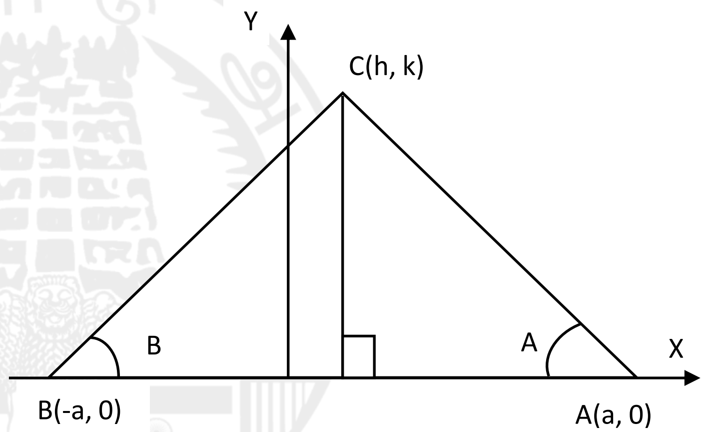
DIRECTORATE OF SCHOOL EDUCATION TAMILNADU

12JPCM09 (2023-24)	JEE PRACTICE QUESTIONS (TEST-9)	Class : XII Time : 1.15 hrs Total Marks : 180

Answer key

12th – MATHS

31. Ans : C)



Let the variable c is (h, k)
From figure
tan A = $\frac{CD}{DA} = \frac{K}{a-h}$
tan B = $\frac{CD}{BD} = \frac{K}{a+h}$
given tan A + tan B = λ
$\frac{K}{a-h} + \frac{k}{a+h} = \lambda$
on simplifying we get,
$2ak + 2h^2 = \lambda a^2$
Hence the locus is $\lambda x^2 + 2ay = \lambda a^2$

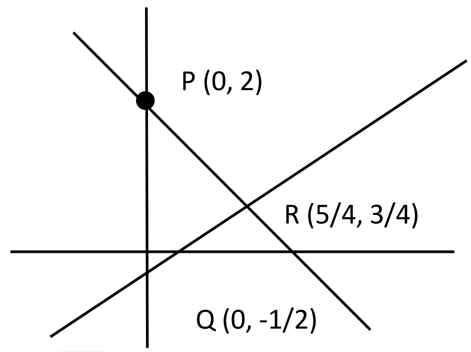
32. Ans : B)
$(x_1, y_1) = (2, 3)$, θ = 45°
Equn of line is $\frac{x-2}{\cos 45°} = \frac{y-3}{\sin 45°}$ = r
From first two parts, x – 2 = y – 3
x – y + 1= 0
co-ordinates of p on this line is $\left(2+\frac{r}{\sqrt{2}}, 3+\frac{r}{\sqrt{2}}\right)$
It lies on the line x + y + 1 = 0
$\therefore \left(2+\frac{r}{\sqrt{2}}\right) + \left(3+\frac{r}{\sqrt{2}}\right) + 1 = 0$

$r = -3\sqrt{2}$

$|r| = 3\sqrt{2} \Rightarrow p = (5, 6)$

$AP = 3\sqrt{2}$

33. Ans: B



Equation of L is

$y - 1 = \frac{-1}{1}(x-1)$

y = -x + 2

Equation of m is

$y = x - \frac{1}{2}$

If these lines meet y - axis at P and Q, then $PQ = \frac{5}{2}$

x-coordinate of their point of their point of intersection $R = \frac{5}{4}$

Area of $\Delta PQR = \frac{1}{2}\left[\frac{5}{2} \times \frac{5}{4}\right] = \frac{25}{16}$

34. Ans : C

P is on the line $2x - 3y = 0 \Rightarrow P = \left(a, \frac{2a}{3}\right)$

Q is on the line $2x + 3y = 0 \Rightarrow Q = \left(a, \frac{-2b}{3}\right)$

Given Area of a ΔOAB = 5

$$\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 0 & \frac{2a}{3} & 1 \\ b & \frac{-2b}{3} & 1 \end{vmatrix} = \pm 5$$

4ab = ±30

Let the mid point of PQ as M(h, k)

$$2h = a+b, \; 2k = \frac{2a-2b}{3}$$

Now

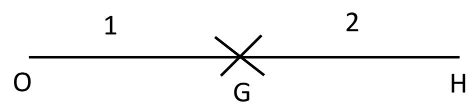
$(a+b)^2 - (a-b)^2 = 4ab$

$4h^2 - 9k^2 = \pm 30$

Hence the locus is $4x^2 - 9y^2 \pm 30 = 0$

35. Ans: A

Circumcenter O = $\left(\frac{-1}{3}, \frac{2}{3}\right)$



Ortho centre H = $\left(\frac{11}{3}, \frac{4}{3}\right)$

A = (1, 10)

Centroid G = $\left(1, \frac{8}{9}\right)$

Let D be the mid point of BC

AG : GD = 2 : 1

D = $\left(1, \frac{-11}{3}\right)$

36. Ans : C

D $(x_1\ y\ ) = 1$

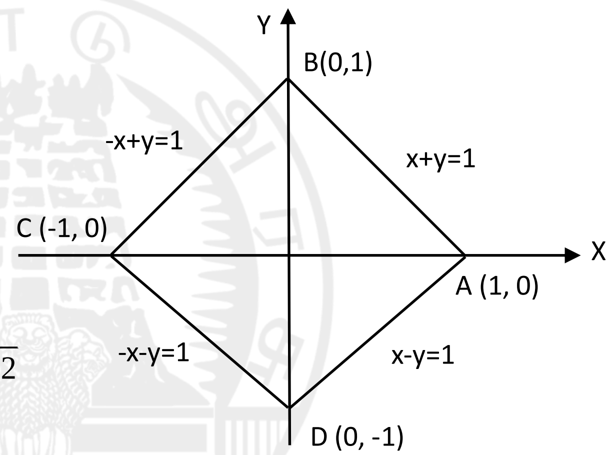
$1 \times 1 + |y| = 1$

From graph

It is a square

Are a = AB ×AD = $\sqrt{2}\ \sqrt{2}$

= 2.59 units



37. Ans : C

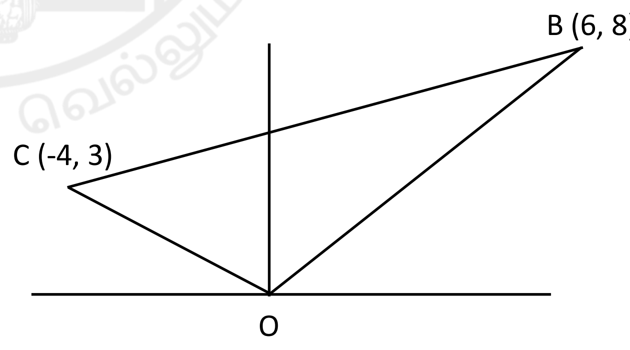
Slope of OB = $\frac{8}{6}$

Slope of OC = $\frac{-3}{4}$

$\frac{8}{6} \times \frac{-3}{4} = -1$

$\therefore$ OB + OC

OBC is right angled triangle



$\therefore$ Circumcenter = mid point of hypotenuse = $\left(1, \frac{11}{2}\right)$

Ortho centre = vertex 0 (0, 0)

Distance = $\sqrt{1 + \frac{121}{4}} = \frac{5\sqrt{5}}{2}$ unit

38. Ans : C

Equation of bisectors of the given lines are

$$\frac{3x+4y-5}{\sqrt{3^2+4^2}} = \pm\left(\frac{5x-2y-10}{\sqrt{5^2+(-2)^2}}\right)$$

14x + 112y – 15 = 0 (or) 64x – 8y – 115 = 0

14x + 112y – 15 = 0

$$\frac{14}{15}x+\frac{112y}{15}-1=0, \frac{64x}{115}-\frac{8y}{115}-1=0$$

$$a=\frac{14}{15}, b=\frac{112}{15} (or)\ a=\frac{64}{115}, b=\frac{-8}{115}$$

39. Ans : A

$$y=\sqrt{3}x = x\tan 60^\circ$$

$$\frac{x-0}{\cos 60^\circ}=\frac{y-0}{\sin 60^\circ}=r$$

$$(x,y)=\left(\frac{r}{2},\frac{r\sqrt{3}}{2}\right)$$

It lies on $x^3 + y^3 + 3xy + 5x^2 + 3y^2 + 4x + 5y - 1 = 0$

$$\frac{r^3}{8}+\frac{3\sqrt{3}r^3}{8}+\frac{3\sqrt{3}r^2}{4}+\frac{5r^2}{4}+\frac{9r^2}{4}+2r+\frac{5\sqrt{3}r}{2}-1=0$$

$$\left(\frac{3\sqrt{3}+1}{8}\right)r^3+\left(\frac{14+3\sqrt{3}}{4}\right)r^2+\left(\frac{4+5\sqrt{3}}{2}\right)r-1=0$$

Here product of the roots = OA.OB. OC

$$r_1r_2r_3=\frac{1}{\frac{3\sqrt{3}+1}{8}}=\frac{4}{13}(3\sqrt{3}-1)$$

40. Ans: B

$$|PA-PB|\le|AB|$$

Maximum value of $|PA-PB|=|AB|$, which is

Possible only when P, A, B are collinear

If P(x, y) then equation of AB is $\frac{x}{2}+\frac{y}{2}=1 \Rightarrow x+y=2$

Solving $x + y = 2$ and $2x + 3y + 1 = 0$ we get $P(x_1 y) = P(7, -5)$

41. Ans: D)

$y = \cos x \cos(x+2) - \cos^2(x+1)$

$= \frac{1}{2}\left[2\cos x\cos(x+2) - 2\cos^2(x+1)\right]$

$= \frac{1}{2}[\cos(2x+2) + \cos 2 - 1 - \cos(2x+2)]$

$= \frac{1}{2}[\cos 2 - 1]$

$= \frac{1}{2}[1 - 2\sin^2 1 - 1]$

y = $-\sin^2 1$. Which is a straight line passing through $\left(\frac{\pi}{2}, -\sin^2 1\right)$ and parallel to the x-axis

42. Ans : D)

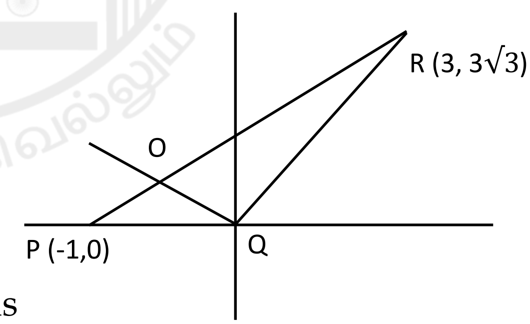
Let A$\left(1,\sqrt{3}\right)$, 0 (0, 0) and B (2, 0) are given P × S

OA = AB = BO

So, it is an equilateral triangle and the in centre co-incides with the centroid

Incentre = centroid = $\left(\frac{0+1+2}{3}, \frac{0+0+\sqrt{3}}{3}\right) = \left(1, \frac{1}{\sqrt{3}}\right)$

43. Ans : C



Here Q (0, 0) is origin

Slope of QR = $\frac{3\sqrt{3}-0}{3-0} = \sqrt{3}$

tan θ = $\sqrt{3}$

θ = 60˚

Clearly $\lfloor PQR$ = 120˚ since P on x – axis

OQ is the angle bisector of angle. So line OQ makes 120 ˚ with +ve direction of x-axis

The equation of the bisector of $\lfloor PQR$ is

y = tan 120˚ x

y = $-\sqrt{3}x$

$\sqrt{3}x + y = 0$

44. Ans : C

$$\begin{vmatrix} 1 & a & a \\ b & 1 & b \\ c & c & 1 \end{vmatrix} = 0$$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$\begin{vmatrix} 1 & a-1 & a-1 \\ b & 1-b & 0 \\ c & 0 & 1-c \end{vmatrix} = 0$$

Let A = a-1, B = b-1, C = c-1

$$\begin{vmatrix} 1 & A & A \\ 1+B & -B & 0 \\ 1+C & 0 & -C \end{vmatrix} = 0$$

$$1(BC-0) - A(-C-BC) + A(B+BC) = 0$$

$$\frac{1}{A} + \frac{1}{B} + \frac{1}{C} = -2$$

$$\frac{a}{a-1} + \frac{b}{b-1} + \frac{c}{c-1} = \frac{1+A}{A} + \frac{1-B}{B} + \frac{1+C}{C} = \frac{1}{A} + \frac{1}{B} + \frac{1}{C} = 3$$

$= -2 + 3 = 1$

45. Ans : B

$bx + cy = a$

$$\frac{x}{a/b} + \frac{y}{a/c} = 1$$

Points on me line are $A(\frac{a}{b}, 0), B(0, \frac{a}{c})$

Area of ΔOAB = $= \frac{1}{8}$ (Given)

$$\frac{1}{2}\left(\frac{a}{b}\right)\left(\frac{a}{c}\right) = \pm\frac{1}{8}$$

$$\frac{a^2}{bc} = \pm\frac{1}{4}$$

$(2a)^2 = \pm bc$

b, ±2a, c are in G.P

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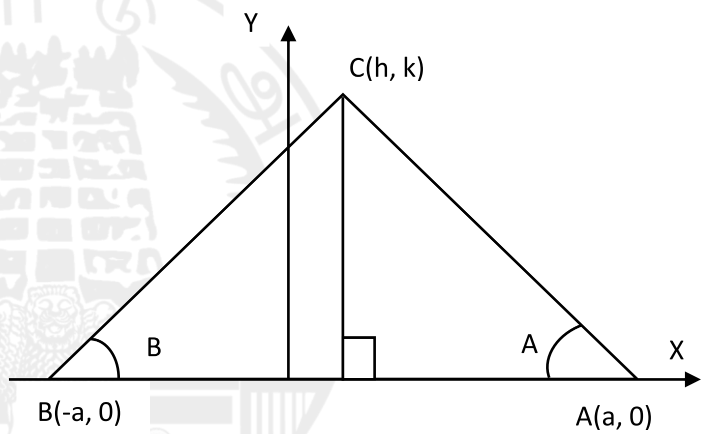

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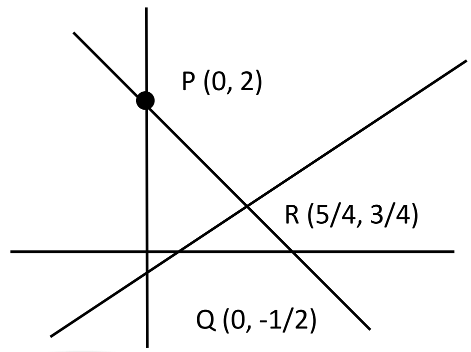

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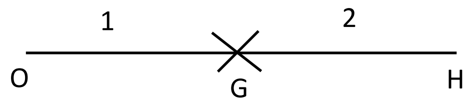

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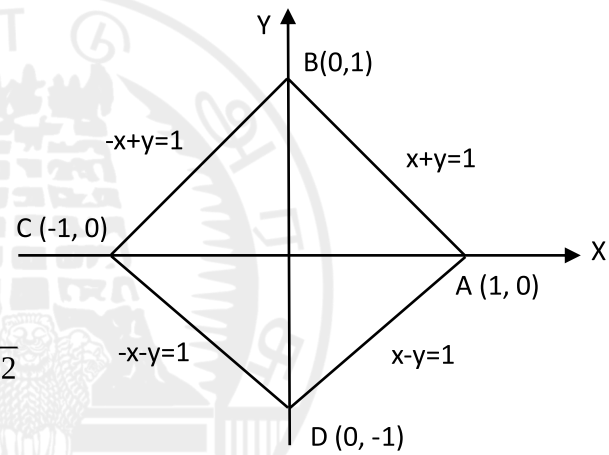

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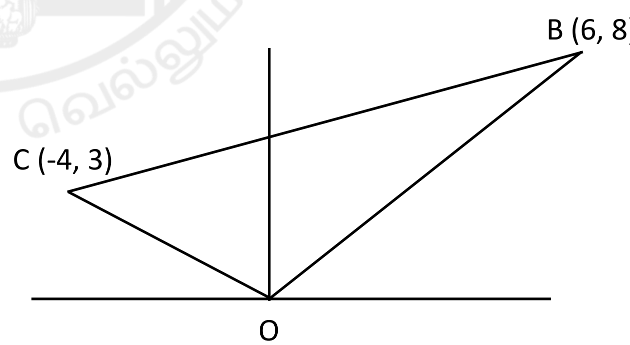

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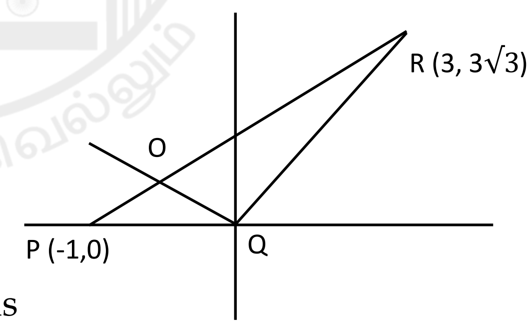

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