac{\frac{dy}{dx}}{\frac{du}{dx}} = \frac{\frac{\pi}{180}\cos x^\circ}{\sin x}$

$$= -\frac{\pi}{180}\cos x^\circ \cos ec\, x$$

45. Ans : C)

$$\sqrt{x^2+y^2} = ae^{\tan^{-1}\left(y/x\right)}$$

$$\frac{1}{2\sqrt{x^2+y^2}}(2x+2yy^1) = a.e^{\tan^{-1}}(y/x) \times \frac{1}{\left(1+\frac{y^2}{x^2}\right)} \times \frac{xy^1 - y}{x^2}$$

$$\Rightarrow \frac{x+yy'}{\sqrt{x^2+y^2}} = \sqrt{x^2+y^2} \times \frac{x^2}{(x^2+y^2)} \times \frac{xy' = y}{x^2}$$ [From Eq. (i)]

$$\therefore \quad x + yy' = xy' - y \Rightarrow y' = \frac{x+y}{x-y}$$

$$\therefore \quad y'' = \frac{2(xy'-y)}{(x-y)^2}$$

$$y''(0) = \frac{2(0-y(0)}{\{0-y(0)\}} = \frac{-2}{y(0)} = \frac{-2}{ae^{\pi/2}} = \frac{-2}{a}e^{-\pi/2}$$

DIRECTORATE OF SCHOOL EDUCATION TAMILNADU

11JPCM13 (2023-24)	**JEE PRACTICE QUESTIONS (TEST-13)**	**Class : XI Time : 1.15 hrs Total Marks : 180**

Answer key

11th – MATHS

31.Ans : C)

Let $x = a\cos^2\theta + b\sin^2\theta$

$\therefore a-x=(a-b)\sin^2\theta, x-b=(a-b)\cos^2\theta$

$\therefore y=(a-b)\sin\theta\cos\theta-(a-b)\theta$

$$=\frac{(a-b)\sin 2\theta}{2}-(a-b)\theta$$

$$\Rightarrow \frac{y}{\theta}=(a-b)2\cos 2\theta-(a-b)$$

$=-(a-b)2\sin^2\theta=-2(a-b)\sin^2\theta$

and $\frac{dx}{d\theta}=(b-a)\sin 2\theta$

$$\therefore \frac{dy}{dx}=\frac{2(a-b)\sin^2\theta}{(b-a)\sin 2\theta}=\tan\theta=\sqrt{\left(\frac{a-x}{x-b}\right)}$$

32. Ans : A)

$$y=\tan^{-1}\sqrt{\left(\frac{1-\cos\left(\frac{\pi}{2}+x\right)}{1+\cos\left(\frac{\pi}{2}+x\right)}\right)}=\tan^{-1}\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right| \quad \text{.... (i}$$

Now, $\frac{\pi}{2}<x<\pi$

$\therefore \quad \frac{\pi}{4}<\frac{x}{2}<\frac{\pi}{2}$

or $\quad \frac{\pi}{2}<\frac{\pi}{4}+\frac{x}{2}<\frac{3\pi}{4}$

$\therefore \quad \left|\tan\left(\frac{\pi}{2}+\frac{x}{2}\right)\right|=-\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$ ($\because$ in II quadrant)

$$= \tan\left\{\pi - \left(\frac{\pi}{4} + \frac{x}{2}\right)\right\}$$

From Eq, (i)

$$y = \tan^{-1}\tan\left\{\pi - \left(\frac{\pi}{4} + \frac{x}{2}\right)\right\}$$

$$= \pi - \left(\frac{\pi}{4} + \frac{x}{2}\right)$$

$$= \frac{3\pi}{4} - \frac{x}{2}$$

($\because$ Principle value of $\tan^{-1}\tan x$ in - $\frac{\pi}{2} to \frac{x}{2}$)

$$\therefore \frac{dy}{dx} = -\frac{1}{2}$$

33. Ans : C)

Let $y = f(x) \Rightarrow x = f^{-1}(y)$

then $f(x) = x + \tan x$

$\Rightarrow$ $y = f^{-1}(y) + \tan(f^{-1}(y)$

$\Rightarrow$ $y = g(y) + \tan(g(y) \;\; or \; x = g(x) + \tan(g(x))$ ……. (i)

Differentiating both sides, then we get

$$1 = g^1(x) + sec^2 g(x).\, g^1(x)$$

$$g^1(x) = \frac{1}{1+\sec^2(g(x))} = \frac{1}{1+1+\tan^2(g(x))}$$

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$$x^2 + y^2 = t - \frac{1}{t}, x^4 + y^4 = t^2 + \frac{1}{t^2}$$

$$= \left(t - \frac{1}{t}\right)^2 + 2$$

$$= X^4 + y^4 + 2x^2y^2 + 2$$

$\therefore$ $x^2y^2 = -1$

$\Rightarrow$ $x^2.2y\frac{dy}{dx} + y^2.2x = 0$

$$\Rightarrow \quad x^3 y\frac{dy}{dx} = -x^2y^2 = 1$$

35. Ans : B)

Let $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}x$

$$\therefore \quad \frac{du}{dx} = \frac{2}{1+x^2}$$

and $v = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1}x$

Differential coefficient

$$\therefore \quad \frac{dv}{dx} = \frac{2}{1+x^2}$$

$$\therefore \quad \frac{du}{dv} = \frac{\left(\frac{du}{dx}\right)}{\left(\frac{dv}{dx}\right)} = 1$$

36. Ans : C)

Since, $y = \sin x^\circ$

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$$= \frac{\pi}{180} = \cos x^2$$

and = $u = \cos x$

$$\therefore \frac{du}{dx} = \sin x$$

Then $\frac{dy}{du} = \frac{\frac{dy}{dx}}{\frac{du}{dx}} = \frac{\frac{\pi}{180}\cos x^\circ}{\sin x}$

$$= -\frac{\pi}{180}\cos x^\circ \cos ec\, x$$

37. Ans : C)

$$\sqrt{x^2+y^2} = ae^{\tan-1\left(y/x\right)}$$

$$\frac{1}{2\sqrt{x^2+y^2}}(2x+2yy^1) = a.e^{\tan^{-1}(y/x)} \times \frac{1}{\left(1+\frac{y^2}{x^2}\right)} \times \frac{xy^1-y}{x^2}$$

$$\Rightarrow \frac{x+yy'}{\sqrt{x^2+y^2}} = \sqrt{x^2+y^2} \times \frac{x^2}{(x^2+y^2)} \times \frac{xy' = y}{x^2}$$ [From Eq. (i)]

$$\therefore \quad x + yy' = xy' - y \Rightarrow y' = \frac{x+y}{x-y}$$

$$\therefore \quad y'' = \frac{2(xy'-y)}{(x-y)^2}$$

$$y''(0) = \frac{2(0-y(0))}{\{0-y(0)\}} = \frac{-2}{y(0)} = \frac{-2}{ae^{\pi/2}} = \frac{-2}{a}e^{-\pi/2}$$

38. Ans : C)

$$x = \frac{\sin y}{\sin(a+y)}$$

$$\Rightarrow \frac{dx}{dy} = \frac{\sin(a)}{\sin^2(a+y)}$$

$$\therefore \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a} = \frac{A}{1+x^2-2x\cos a}$$

Put $x = 0, y = 0$

Then $A = \sin a$

39. Ans : B)

$$\therefore y = \tan^{-1}\left(\frac{ax-b}{bx-a}\right) = \tan^{-1}\left(\frac{x-\frac{b}{a}}{1+\frac{b}{a}.x}\right)$$

$$= \tan^{-1}x - \tan^{-1}\left(\frac{b}{a}\right)$$

$$\therefore \frac{dy}{dx} = \frac{1}{1+x^2} - 0$$

$$\therefore \frac{dy}{dx}\Big|_{x=-1} = 2008 \times \frac{1}{2} = 1004$$

40. Ans : C)

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{a(\text{cos}t-t(-\text{sin}t)-\text{cos}t)}{a(-\text{sin}t+\text{cos}t+\text{sin}t)}$$

$$=tan\ t$$

$$\Rightarrow\frac{d^2y}{dx^2}=\frac{d}{dx}(\tan t)=\frac{d}{dt}(\tan t).\frac{dt}{dx}$$

$$=\sec^2 t\frac{1}{at\cos t}=\frac{\sec^3 t}{at}$$

$$\frac{d^2y}{dx^2}(at\quad t=\frac{\pi}{3})=\frac{8}{\pi a/3}=\frac{24}{\pi a}$$

$$\therefore 120\pi a\frac{d^2y}{dx^2}\bigg|_{t=\pi/3}=120\times 24=2880$$

41. *Ans : D)*

Let x = cos θ

$$\frac{1+x}{1-x}=\frac{1+\cos\theta}{1-\cos\theta}=\frac{2\cos^2\frac{\theta}{2}}{2\sin^2\theta/2}=\cot\theta/2$$

$$\sin^2\left(\cot^{-1}\sqrt{\frac{1+x}{1-x}}\right)=\sin^2\left(\cot^{-1}(\cot 0/2\right)$$

$$=(\sin^2 0/2)$$

$$\frac{d}{dx}\left[\sin^2(\cot^{-1}\sqrt{\frac{1+x}{1-x}}\right]=\frac{d}{dx}\left(\sin^2\frac{\theta}{2}\right)$$

$$=(2\sin\theta/2\cos\theta/2)\left(\frac{d\theta}{dx}\right)$$

$$=\sin\theta\left(\frac{-1}{\sin\theta}\right)=-1$$

42. Ans : B)

$$(a+\sqrt{2}\ b\cos x)\ (a-\sqrt{2}b\cos y)=a^2-b^2$$

$$(a-\sqrt{2}\ b\cos y)\quad\left[0+\sqrt{2}b(-\text{sin}x)\frac{dx}{dy}\right]+(a+\sqrt{2}b\ \text{cos}x)\times[0-\sqrt{2}b(-\text{sin}y)=0$$

$$(a-\sqrt{2}\ b\cos y)\ (-\sqrt{2}b\,din\,x\frac{dx}{dy})+(a+\sqrt{2}b\cos x)(\sqrt{2}\,b\sin y)=0$$

$A+\left(\pi/_4, \pi/_4\right)$

$(a-b)\ (-b\, dx/_{dy})+\left(a+b\right)(b)=0$

$-b(a-b)\frac{dy}{dx}=-b(a+b)$

$\frac{dy}{dx}=\frac{a+b}{a-b}$

43. *Ans : A)*

Log y = tan x log (sin x)

$\frac{1}{y}\frac{dy}{dx}=\tan x\ \frac{1}{\sin x}\cos x+\log(\sin x)\sec^2 x+\tan x$

$\frac{dy}{dx}=(\sin x)^{\tan x}[1+\sec^2 x\log\sin x]$

44. Ans : A)

$y=\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$

$\tan^{-1}\left(\frac{2(3x\,3/_2)}{1-(3x\,3/_2)^2}\right)$

$y=2\tan^{-1}(3x\,3/_2)$ $\qquad \left[\therefore 2\tan^{-1}x=\tan^{-1}\frac{2x}{1-x^2}\right]$

$\frac{dy}{dx}=2.\frac{1}{1+(3x^{3/2})^2}3\times\frac{3}{2}(x)^{1/2}$

$\frac{dy}{dx}=\frac{9}{1+9x^2}.\sqrt{x}$

$\therefore g(x)=\frac{9}{1+9x^2}$

45. Ans : A)

Log (x + y) = 2xy

When x = 0 ⇒ y = 1

$\frac{1}{x+y}\left[1+\frac{dy}{dx}\right]=2y+2x\frac{dy}{dx}$

$$\frac{dy}{dx}=\frac{\frac{1}{x+y}-2y}{2x-\frac{1}{x+y}}$$

$$y^1(0)=\frac{1-2}{0-1}=1$$