SCHOOL EDUCTION DEPARTMENT

CHENNAI   DISTRICT

LEARNING MATERIAL
2023-2024

HIGHER SECONDARY SECOND YEAR

BUSINESS MATHEMATICS & STATISTICS

Preface

We convey our sincere gratitude to our respected Chief Educational Officer, who has given this opportunity to bring out an unique material for the students (XII standard Business Mathematics and Statistics) in the name of Learning Material.

The learning material is prepared based on the selected chapters. This includes classification for selected chapters,solved textbook exercise problems
(2 marks, 3 marks and 5 marks).

Students can prepare the example problems based on the classification. All the text book MCQ problems have to be practiced regularly. Students must practice all the problems in the classification. This material mainly focus on the slow learners to achieve their goals.

Good effort always lead to success

All the best!!!

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CLASSIFICATION OF TEXT BOOK PROBLEMS (Selected Chapters)

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CHAPTER 1
APPLICATION OF MATRICES AND DETERMINATS
(2, 3 AND 5 MARKS)

2 - MARKS

Exercise 1.1

Question 1. Find the rank of the matrix

Solution:

1 (i) Let $A = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$ Order of A is 2×2. $\therefore \rho(A) \leq 2$

Consider the second order minor

$\begin{vmatrix} 5 & 6 \\ 7 & 8 \end{vmatrix} = 40 - 42 = -2 \neq 0$

There is a minor of order 2 , which is not zero.

$\rho(A) = 2$

1(ii) Let $A = \begin{pmatrix} 1 & -1 \\ 3 & -6 \end{pmatrix}$ Order of A is 2×2; $\therefore \rho(A) \leq 2$

Consider the second order minor

$\begin{vmatrix} 1 & -1 \\ 3 & -6 \end{vmatrix} = -6 + 3 = -3 \neq 0$ $\rho(A) = 2$

1iii) Let $A = \begin{pmatrix} 1 & 4 \\ 2 & 8 \end{pmatrix}$ Since A is of order 2×2, $\therefore \rho(A) \leq 2$

Now $\begin{vmatrix} 1 & 4 \\ 2 & 8 \end{vmatrix} = 8 - 8 = 0$

Since second order minor vanishes $\rho(A) \neq 2$

But first order minors, $|1| = 1$ non zero.

$\rho(A) = 1$

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3 - MARKS

Question 1. Find the rank of the matrix

(iv) Let $A = \begin{pmatrix} 2 & -1 & 1 \\ 3 & 1 & -5 \\ 1 & 1 & 1 \end{pmatrix}$

Order of A is 3×3 ; $\therefore$ $\rho(A) \leq 3$

$$\begin{vmatrix} 2 & -1 & 1 \\ 3 & 1 & -5 \\ 1 & 1 & 1 \end{vmatrix} = 2(1+5) + 1(3+5) + 1(3-1) = 2(6) + 8 + 2 = 22 \neq 0$$

There is a minor of order 3 , which is non zero. $\rho(A) = 3$

(v) Let $A = \begin{pmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ -2 & 4 & -4 \end{pmatrix}$

Since order of A is 3×3, $\therefore$ $\rho(A) \leq 3$

$$\begin{vmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ -2 & 4 & -4 \end{vmatrix} = -1(12-16) - 2(-16+8) - 2(16-6) = 4 + 16 - 20 = 0$$

Since the third order minor vanishes, $\rho(A) \neq 3$

Consider $\begin{vmatrix} -1 & 2 \\ 4 & -3 \end{vmatrix} = 3 - 8 = -5 \neq 0$

There is a minor order 2 , which is non zero

$\rho(A) = 2$

(vi) Let $A = \begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{pmatrix}$

$$A = \begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & 0 & 3 & -8 \\ 0 & 0 & 6 & -16 \end{pmatrix} \quad \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & 0 & 3 & -8 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad R_3 \to R_3 - 2R_2$$

The above matrix is in echelon form.

The number of non zero rows is 2 $\Rightarrow \rho(A) = 2$

(vii) Let $A = \begin{pmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{pmatrix}$

Order of A is 3×4 $\therefore \rho(A) \leq 3$

$$A = \begin{pmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & -2 & 1 & -5 \\ 3 & 1 & -5 & -1 \\ 1 & 5 & -7 & 2 \end{pmatrix} \quad R_1 \leftrightarrow R_2$$

$$\sim \begin{pmatrix} 1 & -2 & 1 & -5 \\ 0 & 7 & -8 & 14 \\ 0 & 7 & -8 & 7 \end{pmatrix} \quad \begin{matrix} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\sim \begin{pmatrix} 1 & -2 & 1 & -5 \\ 0 & 7 & -8 & 14 \\ 0 & 0 & 0 & -7 \end{pmatrix} \quad R_3 \to R_3 - R_2$$

The number of non-zero rows is 3 $\therefore \rho(A) = 3$

(viii) Let $A = \begin{pmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{pmatrix}$

$$A = \begin{pmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & -2 & 3 & 4 \\ 0 & 0 & 5 & 5 \\ 0 & 0 & 10 & 10 \end{pmatrix} \quad \begin{matrix} R_2 \to R_2 + 2R_1 \\ R_3 \to R_3 + R_1 \end{matrix}$$

$$\sim \begin{pmatrix} 1 & -2 & 3 & 4 \\ 0 & 0 & 5 & 5 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad R_3 \to R_3 - 2R_2$$

The number of non-zero rows is 2 $\therefore \rho(A) = 2$

Question 2.

If $A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix}$, then find the rank of AB Solution:

Given $A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix}$

$AB = \begin{bmatrix} 1-2-5 & -2+4-1 & 3-6+1 \\ 2+6+20 & -4-12+4 & 6+18-4 \\ 3+4+15 & -6-8+3 & 9+12-3 \end{bmatrix}$

$AB = \begin{pmatrix} -6 & 1 & -2 \\ 28 & -12 & 20 \\ 22 & -11 & 18 \end{pmatrix}$

Consider

$\begin{vmatrix} -6 & 1 & -2 \\ 28 & -12 & 20 \\ 22 & -11 & 18 \end{vmatrix} = -6(-216+220) - 1(504-440) - 2(-308+264)$

$= -6(4) - 1(64) - 2(-56) = 24 \neq 0$

Since the third order minor is not zero, $\rho(AB) = 3$

If $A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix}$, then find rank of BA.

Solution:

Given $A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix}$

$BA = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$

$= \begin{pmatrix} 1-4+9 & 1+6-6 & -1-8+9 \\ -2+8-18 & -2-12+12 & 2+16-18 \\ 5+2-3 & 5-3+2 & -5+4-3 \end{pmatrix}$

$= \begin{pmatrix} 6 & 1 & 0 \\ -12 & -2 & 0 \\ 4 & 4 & -4 \end{pmatrix}$

Consider the third order minor,

$\begin{vmatrix} 6 & 1 & 0 \\ -12 & -2 & 0 \\ 4 & 4 & -4 \end{vmatrix} = 6(8-0) - 1(48-0) + 0(-48+8)$

$= 48 - 48 + 0 = 0$

$\rho(BA) \neq 3$

Take a second order minor, $\begin{vmatrix} -2 & 0 \\ 4 & -4 \end{vmatrix} = 8 \neq 0.$

$\rho(BA) = 2$

Exercise 1.2

Question 1.

Solve the following equations by using Cramer's rule

(i) $2x + 3y = 7; 3x + 5y = 9$

$\Delta = \begin{vmatrix} 2 & 3 \\ 3 & 5 \end{vmatrix} = 1 \neq 0$ {∴ We can apply Cramer's Rule}

$\Delta_x = \begin{vmatrix} 7 & 3 \\ 9 & 5 \end{vmatrix} = 35 - 27 = 8$

$\Delta_y = \begin{vmatrix} 2 & 7 \\ 3 & 9 \end{vmatrix} = 18 - 21 = -3$

So, $x = \frac{\Delta_x}{\Delta} = 8 \qquad y = \frac{\Delta_y}{\Delta} = -3$

$\therefore\ x = 8$ and $y = -3$

(ii) $5x + 3y = 17; 3x + 7y = 31$

$\Delta = \begin{vmatrix} 5 & 3 \\ 3 & 7 \end{vmatrix} = 35 - 9 = 26 \neq 0$

$\Delta_x = \begin{vmatrix} 17 & 3 \\ 31 & 7 \end{vmatrix} = 119 - 93 = 26$

$\Delta_y = \begin{vmatrix} 5 & 17 \\ 3 & 31 \end{vmatrix} = 155 - 51 = 104$

$x = \frac{\Delta_x}{\Delta} = \frac{26}{26} = 1 \ \ \& \ \ y = \frac{\Delta_y}{\Delta} = \frac{104}{26} = 4 \ \therefore\ (x, y) = (1,4)$

Question 2.

A commodity was produced by using 3 units of labour and 2 units of capital, the total cost is ₹ 62. If the commodity had been produced by using 4 units of labour and one unit of capital, the cost is ₹ 56 . What is the cost per unit of labour and capital? (Use determinant method).

Solution:

Let the cost per unit of labour be ₹x and

cost per unit of capital be ₹ y.

$3x + 2y = 62$ & $4x + y = 56$

$\Delta = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix} = 3 - 8 = -5 \neq 0$

Hence the system has a unique solution .

$\Delta_x = \begin{vmatrix} 62 & 2 \\ 56 & 1 \end{vmatrix} = 62 - 112 = -50$

$\Delta_y = \begin{vmatrix} 3 & 62 \\ 4 & 56 \end{vmatrix} = 168 - 248 = -80$

$\therefore x = \frac{\Delta_x}{\Delta} = \frac{-50}{-5} = 10 \qquad y = \frac{\Delta_y}{\Delta} = \frac{-80}{-5} = 16$

Hence the cost per unit of labour is ₹10 and cost per unit of capital is ₹16

Question 3 .

A total of ₹ 8,600 was invested in two accounts. One account earned $4\frac{3}{4}\%$ annual interest and the other earned $6\frac{1}{2}\%$ annual interest. If the total interest for one year was ₹431.25, how much was invested in each account? (Use determinant method).

Solution:

Let ₹x and ₹y be the amounts invested in the two accounts.

Interest for first account $= 4\frac{3}{4}\%x = \frac{19}{4} \times \frac{1}{100} \times x = \frac{19}{400}x$

Interest for second account $= 6\frac{1}{2}\% = \frac{13}{2} \times \frac{1}{100}y = \frac{13}{200}y$

$x + y = 8600$ & $\frac{19}{400}x + \frac{13}{200}y = 431.25$

Multiplying equation by 400, $19x + 26y = 172500$

$\Delta = \begin{vmatrix} 1 & 1 \\ 19 & 26 \end{vmatrix} = 26 - 19 = 7 \neq 0$

$\Delta_x = \begin{vmatrix} 8600 & 1 \\ 172500 & 26 \end{vmatrix} = 223600 - 172500 = 51100$

$\Delta_y = \begin{vmatrix} 1 & 8600 \\ 19 & 172500 \end{vmatrix} = 172500 - 163400 = 9100$

By Cramer's rule,

$x = \frac{\Delta_x}{\Delta} = \frac{51100}{7} = 7300 \ \ \& \ \ y = \frac{\Delta_y}{\Delta} = \frac{9100}{7} = 1300$

Hence the amount invested at $4\frac{3}{4}\%$ is ₹7300 and amount invested at $6\frac{1}{2}\%$ is ₹1300

Question 4.

At marina two types of games viz., Horse riding and Quad Bikes riding are available on hourly rent. Keren and Benita spent ₹ 780 and ₹ 560 during the month of May.

Name	Number of hours		Total amount spent (in ₹)
	Horse Riding	Quad Bike Riding	
Keren	3	4	780
Benita	2	3	560

Find the hourly charges for the two games (rides). (Use determinant method).

Solution:

Let hourly charges for horse riding be ₹x and

hourly charges for Quad bike riding be ₹y.

$3x + 4y = 780$ & $2x + 3y = 560$

$\Delta = \begin{vmatrix} 3 & 4 \\ 2 & 3 \end{vmatrix} = 9 - 8 = 1 \neq 0$

So there exists a unique solution.

$\Delta_x = \begin{vmatrix} 780 & 4 \\ 560 & 3 \end{vmatrix} = 2340 - 2240 = 100$

$\Delta_y = \begin{vmatrix} 3 & 780 \\ 2 & 560 \end{vmatrix} = 1680 - 1560 = 120$

By Cramer's rule,

$x = \frac{\Delta_x}{\Delta} = \frac{100}{1} = 100.$ $y = \frac{\Delta_y}{\Delta} = \frac{120}{1} = 120$

Hourly charges for horse riding and bike riding are ₹ 100 and ₹ 120 respectively.

Exercise 1.3

Question 1.

The subscription department of a magazine sends out a letter to a large mailing list inviting subscriptions for the magazine. Some of the people receiving this letter already subscribe to the magazine while others do not. From this mailing list, 45% of those who already subscribe will subscribe again while 30% of those who do not now subscribe will subscribe. On the last letter, it was found that 40% of those receiving it ordered a subscription. What per cent of those receiving the current letter can be expected to order a subscription?

Solution:

Let X represent people who subscribe for the magazine and Y represent persons who do not subscribe for the magazine.

$(X \to X) = 45\% = 0.45$ & $(X \to Y) = 100 - 45 = 55\% = 0.55$

$(Y \to X) = 30\% = 0.3$ & $(Y \to Y) = (100 - 30) = 70\% = 0.7$

$$T = \begin{array}{c} \\ X \\ Y \end{array} \begin{array}{c} \begin{array}{cc} X & Y \end{array} \\ \begin{pmatrix} 0.45 & 0.55 \\ 0.3 & 0.7 \end{pmatrix} \end{array}$$

Initial Value for $X = 40\% = 0.4$;

$Y = (100 - 40) = 60\% = 0.6$

$$\begin{array}{cc} X & Y \\ (0.4 & 0.6) \end{array} \begin{array}{c} \\ X \\ Y \end{array} \begin{array}{c} \begin{array}{cc} X & Y \end{array} \\ \begin{pmatrix} 0.45 & 0.55 \\ 0.3 & 0.7 \end{pmatrix} \end{array} =$$

$= (0.4 \times 0.45 + 0.6 \times 0.3 \quad 0.4 \times 0.55 + 0.6 \times 0.7)$

$= (0.18 + 0.18 \quad 0.22 + 0.42) = (0.36 \quad 0.64)$

That is $X = 36\%$ and $Y = 64\%$

Thus 36% of those receiving the current letter can be expected to order a subscription.

5 - MARKS

EXERCISE 1.1

Question 3 .

Solve the following system of equations by rank method.

$\mathbf{x + y + z = 9}$, $\mathbf{2x + 5y + 7z = 52}$, $\mathbf{2x + y - z = 0}$

Solution:

The given equations are $x + y + z = 9$, $2x + 5y + 7z = 52$, $2x + y - z = 0$

The matrix equation corresponding to the given system is

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 52 \\ 0 \end{pmatrix}$$

$A \quad X \quad = B$

Augmented matrix

$$[A, B] = \begin{pmatrix} 1 & 1 & 1 & 9 \\ 2 & 5 & 7 & 52 \\ 2 & 1 & -1 & 0 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & -1 & -3 & -18 \end{pmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & 0 & -4 & -20 \end{pmatrix} R_3 \to 3R_3 + R_2$$

$$\text{Now } A \sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 5 \\ 0 & 0 & 4 \end{pmatrix} \Rightarrow \rho(A) = 3$$

$$\text{Augmented matrix } [A,B] \sim \begin{pmatrix} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & 0 & -4 & -20 \end{pmatrix}$$

has three non-zero rows , $\rho([A, B]) = 3$

That is, $\rho(A) = \rho([A, B]) = 3 =$ number of unknowns.

So the given system is consistent and has unique solution.

To find the solution, we rewrite the echelon form into the matrix form.

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 5 \\ 0 & 0 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 34 \\ -20 \end{pmatrix}$$

$x + y + z = 9 \to (1)$

$3y + 5z = 34 \to (2)$

$-4z = -20 \to (3)$

$(3) \Rightarrow z = 5$

$(2) \Rightarrow 3y = 34 - 25 = 9$

$y = 3$

$(1) \Rightarrow x = 9 - 3 - 5$

$(2)\ x = 1$

Question 4.

Show that the equations $5x + 3y + 7z = 4$, $3x + 26y + 2z = 9$, $7x + 2y + 10z = 5$ are consistent and solve them by rank method.

Solution: The given equations are,

$5x + 3y + 7 = 4$

$3x + 26y + 2z = 9$

$7x + 2y + 10z = 5$

The matrix equation corresponding to the given system is

$$\begin{pmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \\ 5 \end{pmatrix}$$

A X = B

Augmented matrix $[A,B] = \begin{pmatrix} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{pmatrix}$

$$\sim \begin{pmatrix} 5 & 3 & 7 & 4 \\ 0 & 121 & -11 & 33 \\ 0 & -11 & 1 & -3 \end{pmatrix} \begin{matrix} R_2 \to 5R_2 - 3R_1 \\ R_3 \to 5R_3 - 7R_1 \end{matrix}$$

$$\sim \begin{pmatrix} 5 & 3 & 7 & 4 \\ 0 & 121 & -11 & 33 \\ 0 & 0 & 0 & 0 \end{pmatrix} R_3 \to 11R_3 + R_2$$

$$\sim \begin{pmatrix} 5 & 3 & 7 & 4 \\ 0 & 11 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix} R_2 \to R_2/11$$

The equivalent matrix is in echelon form. It has two non-zero rows.

$\therefore \rho(A) = \rho([A, B]) = 2 <$ number of unknowns.

So the equations are consistent and have infinitely many solutions

$\Rightarrow 5x + 3y + 7z = 4$……………(1)

$11y - z = 3$……………(2)

Z= k

$(2) \Rightarrow 11y = k + 3 \Rightarrow \quad y = \frac{k+3}{11}$

$(1) \Rightarrow 5x + 3(\frac{k+3}{11}) + 7k = 4$

$\frac{55x + 3k + 9 + 77k}{11} = 4$; $55x = 35 - 80k$; $11x = 7 - 16k$;

$x = \frac{-16}{11}k + \frac{7}{11}$

Let us take $z = k$, $k \in R$. We get, $y = \frac{k+3}{11}$, $x = \frac{-16}{11}k + \frac{7}{11}$

By giving different values for k, we get different solutions. Thus the solutions of the given system are given by

$x = \frac{1}{11}(7 - 16k)$; $y = \frac{1}{11}(3 + k)$; $z = k$

Question 5.

Show that the following system of equations have unique solution:

$x + y + z = 3, x + 2y + 3z = 4, x + 4y + 9z = 6$ by rank method.

Solution:

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix}$$

A X = B

Augmented matrix $[A,B] = \begin{pmatrix} 1 & 1 & 1 & 3 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & 9 & 6 \end{pmatrix}$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 3 & 8 & 3 \end{pmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 2 & 0 \end{pmatrix} R_3 \to R_3 - 3R_2$$

$\rho(A) = \rho([A, B]) = 3 =$ number of unknowns.

The given system is consistent and has a unique solution.

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$$

$x + y + z = 3$…… (1); $y + 2z = 1$……(2); $2z = 0$…… (3)

$(3) \Rightarrow z = 0$ $(2) \Rightarrow y = 1$

$(1) \Rightarrow x + 1 + 0 = 3 \Rightarrow x = 2$

So the unique solution is $x = 2, y = 1, z = 0$

Question 6.

For what values of the parameter X, will the following equations fail to have unique solution:

$3x - y + \lambda z = 1$, $2x + y + z = 2$, $x + 2y - \lambda z = -1$ by rank method.

Solution:

$$\begin{pmatrix} 3 & -1 & \lambda \\ 2 & 1 & 1 \\ 1 & 2 & -\lambda \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

A X = B

Augmented matrix $[A,B] = \begin{pmatrix} 3 & -1 & \lambda & 1 \\ 2 & 1 & 1 & 2 \\ 1 & 2 & -\lambda & -1 \end{pmatrix}$

$$\sim \begin{pmatrix} 1 & 2 & -\lambda & -1 \\ 2 & 1 & 1 & 2 \\ 3 & -1 & \lambda & 1 \end{pmatrix} R_1 \leftrightarrow R_3$$

$$\sim \begin{pmatrix} 1 & 2 & -\lambda & -1 \\ 0 & -3 & 1 + 2\lambda & 4 \\ 0 & -7 & 4\lambda & 4 \end{pmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{pmatrix} 1 & 2 & -\lambda & -1 \\ 0 & -3 & 1 + 2\lambda & 4 \\ 0 & 0 & -2\lambda - 7 & -16 \end{pmatrix} R_3 \to 3R_3 - 7R_2$$

For the system to be inconsistent (or) not to have unique solution, $\rho([A, B]) \neq \rho(A)$

But $\rho([A, B]) = 3$; So $\rho(A) \neq 3 \Rightarrow -2\lambda - 7 = 0 \Rightarrow -2\lambda = 7$

$\lambda = \frac{-7}{2}$

So when $\lambda = \frac{-7}{2}$, the equations fail to have unique solution.

Question 7.

The price of three commodities, X, Y and Z are x , y and z respectively Mr. Anand purchases 6 units of Z and sells 2 units of X and 3 units of Y. Mr.Amar purchases a unit of Y and sells 3 units of X and 2 units of Z. Mr. Amit purchases a unit of X and sells 3 units of Y and a unit of Z. In the process they earn ₹5, 000/– , ₹2, 000/ - and ₹5, 500/– respectively. Find the prices per unit of three commodities by rank method.

Solution:

The price of three commodities X, Y, Z are given as x, y, z.

	X	Y	Z
Anand	sells 2 units(+)	sells 3 units(+)	buys 6 units(-)
Amar	sells 3 units(+)	buys 1 unit(-)	sells 2 units(+)
Amit	buys 1 unit(-)	sells 3 units(+)	sells 1 unit(+)

Anand → $2x + 3y - 6z = 5000$

Amar → $3x - y + 2z = 2000$

Amit → $-x + 3y + z = 5500$

The matrix equation is given by

$$\begin{pmatrix} 2 & 3 & -6 \\ 3 & -1 & 2 \\ -1 & 3 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5000 \\ 2000 \\ 5500 \end{pmatrix}$$

A X = B

Augmented matrix [A,B]= $\begin{pmatrix} 2 & 3 & -6 & 5000 \\ 3 & -1 & 2 & 2000 \\ -1 & 3 & 1 & 5500 \end{pmatrix}$

$$\sim \begin{pmatrix} -1 & 3 & 1 & 5500 \\ 3 & -1 & 2 & 2000 \\ 2 & 3 & -6 & 5000 \end{pmatrix} R_1 \leftrightarrow R_3$$

$$\sim \begin{pmatrix} -1 & 3 & 1 & 5500 \\ 0 & 8 & 5 & 18500 \\ 0 & 9 & -4 & 16000 \end{pmatrix} \begin{matrix} R_2 \to R_2 + 3R_1 \\ R_3 \to R_3 + 2R_1 \end{matrix}$$

$$\sim \begin{pmatrix} -1 & 3 & 1 & 5500 \\ 0 & 72 & 45 & 166500 \\ 0 & 72 & -32 & 128000 \end{pmatrix} \begin{matrix} R_2 \to 9R_2 \\ R_3 \to 8R_3 \end{matrix}$$

$$\sim \begin{pmatrix} -1 & 3 & 1 & 5500 \\ 0 & 72 & 45 & 166500 \\ 0 & 0 & -77 & -38500 \end{pmatrix} R_3 \to R_3 - R_2$$

$\rho(A) = \rho([A, B]) = 3 =$ number of unknowns

So the system has unique solution.

∴ The given system is equivalent to the matrix equation.

$$\begin{pmatrix} -1 & 3 & 1 \\ 0 & 72 & 45 \\ 0 & 0 & -77 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5500 \\ 166500 \\ -38500 \end{pmatrix}$$

$-x + 3y + z = 5500$…………………(1)

$72y + 45z = 166500$………………(2)

$-77z = -38500$……………(3)

(3) ⇒ $z = \frac{-38500}{-77} = 500$ (2) ⇒ $72y = 166500 - 45(500)$

$72y = 166500 - 22500 \Rightarrow y = 2000$

(1) ⇒ $x = 3(2000) + 500 - 5500 \Rightarrow x = 1000$

The prices per unit of the three commodities are Rs.1000, Rs. 2000 and Rs. 500

Question 8.

An amount of ₹5, 000/ - is to be deposited in three different bonds bearing 6%, 7% and 8% per year respectively. Total annual income is ₹358/–. If the income from the first two investments is ₹ 70 /- more than the income from the third, then find the amount of investment in each bond by the rank method.

Solution:

Let the amount of investment in the three different bonds be Rs. x, Rs. y and Rs. z respectively. We get the following equations according to the given conditions,

$x + y + z = 5000$

$\frac{6}{100}x + \frac{7}{100}y + \frac{8}{100}z = 358$ (or) $6x + 7y + 8z = 35800$

$\frac{6}{100}x + \frac{7}{100}y = 70 + \frac{8}{100}z$ (or) $6x + 7y - 8z = 7000$

This can be written as $\begin{pmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5000 \\ 35800 \\ 7000 \end{pmatrix}$

A X = B

Augmented matrix [A,B]

$$\begin{pmatrix} 1 & 1 & 1 & 5000 \\ 6 & 7 & 8 & 35800 \\ 6 & 7 & -8 & 7000 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 5000 \\ 6 & 7 & 8 & 35800 \\ 0 & 0 & -16 & -28800 \end{pmatrix} R_3 \to R_3 - R_2$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 5000 \\ 0 & 1 & 2 & 5800 \\ 0 & 0 & -16 & -28800 \end{pmatrix} R_2 \to R_2 - 6R_1$$

The above equivalent matrix is in echelon form with 3 non-zero rows.

So $\rho(A) = \rho([A, B]) = 3 =$ number of unknowns. the system has a unique solution.

The matrix equation is given by

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -16 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5000 \\ 5800 \\ -28800 \end{pmatrix}$$

$x + y + z = 5000$…(1)

$y + 2z = 5800$…(2)

$-16z = -28800$…(3)

(3) ⇒ $z = 1800$

(2) ⇒ $y = 5800 - 2(1800) = 2200$

(1) ⇒ $x = 5000 - 2200 - 1800 = 1000$

The amount invested in the three bonds are ₹ 1000 , ₹ 2200 and ₹ 1800 .

Exercise 1.2

Question 1.

Solve the following equations by using Cramer's rule

(iii) $2x + y - z = 3, x + y + z = 1, x - 2y - 3z = 4$

$$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & -2 & -3 \end{vmatrix} = 2(-3+2) - 1(-3-1) - 1(-2-1)$$
$$= 2(-1) - 1(-4) - 1(-3) = -2+4+3$$
$$= 5 \neq 0$$

System consistent with unique solution

$$\Delta_x = \begin{vmatrix} 3 & 1 & -1 \\ 1 & 1 & 1 \\ 4 & -2 & -3 \end{vmatrix} = 3(-3+2) - 1(-3-4) - 1(-2-4)$$
$$= 3(-1) - 1(-7) - 1(-6)$$
$$= -3 + 7 + 6 = 10$$

$$\Delta_y = \begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 1 & 4 & -3 \end{vmatrix} = 2(-3-4) - 3(-3-1) + (-1)(4-1)$$
$$= 2(-7) - 3(-4) - 1(3)$$
$$= -14 + 12 - 3 = -5$$

$$\Delta_z = \begin{vmatrix} 2 & 1 & 3 \\ 1 & 1 & 1 \\ 1 & -2 & 4 \end{vmatrix} = 2(4+2) - 1(4-1) + 3(-2-1)$$
$$= 2(6)-1(3)+3(-3) = 12-3-9 = 12-12 = 0$$

$\Delta_x = \frac{10}{5} = 2$; $y = \frac{\Delta_y}{\Delta} = \frac{-5}{5} = -1$; $z = \frac{\Delta_z}{\Delta} = \frac{0}{5} = 0$

$\therefore$ The solution is $(x, y, z) = (2, -1, 0)$

1(iv) $x + y + z = 6, 2x + 3y - z = 5, 6x - 2y - 3z = -7$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & -1 \\ 6 & -2 & -3 \end{vmatrix} = 1(-9-2) - 1(-6+6) + 1(-4-18)$$
$$= 1(-11) - 1(0) + 1(-22) = -11-22 = -33$$

$$\Delta_x = \begin{vmatrix} 6 & 1 & 1 \\ 5 & 3 & -1 \\ -7 & -2 & -3 \end{vmatrix} = 6(-9-2) - 1(-15-7) + 1(-10+21)$$
$$= 6(-11)-1(-22)+1(11)$$
$$= -66+22+11 = -66 + 33 = -33$$

$$\Delta_y = \begin{vmatrix} 1 & 6 & 1 \\ 2 & 5 & -1 \\ 6 & -7 & -3 \end{vmatrix} = 1(-15-7) - 6(-6+6) + 1(-14-30)$$
$$= 1(-22) - 6(0) + 1(-44) = -22-44$$
$$= -66$$

$$\Delta_z = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 3 & 5 \\ 6 & -2 & -7 \end{vmatrix} = 1(-21+10) - 1(-14-30) + 6(-4-18)$$
$$= 1(-11) - 1(-44) + 6(-22)$$
$$= -11 + 44 - 132 = -99$$

$\therefore x = \frac{\Delta_x}{\Delta} = \frac{-33}{-33} = 1$

$y = \frac{\Delta_y}{\Delta} = \frac{-66}{-33} = 2$

$z = \frac{\Delta_z}{\Delta} = \frac{-99}{-33} = 3$

Hence the solution is $(x, y, z) = (1,2,3)$.

1(v) $x + 4y + 3z = 2, 2x - 6y + 6z = -3, 5x - 2y + 3z = -5$

Solution:

$$\Delta = \begin{vmatrix} 1 & 4 & 3 \\ 2 & -6 & 6 \\ 5 & -2 & 3 \end{vmatrix} = 1(-18+12) - 4(6-30) + 3(-4+30)$$
$$= 1(-6) - 4(-24) + 3(26)$$
$$= -6 + 96 + 78 = 168 \neq 0$$

we can use Cramer's rule

$$\Delta_x = \begin{vmatrix} 2 & 4 & 3 \\ -3 & -6 & 6 \\ -5 & -2 & 3 \end{vmatrix} = 2(-18+12) - 4(-9+30) + 3(6-30)$$
$$= 2(-6) - 4(21) + 3(-24)$$
$$= -12 - 84 - 72 = -168$$

$$\Delta_y = \begin{vmatrix} 1 & 2 & 3 \\ 2 & -3 & 6 \\ 5 & -5 & 3 \end{vmatrix} = 1(-9+30) - 2(6-30) + 3(-10+15)$$
$$= 1(21) - 2(-24) + 3(5) = 84$$

$$\Delta_z = \begin{vmatrix} 1 & 4 & 2 \\ 2 & -6 & -3 \\ 5 & -2 & -5 \end{vmatrix} = 1(30-6) + 4(-10+15) + 2(-4+30)$$
$$= 1(24) - 4(5) + 2(26) = 56$$

$\therefore x = \frac{\Delta_x}{\Delta} = \frac{-168}{168} = -1$

$y = \frac{\Delta_y}{\Delta} = \frac{84}{168} = \frac{1}{2}$

$z = \frac{\Delta_z}{\Delta} = \frac{56}{168} = \frac{1}{3}$

Hence the solution is $(x, y, z) = \left(-1, \frac{1}{2}, \frac{1}{3}\right)$

Question 5.

In a market survey three commodities A, B and C were considered. In finding out the index number some fixed weights were assigned to the three varieties in each of the commodities. The table below provides information regarding the consumption of three commodities according to the three varieties and also the total weight received by the commodity.

Commodity variety	Variety			Total weight
	I	II	III	
A	1	2	3	11
B	2	4	5	21
C	3	5	6	27

Find the weights assigned to the three varieties by using Cramer's Rule.

Solution:

Let the weights assigned to the three varieties be x, y and z respectively.

According to the problem,

For variety A , $x + 2y + 3z = 11$

For variety B, $2x + 4y + 5z = 21$

For variety C , $3x + 5y + 6z = 27$

Now $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{vmatrix} = 1(24 - 25) - 2(12 - 15) + 3(10 - 12)$

$= -1 - 2(-3) + 3(-2) = -1 \neq 0$

So there exists a unique solution which can be solved by Cramer's rule.

$\Delta_x = \begin{vmatrix} 11 & 2 & 3 \\ 21 & 4 & 5 \\ 27 & 5 & 6 \end{vmatrix} = 11(24 - 25) - 2(126 - 135) + 3(105 - 108)$

$= -11 + 18 - 9 = -2$

$\Delta_y = \begin{vmatrix} 1 & 11 & 3 \\ 2 & 21 & 5 \\ 3 & 27 & 6 \end{vmatrix} = 1(126 - 135) - 11(12 - 15) + 3(54 - 63)$

$= -9 + 33 - 27 = -3$

$\Delta_z = \begin{vmatrix} 1 & 2 & 11 \\ 2 & 4 & 21 \\ 3 & 5 & 27 \end{vmatrix} = 1(108 - 105) - 2(54 - 63) + 11(10 - 12)$

$= 3 + 18 - 22 = -1$

By Cramer's rule,

$x = \frac{\Delta_x}{\Delta} = \frac{-2}{-1} = 2$;

$y = \frac{\Delta_y}{\Delta} = \frac{-3}{-1} = 3$;

$z = \frac{\Delta_z}{\Delta} = \frac{-1}{-1} = 1$

Hence the weights assigned to the three varieties are 2,3 and 1 units respectively.

Question 6.

A total of ₹ 8,500 was invested in three interest-earning accounts. The interest rates were 2%, 3% and 6% if the total simple interest for one year was ₹ 380 and the amount invested at 6% was equal to the sum of the amounts in the other two accounts, then how much was invested in each account? (use Cramer's rule).

Account	Interest rate
1	2%
2	3%
3	6%

Solution:

Let the amounts invested in the three accounts be Rs. x, Rs. y and Rs. z respectively

Interest for the three accounts are $\frac{2}{100}x, \frac{3}{100}y$ and $\frac{6}{100}z$

According to the problem, x+y+z=8500...... (1)

$\frac{2}{100}x + \frac{3}{100}y + \frac{6}{100}z = 380$

(or) multiplying by 100 ,

$2x + 3y + 6z = 38000$.........(2)

$z = x + y$ or $x + y - z = 0$.........(3)

Now, $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 6 \\ 1 & 1 & -1 \end{vmatrix} = 1(-3 - 6) - 1(-2 - 6) + 1(2 - 3)$

$= (-9) - 1(-8) + 1(-1) = -9 + 8 - 1 = -2 \neq 0$

So there exists a unique solution to the system (1) , (2) and (3)

$\Delta_x = \begin{vmatrix} 8500 & 1 & 1 \\ 38000 & 3 & 6 \\ 0 & 1 & -1 \end{vmatrix}$

$= 8500(-3 - 6) - 1(-38000) + 1(38000)$

$= -76500 + 76000 = -500$

$\Delta_y = \begin{vmatrix} 1 & 8500 & 1 \\ 2 & 38000 & 6 \\ 1 & 0 & -1 \end{vmatrix}$

$= 1(-38000) - 8500(-2 - 6) + 1(-38000)$

$= -38000 + 68000 - 38000 = -8000$

$\Delta_z = \begin{vmatrix} 1 & 1 & 8500 \\ 2 & 3 & 38000 \\ 1 & 1 & 0 \end{vmatrix}$

$= 1(-38000) - 1(-38000) + 8500(2 - 3)$

$= -38000 + 38000 - 8500 = -8500$

So by Cramer's rule,

$x = \frac{\Delta_x}{\Delta} = \frac{-500}{-2} = 250$ & $y = \frac{\Delta_y}{\Delta} = \frac{-8000}{-2} = 4000$

$z = \frac{\Delta_z}{\Delta} = \frac{-8500}{-2} = 4250$

Thus the amount invested at 2% is ₹250, at 3% is ₹4000 and at 6% is ₹4250.

Exercise 1.3

Question 2.

A new transit system has just gone into operation in Chennai. Of those who use the transit system this year, 30% will switch over to using the metro train next year and 70% will continue to use the transit system. Of those who use metro train this year, 70% will continue to use metro train next year and 30% will switch over to the transit system. Suppose the population of Chennai city remains constant and that 60% of the commuters use the transit system and 40% of the commuters use metro train this year.

(i) What per cent of commuters will be using the transit system after one year?

(ii) What per cent of commuters will be using the transit system in the long run?

Solution:

Let T denote transit system and M denote metro train.

From the question,

(T → T) = 70% = 0.7 ; (T → M) = 30% = 0.3

(M → T) = 30% = 0.3 ; (M → M) = 70% = 0.7

The transition probability matrix is given by

$$T = \begin{matrix} \\ T \\ M \end{matrix}\begin{pmatrix} T & M \\ 0.7 & 0.3 \\ 0.3 & 0.7 \end{pmatrix}$$

The current position is given by T = 60% and M = 40%

(T M) = (0.6 0.4)

We have to predict the values of T and M after one year.

(i) $(0.6 \quad 0.4)\begin{pmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{pmatrix} = (0.42 + 0.12 \qquad 0.18 + 0.28)$

$= (0.54 \quad 0.46)$

T = 0.54 = 54% & M = 0.46 = 46%

So after one year, 54% of commuters will use the transit system and 46% of commuters will use the metro train.

(ii) **At equilibrium :** $(T\ M)\begin{pmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{pmatrix} = (T \quad M)$

By matrix multiplications,

$(0.7\,T + 0.3M \qquad 0.3\,T + 0.7M) = (T \quad M)$

Equating the corresponding elements,

$0.7\,T + 0.3M = T \Rightarrow 0.3\,M = T - 0.7\,T = 0.3\,T$

$0.3\,T = 0.3\,M \Rightarrow \frac{T}{M} = \frac{0.3}{0.3} = \frac{1}{1}$

$T = \frac{1}{2} \times 100 = 50\%$ & $M = \frac{1}{2} \times 100 = 50\%$

Thus in the long run, 50% of the commuters will be using transit system and 50% will be using metro train.

Question 3.

Two types of soaps A and B are in the market. Their present market shares are 15% for A and 85% for B. Of those who bought A the previous year, 65% continue to buy it again while 35% switch over to B. Of those who bought B the previous year, 55% buy it again and 45% switch over to A. Find their market shares after one year and when is the equilibrium reached?

Solution:

A and B are the two types of soaps. The current market shares are 15% and 85%.

This is represented as (A B) = (0.15 0.85)

(A → A) = 65% = 0.65 : (A → B) = 35 % = 0.35

(B → A) = 45% = 0.45: (B → B) = 55% = 0.55

$$T = \begin{matrix} \\ A \\ B \end{matrix}\begin{pmatrix} A & B \\ 0.65 & 0.35 \\ 0.45 & 0.55 \end{pmatrix}$$

(i) Their market shares after one year is given by

$(0.15 \quad 0.85)\begin{pmatrix} 0.65 & 0.35 \\ 0.45 & 0.55 \end{pmatrix}$

$= (0.0975 + 0.3825 \qquad 0.0525 + 0.4675) = (0.48 \quad 0.52)$

(i.e) A = 0.48 = 48% & B = 0.52 = 52%

So after one-year market shares of soap A will be 48%

and soap B will be 52%

(ii) $(A \quad B)\begin{pmatrix} 0.65 & 0.35 \\ 0.45 & 0.55 \end{pmatrix} = (A \quad B)$

$(0.65\,A + 0.45\,B \qquad 0.35\,A + 0.55\,B) = (A \quad B)$

$0.65\,A + 0.45\,B = A \Rightarrow 0.45\,B = A - 0.65\,A = 0.35\,A$

$0.35\,A = 0.45\,B \Rightarrow \frac{A}{B} = \frac{0.45}{0.35} = \frac{45}{35}$ *[45+35 = 80]*

$A = \frac{45}{80} \times 100 = 56.25\%$ & $B = \frac{35}{80} \times 100 = 43.75\%$

The equilibrium is reached when the market share of soap A is 56.25% and the market share of soap B is 43.75%

Question 4.

Two products A and B currently share the market with shares 50% and 50% each respectively. Each week some brand switching takes place. Of those who bought A the previous week, 60% buy it again whereas 40% switch over to B. Of those who bought B the previous week, 80% buy it again whereas 20% switch over to A. Find their shares after one week and after two weeks. If the price war continues, when is the equilibrium reached?

Solution:

(A → A) = 60% = 0.6; (A → B) = 40% = 0.4

(B → A) = 20% = 0.2; (B→ B) = 80% = 0.8

The transition probability matrix is given by

$$T = \begin{matrix} \\ A \\ B \end{matrix}\begin{pmatrix} A & B \\ 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix}$$

Current market share : (A B) = (0.5 0.5)

fter one week: The shares of A and B are given by

$$\begin{matrix} A & B \\ (0.5 & 0.5) \end{matrix} \begin{matrix} \\ A \\ B \end{matrix}\begin{pmatrix} A & B \\ 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix} = \begin{matrix} A & B \\ (0.3 + 0.1 & 0.2 + 0.4) \end{matrix} = \begin{matrix} A & B \\ (0.4 & 0.6) \end{matrix}$$

So after one week the market share of A is 0.4 = 40%

and that of B is 0.6 = 60%

After two weeks : The shares of A and B are given by

$$\begin{matrix} A & B \\ (0.4 & 0.6) \end{matrix} \begin{matrix} \\ A \\ B \end{matrix}\begin{pmatrix} A & B \\ 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix} = \begin{matrix} A & B \\ (0.24 + 0.12 & 0.16 + 0.48) \end{matrix} = \begin{matrix} A & B \\ (0.36 & 0.64) \end{matrix}$$

Thus after two weeks, A will have 36% of shares and B will have 64% of shares.

Equilibirium : $(A \quad B)\begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix} = (A \quad B)$

$(0.6\,A + 0.2\,B \qquad 0.4\,A + 0.8\,B) = (A \quad B)$

Equating the corresponding elements,

$0.6\,A + 0.2\,B = A \Rightarrow 0.2\,B = A - 0.6\,A = 0.4\,A$

$0.4\,A = 0.6\,B \Rightarrow \frac{A}{B} = \frac{0.6}{0.3} = \frac{6}{3}$ *[6 + 3 = 9]*

$A = \frac{6}{9} \times 100 = 66.67\% = 67\%$ & $B = \frac{3}{9} \times 100 = 33.33\% = 33\%$

Thus the equilibrium is reached when the share of A is 33% and share of B is 67%.

CHAPTER 3
INTEGRAL CALCULUS - II
(2, 3 and 5 Marks)

2 - Marks

Exercise: 3.1

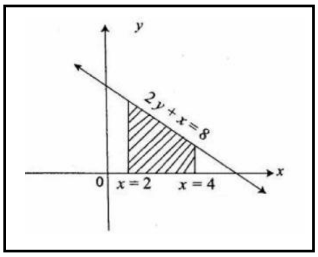
Question 1.

Using Integration, find the area of the region bounded the line $2y + x = 8$, the x-axis and the lines $x = 2, x = 4$

Solution:

The given lines are $2y + x = 8$, x-axis, $x = 2, x = 4$

Required area $= \int_2^4 y dx.$



Now $2y + x = 8 \Rightarrow y = \frac{8-x}{2}$

Area $= \int_2^4 \left(\frac{8-x}{2}\right) dx$

$= \frac{1}{2}\left[8x - \frac{x^2}{x}\right]_2^4$

$= \frac{1}{2}[32 - 8 - 16 + 2]$

$= 5$ sq. units

Question 2.

Find the area bounded by the lines $y - 2x - 4 = 0$, $y = 1, y = 3$ and the y-axis.

Solution:

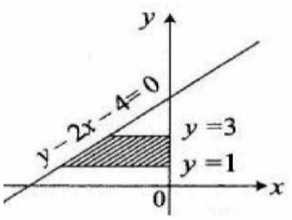
Given lines are $y - 2x - 4 = 0$, $y = 1, y = 3$, y-axis

$y - 2x = 4 = 0 \Rightarrow x = \frac{y - 4}{2}$

We observe that the required area lies to the left to the y-axis

Area $= \int_1^3 - x dy$

$= -\int_1^3 \frac{y-4}{2} dy$



$= -\frac{1}{2}\left[\frac{y^2}{2} - 4y\right]_1^3$

$= -\frac{1}{2}\left[\frac{9}{2} - 12 - \frac{1}{2} + 4\right]$

$= -\frac{1}{2}(-4) = 2$ sq.units

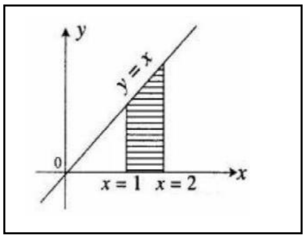
Question 4.

Find the area bounded by the line $y = x$, the x-axis and the ordinates $x = 1, x = 2$.

Solution:

Given lines are $y = x$, x-axis,

$x = 1, x = 2$



Required area $= \int_1^2 y dx$

$= \int_1^2 x dx = \frac{x^2}{2}\int_1^2 = 2 - \frac{1}{2}$

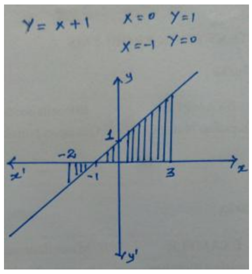
$= \frac{3}{2}$ sq.units

Question 5.

Using integration, find the area of the region bounded by the line $y - 1 = x$, the x axis and the ordinates $x = -2, x = 3$

Solution: Given lines are $y - 1 = x$

$\Rightarrow y = x + 1$; x-axis, $x = -2, x = 3$



Required area

$= \int_{-2}^{-1} - y dx + \int_{-1}^{3} y dx$

$= -\int_{-2}^{-1} (x + 1)dx + \int_{-1}^{3} (x + 1)dx$

$= -\left[\frac{(x+1)^2}{2}\right]_{-2}^{-1} + \left[\frac{(x+1)^2}{2}\right]_{-1}^{3}$

$= -\left[\frac{(-1+1)^2}{2} - \frac{(-2+1)^2}{2}\right] + \left[\frac{(3+1)^2}{2} - \frac{(-1+1)^2}{2}\right]$

$= -\frac{1}{2}[0 - 1] + \frac{1}{2}[16 - 0] = \frac{1}{2} + 8 = \frac{17}{2}$ sq.units

Exercise 3.2

Question 1.

The cost of an overhaul of an engine is ₹10, 000 The operating cost per hour is at the rate of $2x - 240$ where the engine has run x km. Find out the total cost if the engine runs for 300 hours after overhaul.

Solution:

Given that the overhaul cost is Rs. 10,000 .

The marginal cost is $2x - 240$

$MC = 2x - 240 \Rightarrow C = \int MC\, dx + k = \int(2x - 240)dx + k$

$C = x^2 - 240x + k$

k is the overhaul cost $\Rightarrow k = 10,000$

So $C = x^2 - 240x + 10,000$

When x = 300 hours, $C = (300)^2 - 240(300) + 10,000$

$\Rightarrow C = 90,000 - 72000 + 10,000 \Rightarrow C = 28,000$

Question 9.

Determine the cost of producing 200 air conditioners if the marginal cost (is per unit) is $C'(x) = \left(\frac{x^2}{200} + 4\right)$

Solution:

Given $MC = C'(x) = \left(\frac{x^2}{200} + 4\right)$

$\Rightarrow$ Total cost $C = \int\left(\frac{x^2}{200} + 4\right) dx + k = \frac{x^3}{600} + 4x + k$

When $x = 0, c = 0 \Rightarrow k = 0 \Rightarrow C = \frac{x^3}{600} + 4x$

When $x = 200$, $C = \frac{(200)^3}{600} + 4(200) = \frac{8,000,000}{600} + 800$

$C = 14133.33$

So the cost of producing 200 air conditioners is ₹14133.33

Question 11.

If the marginal revenue function for a commodity is $MR = 9 - 4x^2$. Find the demand function.

Solution:

Given, marginal Revenue function $MR = 9 - 4x^2$

Revenue function, $R = \int(MR)dx + k$

$R = \int (9 - 4x^2)dx + k = 9x - \frac{4}{3}\mathbf{x^3} + k$

Since $R = 0$ when $x = 0, k = 0 \Rightarrow R = 9x - \frac{4}{3}\mathbf{x^3}$

Demand function $P = \frac{R}{x} \Rightarrow P = 9 - \frac{4}{3}x^2$

3 - MARKS

EXERCISE 3.1

Question 3.

Calculate the area bounded by the parabola $y^2 = 4ax$ and its latus rectum.

Solution:

Given parabola is $y^2 = 4ax$,

Equation of latus rectum is $x = a$

Required area = 2 [Area in the first quadrant]

limits $x = 0$ and $x = a$

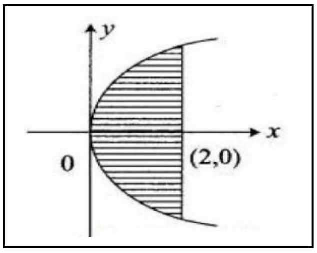
$= 2\int_0^a ydx$

$= 2\int_0^a \sqrt{4ax}dx$

$= 2(2\sqrt{a})\int_0^a x^{\frac{1}{2}}dx$

$= 4\sqrt{a}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^a$

$= (4\sqrt{a})\frac{2}{3}a^{\frac{3}{2}} = \frac{8}{3}a^2$ sq.units



Question 6.

Find the area of the region lying in the first quadrant bounded by the region $y = 4x^2, x = 0, y = 0$ and $y = 4$.

Solution:

The given parabola is $y = 4x^2$

$x^2 = \frac{y}{4}$

comparing with the standared form $x^2 = 4\,ay$

$4a = \frac{1}{4} \Rightarrow a = \frac{1}{16}$

The parabola is symmetric about y-axis

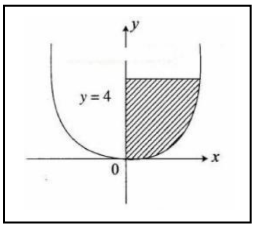
We require the area in the first quadrant.

Area $= \int_0^4 xdy = \int_0^4 \sqrt{\frac{y}{4}}dy$

$= \frac{1}{2}\int_0^4 \sqrt{y}dy$

$= \frac{1}{2}\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$

$= \frac{1}{3}(4)^{\frac{3}{2}} = \frac{8}{3}$ sq.units



Question 7.

Find the area bounded by the curve $y = x^2$ and the line $y = 4$

Solution:

Given the parabola is $y = x^2$ and line $y = 4$

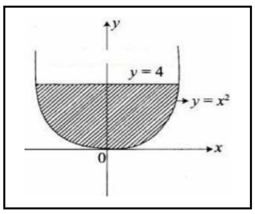
The parabola is symmetrical about the y-axis.

So required area = 2 [Area in the first quadrant between limits $y = 0$ and $y = 4$]

$= 2\int_0^4 xdy = 2\int_0^4 \sqrt{y}dy$

$= 2\left[\frac{2}{3}y^{\frac{3}{2}}\right]_0^4 = \frac{4}{3}\cdot 8$

$= \frac{32}{3}$ sq.units



EXERCISE 3.2

Question 3.

The elasticity of demand with respect to price for a commodity is given by $\frac{(4-x)}{x}$ where p is the price when demand is x. Find the demand function when the price is 4 and the demand is 2. Also, find the revenue function.

Solution:

Given $\eta_d = \frac{4-x}{x} \Rightarrow$ (i.e) $\frac{-p}{x}\frac{dx}{dp} = \frac{4-x}{x}$

$\Rightarrow \frac{-dx}{4-x} = \frac{dp}{p} \Rightarrow \int \frac{dx}{x-4} = \int \frac{dp}{p}$

$\log(x-4) = \log p + \log k \Rightarrow x - 4 = pk$

When $p = 4, x = 2$ gives $\Rightarrow 2 - 4 = 4k \Rightarrow k = -\frac{1}{2}$

Hence $p = \frac{x-4}{\left(-\frac{1}{2}\right)} = 8 - 2x$ is the demand function

The Revenue function $R = px = \mathbf{8x - 2x^2}$

Question 4.

A company receives a shipment of 500 scooters every 30 days. From experience, it is known that the inventory on hand is related to the number of days x. Since the shipment, $I(x) = 500 - 0.03x^2$, the daily holding cost per scooter is ₹ 0.3 . Determine the total cost for maintaining inventory for 30 days.

Solution: inventory $I(x) = 500 - 0.03x^2$

Unit holding cost C_1 = ₹ 0.3 & $T = 30$ days

So total inventory carrying cost

$= C_1\int_0^T I(x)dx = 0.3\int_0^{30}(500 - 0.03x^2)dx$

$= 0.3\left(500x - \frac{0.03x^3}{3}\right)_0^{30} = 0.3\left[500(30) - \frac{0.03}{3}(30)^3\right]$

$= 0.3[15000 - 270] = 4419$

The total cost for maintaining inventory for 30 days is ₹4, 419.

Question 5.

An account fetches interest at the rate of 5% per annum compounded continuously. An individual deposits ₹ 1,000 each year in his account. How much will be in the account after 5 years. $\left(e^{0.25} = 1.284\right)$

Solution: $p = 1000, N = 5, r = 5\% = 0.05$

Annuity $= \int_0^5 1000e^{0.05t}dt = \frac{1000}{0.05}\left(e^{0.05t}\right)_0^5$

$= 20000\left[e^{0.25} - e^0\right] = 20000(1.284 - 1] = 5680$

After 5 years ₹ 5680 will be in the account

Question 6.

The marginal cost function of a product is given by $\frac{dc}{dx} = 100 - 10x + 0.1x^2$ where x is the output. Obtain the total and the average cost function of the firm under the assumption, that its fixed cost is ₹ 500 .

Solution: Given $MC = \frac{dc}{dx} = 100 - 10x + 0.1x^2$

$C = \int MCdx + k \Rightarrow C = \int(100 - 10x + 0.1x^2)dx + k$

$C = 100x - 5x^2 + \frac{0.1x^3}{3} + k$

The fixed cost is $500 \Rightarrow k = 500$

Hence total cost function $= 100x - 5x^2 + \frac{0.1x^3}{3} + 500$

Average cost function $AC = \frac{c}{x} = 100 - 5x + \frac{x^2}{30} + \frac{500}{x}$

Question 7.

The marginal cost function is $MC = 300x^{\frac{2}{5}}$ and fixed cost is zero. Find out the total cost and average cost functions.

Solution:

Given $MC = 300x^{\frac{2}{5}}$

$C = \int 300x^{\frac{2}{5}}dx + k = 300\frac{x^{\frac{7}{5}}}{\frac{7}{5}} + k \Rightarrow$ So $C = \frac{1500}{7}x^{\frac{7}{5}}$

Average cost $= \frac{C}{x} = \frac{1500}{7}x^{\frac{2}{5}}$

Question 8.

If the marginal cost function of x units of output is $\frac{a}{\sqrt{ax+b}}$ and if the cost of output is zero. Find the total cost as a function of x.

Solution:

Given $MC = \frac{a}{\sqrt{ax+b}}$

Total cost $= \int \frac{a}{\sqrt{ax+b}}dx + k \Rightarrow \quad C = 2\sqrt{ax+b} + k$

The cost of output is zero $\Rightarrow x = 0, C = 0$

$0 = 2\sqrt{b} + k \Rightarrow k = -2\sqrt{b}$

So total cost function is $2\sqrt{ax+b} - 2\sqrt{b}$

Question 10.

The marginal revenue (in thousands of Rupees) functions for a particular commodity is $5 + 3e^{-0.03x}$ where x denotes the number of units sold. Determine the total revenue from the sale of 100 units.

(Given $e^{-3} = 0.05$ approximately)

Solution:

Given, marginal Revenue $R''(x) = 5 + 3e^{-0.03x}$

Total revenue from the sale of 100 units is

$R = \int_0^{100} (5 + 3e^{-0.03x})dx$

$R = \left[5x + \frac{3e^{-0.03x}}{-0.03}\right]_0^{100}$ $R = \left(500 + \frac{3e^{-0.03(100)}}{-0.03}\right) - \left(0 - \frac{3}{0.03}\right)$

$R = 500 - 100e^{-3} + 100$

$R = 600 - 100(0.05) = 595$

Total revenue $= 595 \times 1000 =$ ₹5,95,000

Question 12.

Given the marginal revenue function $\frac{4}{(2x+3)^2} - 1$, show that the average revenue function is $P = \frac{4}{6x+9} - 1$

Solution:

$MR = \frac{4}{(2x+3)^2} - 1 \Rightarrow$

$R = \int \frac{4}{(2x+3)^2}dx - \int dx = \frac{2}{-(2x+3)} - x + k$

Since $R = 0$ when $x = 0$

$0 = \frac{2}{-3} + k \Rightarrow k = \frac{2}{3} \Rightarrow R = \frac{-2}{2x+3} - x + \frac{2}{3}$

Average revenue function $P = \frac{R}{x}$

$P = \frac{-2}{x(2x+3)} - 1 + \frac{2}{3x} = \frac{2}{x}\left[\frac{1}{3} - \frac{1}{2x+3}\right] - 1 = \frac{2}{x}\left[\frac{2x+3-3}{3(2x+3)}\right] - 1$

$= \frac{2}{x}\left(\frac{2x}{3(2x+3)}\right) - 1 = \frac{4}{6x+9} - 1$

which is the required answer.

Question 13.

A firm's marginal revenue function is $MR = 20e^{\frac{-x}{10}}\left(1 - \frac{x}{10}\right)$. Find the corresponding demand function.

Solution:

$MR = 20e^{\frac{-x}{10}}\left(1 - \frac{x}{10}\right)$ **$[\int e^x [f(x) + f''(x)) dx = e^x [f(x) + c\}$**

$R = \int 20e^{\frac{-x}{10}}\left(1 - \frac{x}{10}\right)dx + k = 20\int \left(e^{\frac{-x}{10}} - \frac{x}{10}e^{\frac{-x}{10}}\right)dx + k$

$R = 20\int d\left(xe^{\frac{-x}{10}}\right) + k = 20xe^{\frac{-x}{10}} + k$

When $x = 0, R = 0$, so $k = 0$

$R = 20xe^{\frac{-x}{10}}$

The demand function $P = \frac{R}{x} = 20e^{\frac{-x}{10}}$

Question 15.

If the marginal revenue function is $R'(x) = 1500 - 4x - 3x^2$. Find the revenue function and average revenue function.

Solution:

$MR = R'(x) = 1500 - 4x - 3x^2$

Revenue function $R(x) = \int R'(x)dx + c$

$R = \int(1500 - 4x - 3x^2)dx + c$

$R = 1500x - 2x^2 - x^3 + c$

When $x = 0, R = 0 \Rightarrow c = 0$

So $R = 1500x - 2x^2 - x^3$

Average revenue function $P = \frac{R}{x} \Rightarrow 1500 - 2x - x^2$

Question 16.

Find the revenue function and the demand function if the marginal revenue for x units is $MR = 10 + 3x - x^2$

Solution:

Given $MR = 10 + 3x - x^2$

Revenue function $R(x) = \int (MR)dx + k$

$R = \int (10 + 3x - x^2)dx + k = 10x + \frac{3}{2}x^2 - \frac{x^3}{3} + k$

When $x = 0, R = 0, \Rightarrow k = 0 \Rightarrow \quad R = 10x + \frac{3}{2}x^2 - \frac{x^3}{3}$

Demand function $P = \frac{R}{x} = 10 + \frac{3}{2}x - \frac{x^2}{3}$

Question 19.

If $MR = 20 - 5x + 3x^2$, find total revenue function .

Solution:

$MR = 20 - 5x + 3x^2$

$R = \int (MR)dx + k = \int (20 - 5x + 3x^2)dx + k$

$R = 20x - \frac{5x^2}{2} + x^3 + k$ [Since $R = 0$, when $x = 0, k = 0$]

$\Rightarrow R = 20x - \frac{5x^2}{2} + x^3$ is the total revenue function

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Question 20.

If $MR = 14 - 6x + 9x^2$, find the demand function.

Solution: $MR = 14 - 6x + 9x^2$

$R = \int (14 - 6x + 9x^2)dx + k = 14x - 3x^2 + 3x^3 + k$

Since $R = 0$, when $x = 0, k = 0$

So revenue function $R = 14x - 3x^2 + 3x^3$

Demand function $P = \frac{R}{x} = 14 - 3x + 3x^2$

Exercise- 3.3

Question 1.

Calculate consumer's surplus if the demand function $p = 50 - 2x$ and $x = 20$

Solution:

Given demand function $p = 50 - 2x$, $x_0 = 20$

$CS = \int_0^{x_0} p(x)dx - x_0p_0$

When $x = 20$, $p_0 = 50 - 2(20) = 10$

$CS = \int_0^{20} (50 - 2x)dx - (20)(10)$

$= [50x - x^2]_0^{20} - 200 = [1000 - 400] - 200 = 400$

Hence the consumer's surplus is 400 units.

Question 2.

Calculate consumer's surplus if the demand function $p = 122 - 5x - 2x^2$, and $x = 6$

Solution:

Demand function $p = 122 - 5x - 2x^2$ and $x = 6$

when $x = x_0 = 6$

$p_0 = 122 - 5(6) - 2(36) = 122 - 30 - 72 = 20$

$CS = \int_0^{6} (122 - 5x - 2x^2)dx - (20)(6)$

$= \left[122x - \frac{5x^2}{2} - \frac{2x^3}{3}\right]_0^6 - 120$

$= (122)(6) - \frac{5}{2}(36) - \frac{2}{3}(216) - 120$

$= 732 - 90 - 144 - 120 = 378$

Hence the consumer's surplus is 378 units

Question 3.

The demand function $p = 85 - 5x$ and supply function $p = 3x - 35$. Calculate the equilibrium price and quantity demanded. Also, calculate consumer's surplus.

Solution:

Given $p_d = 85 - 5x$ and $p_s = 3x - 35$

At equilibrium prices $p_d = p_s$

$85 - 5x = 3x - 35 \Rightarrow 8x = 120 \Rightarrow x = 15$

$p_0 = 85 - 5(15) = 85 - 75 = 10$

$CS = \int_0^{x_0} p dx - x_0p_0$, $x_0 = 15$

$CS = \int_0^{15} (85 - 5x)dx - (15)(10)$

$= \left(85x - \frac{5x^2}{2}\right)_0^{15} - 150 = 85(15) - \frac{5(225)}{2} - 150 = 562.5$

The equilibrium price is ₹10, the quantity demanded is 15 .

The consumer surplus is 562.50 units.

Question 4.

The demand function for a commodity is $p = e^{-x}$.Find the consumer's surplus when $p = 0.5$.

Solution: Given demand function $p = e^{-x}$

At $p = 0.5$, (i.e) $p_0 = 0.5$; $p_0 = e^{-x_0} \Rightarrow 0.5 = e^{-x_0}$

Taking $\log_e$ on both sides

$\log_e(0.5) = -x_0 \Rightarrow \log_e\left(\frac{1}{2}\right) = -x_0 \Rightarrow -\log_e 2 = -x_0$

$\Rightarrow x_0 = \log_e 2$

$CS = \int_0^{\log_e 2} e^{-x}dx - (\log_e 2)(0.5) = [-e^{-x}]_0^{\log_e 2} - \frac{\log_e 2}{2}$

$= \frac{-1}{2} + 1 - \frac{\log_e 2}{2} = \frac{1}{2} - \frac{\log_e 2}{2}$

$CS = \frac{1}{2}[1 - \log_e 2]$ units

Question 5.

Calculate the producer's surplus at $x = 5$ for the supply function $p = 7 + x$.

Solution: Given supply function is $p = 7 + x$, $x_0 = 5$

$p_0 = 7 + x_0 = 7 + 5 = 12$

$PS = x_0p_0 - \int_0^{x_0} p(x)dx = 5(12) - \int_0^{5} (7 + x)dx$

$= 60 - \left(7x + \frac{x^2}{2}\right)_0^5 = 60 - 35 - \frac{25}{2} = \frac{25}{2}$

Hence the producer's surplus is $\frac{25}{2}$ units

Question 6.

If the supply function for a product is $p = 3x + 5x^2$. Find the producer's surplus when $x = 4$.

Solution: $p_s = 3x + 5x^2$ when $x = 4$, (i.e) $x_0 = 4$,

$p_0 = 3(4) + 5(4)^2 = 12 + 80 = 92$

$PS = x_0p_0 - \int_0^{x_0} p_s(x)dx$

$= 4(92) - \int_0^4 (3x + 5x^2)dx = 368 - \left[\frac{3x^2}{2} + \frac{5x^3}{3}\right]_0^4$

$= 368 - \left[\frac{48}{2} + \frac{5}{3}(64)\right] = 368 - 24 - 106.67 = 237.33$

the producer's surplus is 237.3 units.

Question 7.

The demand function for a commodity is $p = \frac{36}{x+4}$. Find the consumer's surplus when the prevailing market price is ₹ 6.

Solution: Given $p = \frac{36}{x+4}$

The marker price is ₹ 6 (i.e) $p_0 = 6$

$p_0 = \frac{36}{x_0 + 4} \Rightarrow 6 = \frac{36}{x_0 + 4} \Rightarrow x_o = 2$

$CS = \int_0^2 \left(\frac{36}{x+4}\right) dx - p_0x_0 = 36\int_0^2 \left(\frac{1}{x+4}\right) dx - (6)(2)$

$= 36[\log(x + 4)]_0^2 - 12 = 36[\log 6 - \log 4] - 12$

$= 36\log\frac{3}{2} - 12$

So the consumer's surplus when the prevailing market price is ₹ 6 is $\left(36\log\frac{3}{2} - 12\right)$ units.

Question 8.

The demand and supply functions under perfect competition are $p_d = 1600 - x^2$ and $p_s = 2x^2 + 400$ respectively. Find the producer's surplus.

Solution:

Given demand function $p_d = 1600 - x^2$ and

Supply function $p_S = 2x^2 + 400$

$p_s = p_d \Rightarrow 1600 - x^2 = 2x^2 + 400 \Rightarrow 3x^2 = 1200$

$\Rightarrow x^2 = 400 \Rightarrow x = \pm 20$

The value of x cannot be negative. So $x = 20$

$x_0 = 20. \Rightarrow p_0 = 1600 - (20)^2 = 1600 - 400 = 1200$

$PS = x_0p_0 - \int_0^{x_0} p_s dx = (20)(1200) - \int_0^{20} (2x^2 + 400)dx$

$= 24000 - \left[\frac{2x^3}{3} + 400x\right]_0^{20} = 24000 - \left[\frac{16000}{3} + 8000\right]$

$= 16000 - \frac{16000}{3} = \frac{32000}{3}$.

The producer's surplus is $\frac{32000}{3}$ units.

5 -MARKS

Exercise 3.2

Question 2.

Elasticity of a function $\frac{E_y}{E_x}$ is given by $\frac{E_y}{E_x} = \frac{-7x}{(1-2x)(2+3x)}$

Find the function when $x = 2, y = \frac{3}{8}$

Solution:

Given $\eta = \frac{E_y}{E_x} = \frac{-7x}{(1-2x)(2+3x)} \Rightarrow \frac{x}{y}\frac{dy}{dx} = \frac{-7x}{(1-2x)(2+3x)}$

$\frac{dy}{y} = \frac{-7x}{(1-2x)(2+3x)}\frac{dx}{x}$

$\int \frac{dy}{y} = 7\int \frac{dx}{(2x-1)(3x+2)}$ ------(1)

$\frac{1}{(2x-1)(3x+2)} = \frac{A}{(2x-1)} + \frac{B}{(3x+2)}$

$1 = A(3x+2) + B(2x-1)$

Let $x = \frac{1}{2} \Rightarrow 1 = A\left(\frac{3}{2}+2\right)$

$1 = A\left(\frac{7}{2}\right) \Rightarrow A = \frac{2}{7}$

Let $x = \frac{-2}{3} \Rightarrow 1 = B\left[2\left(\frac{-2}{3}\right) - 1\right]$

$1 = B\left(\frac{-4}{3} - 1\right)$

$1 = B\left(\frac{-7}{3}\right) \Rightarrow B = \frac{-3}{7}$

Using these values in (1) we get

$\int \frac{dy}{y} = 7\int \frac{\frac{2}{7}}{2x-1}dx - 7\int \frac{\frac{3}{7}}{3x+2}dx$

$\int \frac{dy}{y} = \int \frac{2dx}{2x-1} - \int \frac{3dx}{3x+2}$

$\log y = \log(2x-1) - \log(3x+2) + \log k$

$y = \left(\frac{2x-1}{3x+2}\right)k$

when $x = 2$, $y = \frac{3}{8} \Rightarrow \frac{3}{8} = \frac{3}{8}k$

$\Rightarrow k = 1$

Hence the function is $y = \frac{2x-1}{3x+2}$

Question 14.

The marginal cost of production of a firm is given by $C'(x) = 5 + 0.13x$, the marginal revenue is given by $R'(x) = 18$ and the fixed cost is ₹120 . Find the profit function.

Solution:

$MC = C'(x) = 5 + 0.13x$

$C(x) = \int C'(x)dx + k_1$

$= \int (5 + 0.13x)dx + k_1 = 5x + \frac{0.13}{2}x^2 + k_1$

When quantity produced is zero, fixed cost is 120

(i.e) When $x = 0, C = 120 \Rightarrow k_1 = 120$

Cost function is $5x + 0.065x^2 + 120$

Now given $MR = R'(x) = 18$

$\Rightarrow R(x) = \int 18dx + k_2 = 18x + k_2$

When $x = 0, R = 0 \Rightarrow k_2 = 0$

Revenue $= 18x$

Profit P = Total Revenue − Total cost

$= 18x - (5x + 0.065x^2 + 120)$

Profit function $= 13x - 0.065x^2 - 120$

Question 17.

The marginal cost function of a commodity is given by $MC = \frac{14000}{\sqrt{7x+4}}$ and the fixed cost is ₹ 18,000 . Find the total cost and average cost.

Solution:

Given $MC = \frac{14000}{\sqrt{7x+4}}$ fixed cost = ₹18,000

Total cost $= \int (MC)dx + k = \int \frac{14000}{\sqrt{7x+4}}dx + k$

$= 14000\left(\frac{2}{7}\sqrt{7x+4}\right) + k = 4000\sqrt{7x+4} + k$

Since the fixed cost is ₹18,000, when $x = 0, k = 18,000$

$\Rightarrow$ Total cost $C = 4000\sqrt{7x+4} + 18000$

Average cost A.C $= \frac{C}{x}$

$= \frac{4000}{x}\sqrt{7x+4} + \frac{18000}{x}$

Question 18.

If the marginal cost (MC) of production of the company is directly proportional to the number of units (x) produced, then find the total cost function, when the fixed cost is ₹ 5,000 and the cost of producing 50 units is ₹ 5,625 .

Solution:

Given that the marginal cost MC is directly proportional to the number of units x.

That is, $MC \propto x$

$MC = kx$, where k is the constant of proportionality Total cost $C = \int(MC)dx + c_1 = \int (kx)dx + c_1$ $C = \frac{kx^2}{2} + c_1$

The fixed cost is given as 5000 . So $c_1 = 5000$

$C = \frac{kx^2}{2} + 5000$

When $x = 50, C = 5625$

So $5625 = \frac{k}{2}(50)^2 + 5000$

$625 = \frac{2500}{2}k \quad \Rightarrow k = \frac{1}{2}$

Thus total cost function $C = \frac{1}{2}\left(\frac{x^2}{2}\right) + 5000$

$C = \frac{x^2}{4} + 5000$

Exercise 3.3

Question 9.

Under perfect competition for a commodity the demand and supply laws are $p_d = \frac{8}{x+1} - 2$ and $p_s = \frac{x+3}{2}$ respectively. Find the consumer's and producer's surplus.

Solution:

Given $p_d = \frac{8}{x+1} - 2$ and $p_s = \frac{x+3}{2}$

Here, since there is perfect competition, there is equilibrium, that is $p_d = p_s$

$\frac{8}{x+1} - 2 = \frac{x+3}{2}$

$\frac{8-2x-2}{x+1} = \frac{x+3}{2}$

$\frac{6-2x}{x+1} = \frac{x+3}{2}$

$(x+1)(x+3) = 12 - 4x$

$x^2 + 4x + 3 = 12 - 4x$

$x^2 + 8x - 9 = 0$

$(x+9)(x-1) = 0$

$x = -9, 1$

Since the value of x cannot be negative, $x = 1$ we take this value as x_0

$p_0 = \frac{8}{x_0+1} - 2 = \frac{8}{2} - 2 = 2$

$CS = \int_0^1 p_d dx - x_0 p_0$

$= \int_0^1 \left(\frac{8}{x+1} - 2\right) dx - (1)(2)$

$= [8\log(x+1) - 2x]_0^1 - 2$

$= 8\log 2 - 2 - [8\log 1 - 0] - 2$

$= 8\log 2 - 4$

$PS = x_0 p_0 - \int_0^{x_0} p_s dx$

$= 2 - \int_0^1 \frac{x+3}{2} dx = 2 - \frac{1}{2}\left(\frac{x^2}{2} + 3x\right)_0^1$

$= 2 - \frac{1}{2}\left(\frac{1}{2} + 3\right) = 2 - \frac{7}{4} = \frac{1}{4}$

Hence under perfect competition,

(i) The consumer's surplus is $(8\log 2 - 4)$ units

(ii) The producer's surplus is $\frac{1}{4}$ units.

Question 10.

The demand equation for a products is $x = \sqrt{100 - p}$ and the supply equation is $x = \frac{p}{2} - 10$. Determine the consumer's surplus and producer's surplus, under market equilibrium.

Solution:

Given demand equation is $x = \sqrt{100 - p}$ and supply equation is $x = \frac{p}{2} - 10$. So the demand law is $x^2 = 100 - p$

$\Rightarrow p_d = 100 - x^2$

Supply law is given by $x + 10 = \frac{p}{2}$

$\Rightarrow p_s = 2(x + 10)$

Under equilibrium $p_d = p_s$

$\Rightarrow 100 - x^2 = 2(x + 10)$

$\Rightarrow 100 - x^2 = 2x + 20$

$\Rightarrow x^2 + 2x - 80 = 0$

$\Rightarrow (x + 10)(x - 8) = 0$

$\Rightarrow x = -10, 8$

The value of x cannot be negative, So $x = 8$

When $x_0 = 8, p_0 = 100 - 8^2 = 100 - 64 = 36$

$CS = \int_0^8 (100 - x^2)dx - (8)(36)$

$= \left(100x - \frac{x^3}{3}\right)_0^8 - 288 = 800 - \frac{512}{3} - 288 = \frac{1024}{3}$

so consumer surplus $= \frac{1024}{3}$ units

$PS = 8(36) - \int_0^8 2(x + 10)dx$

$= 288 - 2\left(\frac{x^2}{2} + 10x\right)_0^8$

$= 288 - 2\left(\frac{64}{2} + 80\right)$

$= 288 - 2(112)$

$= 64$

So the producer's surplus is 64 units.

Question 11.

Find the consumer's surplus and producer's surplus for the demand function $p_d = 25 - 3x$ and supply function $p_S = 5 + 2x$.

Solution:

Given $p_d = 25 - 3x$ and $p_S = 5 + 2x$

At market equilibrium, $p_d = p_S$

$\Rightarrow 25 - 3x = 5 + 2x$

$\Rightarrow 5x = 20 \Rightarrow x = 4$

When $x_0 = 4, p_0 = 25 - 12 = 13$

$CS = \int_0^4 (25 - 3x)dx - 13(4) = \left(25x - \frac{3x^2}{2}\right)_0^4 - 52$

$= 100 - \frac{3}{2}(16) - 52 = 24$

So the consumer's surplus is 24 units.

$PS = 13(4) - \int_0^4 (2x + 5)dx$

$= 52 - (x^2 + 5x)_0^4 = 52 - 16 - 20 = 16$

So the producer's surplus is 16 units.

Chapter - 5
Numerical Methods
(2, 3 and 5 Marks)

2 - Marks

Exercise - 5.1

Question 1.

Evaluate Δ(log ax)

Solution:

$\Delta \mathbf{f}(\mathbf{x}) = \mathbf{f}(\mathbf{x}+\mathbf{h}) - \mathbf{f}(\mathbf{x})$

$\Delta \log ax = \log a(x+h) - \log ax = \log \left[\frac{a(x+h)}{ax}\right] = \log \left(1+\frac{h}{x}\right)$

Exercise 5.2

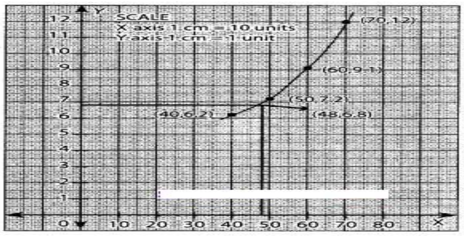
Question 1.

Using graphic method, find the value of y when $x = 48$ from the following data:

x	40	50	60	70
y	6.2	7.2	9.1	12

Solution:

The given points are (40,6.2), (50,7.2)(60,9.1) and (70,12).
We plot the points on a graph with suitable scale



The value of y when x = 48 is 6.8

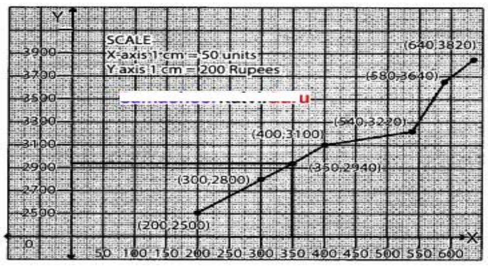
Question 2.

The following data relates to indirect labour expenses and the level of output

Estimate the expenses at a level of output of 350 units, by using the graphic method.

Solution:

Take the units of output along the x-axis, labour expenses along the y-axis.
The points to be plotted are
(200,2500), (300,2800)(400,3100), (640,3820),
(540,3220), (580, 3640)



From the graph, the expenses at a level of output of 350 units are ₹ 2940.

3 - Marks

Exercise 5.1

Question 2.

If $y = x^3 - x^2 + x - 1$ calculate the values of y for $x = 0, 1, 2, 3, 4, 5$ and form the forward differences table.

Solution:

Given $y = x^3 - x^2 + x - 1$

$x = 0, y = 0 - 0 + 0 - 1 = -1$

$x = 1, y = 1 - 1 + 1 - 1 = 0$

$x = 2, y = 8 - 4 + 2 - 1 = 5$

$x = 3, y = 27 - 9 + 3 - 1 = 20$

$x = 4, y = 64 - 16 + 4 - 1 = 51$

$x = 5, y = 125 - 25 + 5 - 1 = 104$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
0	−1					
		1				
1	0		4			
		5		6		
2	5		10		0	
		15		6		0
3	20		16		0	
		31		6		
4	51		22			
		53				
5	104					

Question 3.

If $h = 1$ then prove that $(\mathbf{E}^{-1}\Delta)x^3 = 3x^2 - 3x + 1$

Solution:

h = 1

To prove $(\mathbf{E}^{-1}\Delta)x^3 = 3x^2 - 3x + 1$

L.H.S : $(E^{-1}\Delta)x^3 = E^{-1}(\Delta x^3)$

$= E^{-1}[(x+h)^3 - x^3] = E^{-1}(x+h)^3 - E^{-1}(x^3)$

$= (x - h + h)^3 - (x-h)^3$

$= x^3 - (x-h)^3$

But given h = 1

$(E^{-1}\Delta)x^3 = x^3 - (x-1)^3 = x^3 - [x^3 - 3x^2 + 3x - 1]$

$= 3x^2 - 3x + 1 =$ RHS

So $(E^{-1}\Delta)x^3 = x^3 - (x-1)^3$

Question 4.

If $f(x) = x^2 + 3x$ then show that $\Delta f(x) = 2x + 4$

Solution:

$f(x) = x^2 + 3x$

$\Delta f(x) = f(x+h) - f(x)$

$= (x+h)^2 + 3(x+h) - x^2 - 3x$

$= x^2 + 2xh + h^2 + 3x + 3h - x^2 - 3x = 2xh + 3h + h^2$

Put h = 1, $\Delta f(x) = 2x + 4$

Question 5.

Evaluate $\Delta\left[\frac{1}{(x+1)(x+2)}\right]$ by taking ' 1 ' as the interval of differencing.

Solution:

$\Delta\left[\frac{1}{(x+1)(x+2)}\right], h=1$

By Partial fraction,

$\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$

A = 1, B=-1

So $\Delta\left[\frac{1}{(x+1)(x+2)}\right] = \Delta\left[\frac{1}{x+1} - \frac{1}{x+2}\right] = \Delta\left(\frac{1}{x+1}\right) - \Delta\left(\frac{1}{x+2}\right)$

$= \left[\frac{1}{x+1+1} - \frac{1}{x+1}\right] - \left[\frac{1}{x+1+2} - \frac{1}{x+2}\right]$

$= \left[\frac{1}{x+2} - \frac{1}{x+1}\right] - \left[\frac{1}{x+3} - \frac{1}{x+2}\right]$

$= \left[\frac{-1}{(x+2)(x+1)}\right]\left[\frac{1}{(x+3)(x+2)}\right]$

$= \frac{-1}{(x+2)}\left[\frac{1}{x+1} - \frac{1}{x+3}\right] = \frac{-2}{(x+1)(x+2)(x+3)}$

Question 6 :

Find the missing entry in the following table

x	0	1	2	3	4
y_x	1	3	9	-	81

Solution: Since 4 values are given

$\Delta^4 y_0 = 0, \Rightarrow \therefore (E-1)^4 y_0 = 0$

$(E^4 - 4E^3 + 6E^2 - 4E + 1)y_0 = 0$

$E^4 y_0 - 4E^3 y_0 + 6E^2 y_0 - 4E y_0 + y_0 = 0$

$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$

Given $y_0 = 1, y_1 = 3, y_2 = 9, y_4 = 81$

So we get $81 - 4y_3 + 6(9) - 4(3) + 1 = 0$

$81 - 4y_3 + 54 - 12 + 1 = 0$

$4y_3 = 124 \Rightarrow y_3 = 31$

Question 7.

Following are the population of a district

Year (x)	1881	1891	1901	1911	1921	1931
Population (y) Thousands	363	391	421	–	467	501

Find the population of the year 1911?

Solution:

$y_0 = 363, y_1 = 391, y_2 = 421, y_4 = 467$ and $y_5 = 501$

$\Delta^5 y_0 = 0, \Rightarrow (E-1)^5 y_0 = 0$

$E^5 y_0 - 5E^4 y_0 + 10E^3 y_0 - 10E^2 y_0 + 5E y_0 - y_0 = 0$

$y_5 - 5y_4 + 10y_3 - 10y_2 + 5y_1 - y_0 = 0$

$501 - 5(467) + 10y_3 - 10(421) + 5(391) - 363 = 0$

$501 - 2335 + 10y_3 - 4210 + 1955 - 363 = 0$

$-501 + 2335 + 4210 - 1955 + 363 = 10y_3$

$10y_3 = 4452 \Rightarrow y_3 = 445.2$

The population of the year 1911 is 445 thousand

Exercise 5.2

Question 3.

Using Newton's forward interpolation formula find the cubic polynomial

x	0	1	2	3
f(x)	1	2	1	10

Solution: Newton's forward interpolation formula is

$y_{(x=x_0+nh)} = y_0 + \frac{n}{1!}\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \ldots$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1			
		1		
1	2		-2	
		-1		12
2	1		10	
		9		
3	10			

$x_0 + nh = x.\ x_0 = 0, h = 1 \Rightarrow n = x$

So $y(x) = 1 + \frac{x}{1!}(1) + \frac{x(x-1)}{2!}(-2) + \frac{x(x-1)(x-2)}{3!}(12)$

$y(x) = 1 + x - (x^2 - x) + 2[x^3 - 3x^2 + 2x]$

$y(x) = 1 + x - x^2 + x + 2x^3 - 6x^2 + 4x$

$f(x) = y = 2x^3 - 7x^2 + 6x + 1$ is the required cubic polynomial

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Question 4.

The population of a city in a census taken once in 10 years is given below.

Year	1951	1961	1971	1981
Population in lakhs	35	42	58	84

Estimate the population in the year 1955.

Solution:

x	1951	1961	1971	1981
y	35	42	58	84

$y_{(x=x_0+nh)} = y_0 + \frac{n}{1!}\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \ldots$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
1951	35			
		7		
1961	42		9	
		16		1
1971	58		10	
		26		
1981	84			

Now $x = 1955$, $x_0 = 1951$; $h = 10$

$\Rightarrow n = \frac{x - x_0}{h} = \frac{1955-1951}{10} = \frac{4}{10} = 0.4$

$\Rightarrow$ **n -1 = 0.4 -1 = -0.6; n - 2 = 0.4 - 2 = -1.6**

$y_{(x=1955)} = 35 + \frac{0.4}{1!}(7) + \frac{(0.4)(-0.6)}{2!}(9) + \frac{(0.4)(-0.6)(-1.6)}{3!}(1)$

$= 35 + 2.8 + \frac{(0.4)(-0.6)(9)}{2} + \frac{(0.4)(-0.6)(-1.6)}{6}$

$= 37.8 - 1.08 + 0.064 = 36.784$

Thus the estimated population in the year 1955 is 36.784 lakhs

5 - Marks

Exercise 5.1

Question 8.

Find the missing entries from the following.

(x)	0	1	2	3	4	5
$y = f(x)$	0	–	8	15	–	35

Solution:

$y_0 = 0$, $y_2 = 8, y_3 = 15, y_5 = 35$

Since 4 values are given

$\Delta^4 y_0 = 0$, $\therefore (E-1)^4 y_0 = 0$

$(E^4 - 4E^3 + 6E^2 - 4E + 1)y_0 = 0$

$E^4 y_0 - 4E^3 y_0 + 6E^2 y_0 - 4E y_0 + y_0 = 0$

$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$

$\Rightarrow y_4 - 4(15) + 6(8) - 4y_1 + 0 = 0$

$\Rightarrow y_4 - 4y_1 = 12$

$\Delta^4 y_1 = 0$ $\therefore (E-1)^4 y_1 = 0$

$E^4 y_1 - 4E^3 y_1 + 6E^2 y_1 - 4E y_1 + y_1 = 0$

$y_5 - 4y_4 + 6y_3 - 4y_2 + y_1 = 0$

Given $y_0 = 0, y_2 = 8, y_3 = 15, y_5 = 35$
we get $35 - 4y_4 + 6(15) - 4(8) + y_1 = 0$

$-4y_4 + y_1 = -35 - 90 + 32$

$-4y_4 + y_1 = -93$

$4y_4 - y_1 = 93$

Solving (1) and (2)
$(1) \times 4$ gives
$4y_4 - 16y_1 = 48$

$4y_4 - y_1 = 93$

Subtracting,
$-15y_1 = -45 \Rightarrow y_1 = 3$

Substituting $y_1 = 3$ in (2)

$4y_4 - 3 = 93 \Rightarrow 4y_4 = 96 \Rightarrow y_4 = 24$
$y_1 = f(x_1) = 3$ and $y_4 = f(x_4) = 24$

Question 5.

In an examination the number of candidates who secured marks between certain intervals was as follows:

Marks	0 – 19	20 – 39	40 – 59	60 – 79	80 – 99
No. of candidates	41	62	65	50	17

Estimate the number of candidates whose marks are less than 70 .

Solution:

Since we have to find marks less than 70 we have to find cumulative frequency and also make the class interval continuous

Marks (x)	No. of candidates (y)	Cumulative frequency
$-0.5 - 19.5$	41	41
$19.5 - 39.5$	62	103
$39.5 - 59.5$	65	168
$59.5 - 79.5$	50	218
$79.5 - 99.5$	17	235

The difference table is as follows

Marks (x)	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
Less than 19.5	41				
		62			
Less than 39.5	103		3		
		65		-18	
Less than 59.5	168		-15		0
		50		-18	
Less than 79.5	218		-33		
		17			
Less than 99.5	235				

$y_{(x=x_n+nh)} = y_n + \frac{n}{1!}\nabla y_n + \frac{n(n+1)}{2!}\nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_n + \ldots$

$n = \frac{x - x_n}{h} = \frac{70 - 99.5}{20} = \frac{-30}{20} = -1.475$

n	n +1	n + 2
-1.475	-0.475	0.525

$y = 235 + (-1.475)(17) + \frac{(-1.475)(-0.475)}{2}(-33) + \frac{(-1.475)(-0.475)(0.525)}{6}(-18)$

$= 235 - 25.075 - 11.5603 - 1.1034 = 197.2$

Hence the estimated value of the number of candidates whose marks are less than 70 is 197

Question 6.

Find the value of $f(x)$ when $x = 32$ from the following table

x	30	35	40	45	50
$f(x)$	**15.9**	**14.9**	**14.1**	**13.3**	**12.5**

Solution:

$y_{(x=x_0+nh)} = y_0 + \frac{n}{1!}\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + ..$

The difference table is as follows

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
30	**15.9**				
		−1			
35	14.9		**0.2**		
		−0.8		**−0.2**	
40	14.1		0		**0.2**
		−0.8		0	
45	13.3		0		
		−0.8			
50	12.5				

$n = \frac{x - x_0}{h} = \frac{32-30}{5} = \frac{2}{5} = 0.4$

n	n -1	n - 2	n - 3
0.4	**-0.6**	**-1.6**	**-2.6**

$y_{(x=x_0+nh)} = y_0 + \frac{n}{1!}\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \ldots$

$y_{(x=32)} = 15.9 + \frac{0.4}{1!}(-1) + \frac{(0.4)(-0.6)}{2!}(0.2) +$

$\frac{(0.4)(-0.6)(-1.6)}{3!}(-0.2) + \frac{(0.4)(-0.6)(-1.6)(-2.6)}{4!}(0.2)$

$y = 15.9 - 0.4 - 0.024 - 0.0128 - 0.00832$

$y = 15.45488$

Hence the value of f(x) when x=32 is 15.45

Question 7.

The following data gives the melting point of an alloy of lead and zinc where ' t ' is the temperature in degree c and P is the percentage of lead in the alloy

P	40	50	60	70	80	90
T	**180**	**204**	**226**	**250**	**276**	**304**

Find the melting point of the alloy containing 84 per cent lead.

Solution:

$t_{(p=p_n+nh)=t_n} + \frac{n}{1!}\nabla t_n + \frac{n(n+1)}{2!}\nabla^2 t_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 t_n + \ldots.$

The difference table is given below

p	t	∇t	$\nabla^2 t$	$\nabla^3 t$	$\nabla^4 t$	$\nabla^5 t$
40	180					
		24				
50	204		−2			
		22		4		
60	226		2		−4	
		24		0		**4**
70	250		2		**0**	
		26		**0**		
80	276		**2**			
		28				
90	**304**					

Now ⇒ $n = \frac{x - x_n}{h} = \frac{84-90}{10} = \frac{-6}{10} = -0.6$

n	n +1	n + 2	n + 3	n + 4
- 0.6	**0.4**	**1.4**	**2.4**	**3.4**

$t_{(p=p_n+nh)=t_n} + \frac{n}{1!}\nabla t_n + \frac{n(n+1)}{2!}\nabla^2 t_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 t_n + \ldots.$

$t_{(p=84)} = 304 + \frac{(-0.6)}{1!}(28) + \frac{(-0.6)(0.4)}{2!}(2) + \frac{(-0.6)(0.4)(1.4)}{3!}(0)$

$+ \frac{(-0.6)(0.4)(1.4))(2.4)}{4!}(0) + \frac{(-0.6)(0.4)(1.4))(2.4)(3.4)}{5!}(4)$

$t = 304 - 16.8 - 0.24 - 0.0914$

$t = 286.8686$

Hence the melting point of the alloy containing 84 per cent lead is 286.9°C

Question 8.

Find $f(2.8)$ from the following table.

x	0	1	2	3
$f(x)$	1	2	11	34

Solution:

To find $y = f(x)$ at $x = 2.8$

We use Newton's backward interpolation formula **since the required value is near the end of the table.**

$$y_{(x=x_n+nh)} = y_n + \frac{n}{1!}\nabla y_n + \frac{n(n+1)}{2!}\nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_n + ..$$

The difference table given below

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$
0	1			
		1		
1	2		8	
		9		6
2	11		14	
		23		
3	34			

$$y_{(x=x_n+nh)} = y_n + \frac{n}{1!}\nabla y_n + \frac{n(n+1)}{2!}\nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_n + ...$$

Now $x_n = 3, h = 1, x = 2.8$

$n = \frac{x - x_n}{h} = \frac{2.8 - 3}{1} = \frac{-0.2}{1} = -0.2$

n	n +1	n + 2
-0.2	**0.8**	**1.8**

$$y = 34 + \frac{(-0.2)}{1!}(23) + \frac{(-0.2)(0.8)}{2!}(14) + \frac{(-0.2)(0.8)(1.8)}{3!}(6)$$

$y = 34 - 4.6 - 1.12 - 0.288$

$y = 27.992$

Hence the value of f(x) at x = 2.8 is 27.992

Question 9.

Using interpolation estimate the output of a factory in 1986 from the following data

Year	1974	1978	1982	1990
Output in 1000 tones	25	60	90	170

Solution:

Let x denote the year and y represent the output.

The x values are not equidistant. So we use Lagrange's formula

$\boldsymbol{x_0 = 1974, x_1 = 1978, x_2 = 1982, x_3 = 1990,}$

$\boldsymbol{y_0 = 25, y_1 = 60, y_2 = 80, y_3 = 170}$

For $x = 1986$ we have to find y value

$$\boldsymbol{y = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1}$$

$$\boldsymbol{+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \times y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3}$$

We find the different values separately and substitute in the formula.

x	x_0	x_1	x_2	x_3
1986	1974	1978	1982	1990

$x - x_0$	1986 − 1974	12	$x_0 - x_1$	1974 − 1978	−4
$x - x_1$	1986 − 1978	8	$x_0 - x_2$	1974 − 1982	−8
$x - x_2$	1986 − 1982	4	$x_0 - x_3$	1974 − 1990	−16
$x - x_3$	1986 − 1990	−4	$x_1 - x_2$	1978 − 1982	−4
$x_2 - x_3$	1982 − 1990	−8	$x_1 - x_3$	1978 − 1990	−12

$$y = \frac{(8)(4)(-4)}{(-4)(-8)(-16)}(25) + \frac{(12)(4)(-4)}{(4)(-4)(-12)}(60) + \frac{(12)(8)(-4)}{(8)(4)(-8)}(80) + \frac{(12)(8)(4)}{(16)(12)(8)}(170)$$

$y = 6.25 - 60 + 120 + 42.5 = 108.75$

The output of the factory in 1986 is 109 (thousand tonnes)

Question 10.

Use Lagrange's formula and estimate from the following data the number of workers getting income not exceeding Rs. 26 per month.

Income not exceeding (₹)	15	25	30	35
No. of workers	36	40	45	48

Solution:

Let x represent the income per month and y denote the number of workers.

$x_0 = 15, x_1 = 25, x_2 = 30, x_3 = 35,$

$y_0 = 36, y_1 = 40, y_2 = 45, y_3 = 48$

We have to find the value of y at $x = 26$

x	x_0	x_1	x_2	x_3
26	15	25	30	35

$x - x_0$	26 − 15	11	$x_0 - x_1$	15 − 25	−10
$x - x_1$	26 − 25	1	$x_0 - x_2$	15 − 30	−15
$x - x_2$	26 − 30	−4	$x_0 - x_3$	15 − 35	−20
$x - x_3$	26 − 35	−9	$x_1 - x_2$	25 − 30	−5
$x_2 - x_3$	30 − 35	−5	$x_1 - x_3$	25 − 35	−10

By Lagrange's interpolation formula,

$$\boldsymbol{y = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1}$$

$$\boldsymbol{+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \times y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3}$$

The different values are given in the table below

$$y = \frac{(1)(-4)(-9)}{(-10)(-15)(-20)}(36) + \frac{(11)(-4)(-9)}{(10)(-5)(-10)}(40) + \frac{(11)(1)(-9)}{(15)(5)(-5)}(45) + \frac{(11)(1)(-4)}{(20)(10)(5)}(48)$$

$y = -0.432 + 31.68 + 11.88 - 2.112$

$y = 41.016$

Thus the number of workers getting income not exceeding Rs. 26 per month is 41

Question 11.
Using interpolation estimate the business done in 1985 from the following data.

Year	1982	1983	1984	1986
Business done (in lakhs)	150	235	365	525

Solution:
Let x denote the year of business and
y (in lakhs) denote the amount of business.
$x_0 = 1982, x_1 = 1983, x_2 = 1984, x_3 = 1986$
$y_0 = 150;\ y_1 = 235\ ;\ y_2 = 365\ ;\ y_3 = 525$
We have to find the value of y when $x = 1985$.

x	x_0	x_1	x_2	x_3
1985	1982	1953	1984	1986

$x - x_0$	$1985 - 1982$	3	$x_0 - x_1$	$1982 - 1983$	-1
$x - x_1$	$1985 - 1983$	2	$x_0 - x_2$	$1982 - 1984$	-2
$x - x_2$	$1985 - 1984$	1	$x_0 - x_3$	$1982 - 1986$	-4
$x - x_3$	$1985 - 1986$	-1	$x_1 - x_2$	$1983 - 1984$	-1
$x_2 - x_3$	$1984 - 1986$	-2	$x_1 - x_3$	$1983 - 1986$	-3

By Lagrange's interpolation formula,

$$y = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \times y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$y = \frac{(2)(1)(-1)}{(-1)(-2)(-4)}(150) + \frac{(3)(1)(-1)}{(1)(-1)(-3)}(235) + \frac{(3)(2)(-1)}{(2)(1)(-2)}(365) + \frac{(3)(2)(1)}{(4)(3)(2)}(525)$$

$y = 37.5 - 235 + 547.5 + 131.25 = 481.25$

Thus the business done in the year 1985 is estimated as 481.25 lakhs

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Question 12.
Using interpolation, find the value of $f(x)$ when $x = 15$

x	3	7	11	19
$f(x)$	42	43	47	60

Solution:
We have to find the value of y when $x = 15$.

$x_0 = 3$	$x_1 = 7$	$x_2 = 11$	$x_3 = 19$
$y_0 = 42$	$y_1 = 43$	$y_2 = 47$	$y_3 = 60$

x	x_0	x_1	x_2	x_3
15	3	7	11	19

$x - x_0$	$15 - 3$	12	$x_0 - x_1$	$3 - 7$	-4
$x - x_1$	$15 - 7$	8	$x_0 - x_2$	$3 - 11$	-8
$x - x_2$	$15 - 11$	4	$x_0 - x_3$	$3 - 19$	-16
$x - x_3$	$15 - 19$	-4	$x_1 - x_2$	$7 - 11$	-4
$x_2 - x_3$	$11 - 19$	-8	$x_1 - x_3$	$7 - 19$	-12

$$y = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \times y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

The different values are given in the table below.

$$y = \frac{(8)(4)(-4)}{(-4)(-8)(-16)}(42) + \frac{(12)(4)(-4)}{(4)(-4)(-12)}(43) + \frac{(12)(8)(-4)}{(8)(4)(-8)}(47) + \frac{(12)(8)(4)}{(16)(12)(8)}(60)$$

$y = 10.5 - 43 + 70.5 + 15 = 53$

Hence the value of f(x) when $x = 15$ is 53

CHAPTER 8

Sampling Techniques and Statistical Inference

(2, 3 and 5 Marks)

2 Marks

Exercise 8.1

Question 1. What is the population?

Answer:

Population refers to all individuals under the study is called as population.

Examples of population:1. The number of students in a class, 2. The number of boys and girls in a tuition centre etc.

Question 2.What is the sample?

Answer:

A group of individuals selected from the population to make representation to the entire population under study is called a sample.

Question 3. What is statistic?

Answer:

Any statistical measure such as mean, variance, standard deviation, etc., computed from the sample is known as statistic.

Question 4. Define parameter.

Answer:

The statistical constants of the population like mean (μ), variance(σ^2) is referred as parameter.

Question 5. What is the sampling distribution of a statistic?

Answer:

Sampling distribution of a statistic is the frequency distribution which is formed with various values of a statistic computed from different samples of the same size drawn from the same population.

Question 6. What is the standard error?

Answer :

The standard deviation of the sampling distribution of a statistic is known as its standard error (S.E).

S.NO	Statistic	Standard Error
1	Sample mean	$\sigma/\sqrt{n}$
2	Observed sample proportion	$\sqrt{PQ/n}$
3	Sample standard deviation	$\sqrt{\sigma^2/2n}$
4	Sample variance	$\sigma^2\sqrt{2/n}$
5	Sample quartiles	$1.36263\sigma/\sqrt{n}$
6	Sample correlation coefficient	$1.25331\sigma/\sqrt{n}$
7	Samplian	$(1-\rho^2)/\sqrt{n}$

Question 12: State any two merits of simple random sampling.

Solution:

- In simple random sampling personal bias is completely eliminated.
- This method is economical as it saves time, money and labour.

Question 14:

State any two demerits of systematic random sampling.

Solution:

- Systematic samples are not random samples.
- If N is not a multiple of n, then the sampling interval (k) cannot be an integer, thus sample selection becomes difficult.

Exercise 8.2

Question 1. Mention two branches of statistical inference?

Answer:

The two branches of statistical inference are estimation and testing of hypothesis.

Question 2. What is an estimator?

Answer:

An estimator is a statistic that is used to infer the value of an unknown population parameter in a statistical model. The estimator is a function of the data arid so it is also a random variable.

Question 3. What is an estimate?

Answer:

Any specific numerical value of the estimator is called an estimate. For example, sample means are used to estimate population means.

Question 4. What is point estimation?

Answer:

Point estimation involves the use of sample data to calculate a single value which is to serve as a best estimate of an unknown population parameter. For example the mean height of 145 cm from a sample of 15 students is‘a point estimate for the mean height of the class of 100 students.

Question 5. What is interval estimation?

Answer:

Interval estimation is the use of sample data to calculate an interval of possible values of an unknown population parameter. For example the interval estimate for the population mean is (101.01, 102.63).This gives a range within which the population mean is most likely to be located.

Question 6. What is confidence interval?

Answer:

A confidence interval L a type of interval estimate, computed from the statistics of the observed data, that might contain the true value of an unknown population parameter. The numbers at the upper and lower end of a confidence interval are called confidence limits. For example, if mean is 7.4 with confidence interval (5.4,9.4), then the numbers 5.4 and 9.4 are the confidence limits.

Question 7. What is null hypothesis? Give an example.

Answer:

A null hypothesis is a type of hypothesis, that proposes that no statistical significance exists in a set of given observations. For example, let the average time to cook a specific dish is 15 minutes. The null hypothesis would be stated as "The population mean is equal to 15 minutes", (i.e) $H_0: \mu = 15$

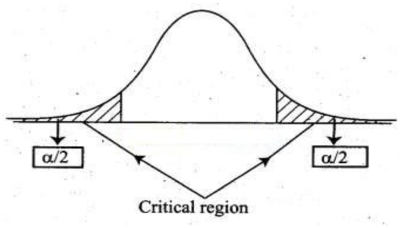
Question 8 . Define the alternative hypothesis.
Answer:

The alternative hypothesis is the hypothesis that is contrary to the null hypothesis and it is denoted by H_1.

For example if $H_1: \mu = 15$, then the alternative hypothesis will be : $H_1: \mu \neq 15$, (or) $H_1 : \mu < 15$ (or) $H_1: \mu > 15$.

Question 9. Define the critical region.
Answer:

The critical region is the region of values that corresponds to the rejection of the null hypothesis at some chosen probability level. For the two-tailed test, the critical region is given below.



where α is the level of significance.

Question 10. Define critical value.

Answer:

A critical value is a point on the test distribution that is compared to the test statistic to determine whether to reject the null hypothesis. It depends on the level of significance.

For example, if the confidence level is 90% then the critical value is 1.645.

Question 11. Define the level of significance

Answer:

The level of significance is defined as the probability of rejecting a null hypothesis by

the test when it is really true, which is denoted as α. That is P(Type 1 error) = α.

For example, the level of significance 0.1 is related to the 90% confidence level.

Question 12. What is a type I error?

Answer:

In statistical hypothesis testing, a Type f error is the rejection of a true null hypothesis.

Example of Type I errors includes a test that shows a patient to have a disease when he does not have the disease, a fire alarm going on indicating a fire when there is no fire (or) an experiment indicating that medical treatment should cure a disease when in fact it does not.

Question 13 .What is the single-tailed test?
Answer:

A single-tailed test or a one-tailed test is a statistical test in which the critical area of a distribution is one-sided so that it is either greater than or less than a certain value, but not both. For the null hypothesis $H_0: \mu = 16.91$, the alternative hypothesis $H_1: \mu > 16.91$ or $H_1: \mu < 16.91$ are one-tailed tests.

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3 - Marks

Execise 8.1

Question 10. Explain in detail about sampling error.
Answer:

Sampling Errors: Errors, which arise in the normal course of investigation or enumeration on account of chance, are called sampling errors. Sampling errors are inherent in the method of sampling. They may arise accidentally without any bias or prejudice.
Sampling Errors arise primarily due to the following reasons:

- Faulty selection of the sample instead of the correct sample by defective sampling technique.
- The investigator substitutes a convenient sample if the original sample is not available while investigation.
- In area surveys, while dealing with borderlines it depends upon the investigator whether to include them in the sample or not. This is known as Faulty demarcation of sampling units.

Question 11. Explain in detail about the non-sampling error.
Answer: Non-Sampling Errors:
The errors that arise due to human factors which always vary from one investigator to another in selecting, estimating or using measuring instruments(tape, scale) are called Non-Sampling errors. It may arise in the following ways:

- Due to negligence and carelessness of the part of either investigator or respondents.
- Due to the lack of trained and qualified investigators.
- Due to the framing of a wrong questionnaire.
- Due to applying the wrong statistical measure
- Due to incomplete investigation and sample survey.

Question 13.

State any three merits of stratified random sampling.

Answer:

- A random stratified sample is superior to a simple random sample because it ensures

  representation of all groups and thus it is more representative of the population which is being sampled.
- A stratified random sample can be kept small in size without losing its accuracy.
- It is easy to administer if the population under study is sub-divided.

Question 17.
A wholesaler in apples claims that only 4% of the apples supplied by him are defective. A random sample of 600 apples contained 36 defective apples. Calculate the standard error concerning good apples.
Solution:
Sample size = 600 No. of defective apples = 36

Sample proportion $p = \frac{36}{600} = 0.06$

Population proportion

P = probability of defective apples = 4% = 0.04

$Q = 1 - P = 1 - 0.04 = 0.96$

The S.E for sample proportion is given by S.E

$$= \sqrt{\frac{PQ}{N}} = \sqrt{\frac{(0.04)(0.96)}{600}} = \sqrt{0.000064} = 0.008$$

Question 16.

Using the following Tippet's random number table.

2952	6641	3992	9792	7969	5911	3170	5624
4167	9524	1545	1396	7203	5356	1300	2693
2670	7483	3408	2762	3563	1089	6913	7991
0560	5246	1112	6107	6008	8125	4233	8776
2754	9143	1405	9025	7002	6111	8816	6446

Draw a sample of 10 three-digit numbers which are even numbers.

Solution:

There are many ways to select a sample of 10 3-digit even numbers. From the table, start from the first

number and move along the column. Select the first three digits as the number. If it is an odd number,

move to the next number. The selected sample is 416, 664, 952, 748, 524, 914, 154, 340, 140, 276.

STANDARD ERROR	FORMULA
Standard deviation	$\sqrt{\sigma^2/2n}$
Mean	$\frac{\sigma}{\sqrt{n}}$
Population proportion	$\sqrt{\frac{PQ}{N}}$

Exercise 8.2

Question 14.

A sample of 100 items, draw from a universe with mean value 4 and S.D 3, has a mean value 3.5. Is the difference in the mean significant?

Solution:

Given Sample size $n = 100$

POPULATION DATA	SAMPLE DATA
Popuation mean $= \mu = 4$	Sample mean $= \bar{x} = 3.5$
Population S.D. $= \sigma = 3$	-

Now, null hypothesis $H_0: \mu = 4$
Alternative hypothesis $H_1: \mu \neq 4$ (Two tail)
level of significance $\alpha = 5\% = 0.05$
The table value $Z_{\alpha/2} = 1.96$

Test statistic: $Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{3.5-4}{\frac{3}{\sqrt{100}}} = \frac{-0.5}{0.3} = -1.667$

$|Z| = |-1.667| = 1.667$

$|Z| = 1.667 < 1.96$ (i.e) $|Z| < Z_{\alpha/2}$.

the null hypothesis H_0 is accepted.

Therefore, we conclude that there is no significant difference between the sample mean and the population mean.

5 - Marks

Exercise 8.1

Question 7.

Explain in detail about simple random sampling with a suitable example.

Answer:

(i) Simple random sampling:

In this technique, the samples are selected in such a way that each and every unit in the population has an equal and independent chance of being selected as a sample. Simple random sampling may be done, with or without replacement of the samples selected. In a simple random sampling with replacement, there is a possibility of selecting the same sample any number of times. So, simple

random sampling without replacement is followed.

Thus in simple random sampling from a population of N units, the probability of drawing any unit at the first draw is $\frac{1}{N}$, the probability of drawing any unit in the second draw from among the available (N – 1) units is $\frac{1}{(N-1)}$, and so on. Several methods have been adopted for random selection of the samples from the population. Of those, the following two methods are generally used and which are described below.

1. Lottery method

This is the most popular and simplest method when the population is finite. In this method, all the items of the population are numbered on separate slips of paper of the same size, shape and colour. They are folded and placed in a container and shuffled thoroughly. Then the required numbers of slips are selected for the desired sample size. The selection of items thus depends on chance.

For example, if we want to select 10 students, out of 100 students, then we must write the names/roll number of all the 100 students on slips of the same size and mix them, then we make a blindfold selection of 10 students. This method is called unrestricted random sampling because units are selected from the population without any restriction. This method is mostly used in lottery draws. If the population or universe is infinite, this method is inapplicable.

2. Table of Random number

When the population size is large, it is difficult to number all the items on separate slips of paper of

same size, shape and colour. The alternative method is that of using the table of random numbers. The most practical, easy and inexpensive method of selecting a random sample can be done through "Random Number Table". The random number table has been so constructed that each of the digits 0, 1, 2,..., 9 will appear approximately with the same frequency and independently of each other.

The various random number tables available are

- L.H.C. Tippett random number series
- Fisher and Yates random number series
- Kendall and Smith random number series
- Rand Corporation random number series.

Tippett's table of random numbers is most popularly used in practice.

Question 8.
Explain the stratified random sampling with a suitable example.
Answer:

Stratified Random Sampling
In stratified random sampling, first divide the population into subpopulations, which are called strata. Then, the samples are selected from each of the strata through random techniques. The collection of all the samples from all strata gives the stratified random samples.

When the population is heterogeneous or different segments or groups with respect to the variable or characteristic under study, then the Stratified Random Sampling method is studied.. First, the
population is divided into the homogeneous number of sub-groups or strata before the sample is drawn. A sample is drawn from each stratum at random. Following steps are involved in selecting a random sample in a stratified random sampling method.
(a) The population is divided into different classes so that each stratum will consist of more or less homogeneous elements. The strata are so designed that they do not overlap each other.
(b) After the population is stratified, a sample of a specified size is drawn at random from each stratum using Lottery Method or Table of Random Number Method.

Stratified random sampling is applied in the field of the different legislative areas as strata in election polling, division of districts (strata) in a state etc...

Ex: From the following data, select 68 random samples from the population of the heterogeneous group with a size of 500 through stratified random sampling, considering the following categories as strata.

- Category 1: Lower income class −39%
- Category 2: Middle income class - 38%
- Category 3: Upper income class −23%

Solution:

Stratum	Homogenous group	Percentage From population	No.of ppl in each starta	Random Samples
Category 1	Lower income class	39	$\frac{39}{100}\times 500$ = 195	$195\times\frac{68}{500}$ = 26.5~26
Category 2	Middle income class	38	$\frac{38}{100}\times 500$ = 190	$190\times\frac{68}{500}$ = 26.5~26
Category 3	Upper income class	23	$\frac{23}{100}\times 500$ = 115	$115\times\frac{68}{500}$ = 15.6~16
Total		100	500	

Question 9.
Explain in detail about systematic random sampling with example.
Answer:

Systematic sampling:
In systematic sampling, randomly select the first sample from the first k units. Then every kth member, starting with the first selected sample, is included in the sample.
Systematic sampling is a commonly used technique if the complete and up-to-date list of the sampling units is available. We can arrange the items in numerical, alphabetical, geographical or in any other order. The procedure of selecting the samples starts with selecting the first sample at random, the rest being automatically selected according to some pre-determined (pattern. A systematic sample is formed by selecting every item from the population, where k refers to the sample interval. The sampling interval can be determined by dividing the size of the population by the size of the sample to be chosen.
That is $k = \frac{N}{n}$, where k is an integer.
k = Sampling interval, N = Size of the population,
n = Sample size.
Procedure for selection of samples by systematic sampling method
(i) If we want to select a sample of 10 students from a class of 100 students,
the sampling interval is calculated as $k = \frac{N}{n} = \frac{100}{10} = 10$
Thus sampling interval = 10 denotes that for every 10 samples one sample has to be selected.
(ii) The first sample is selected from the first 10 (sampling interval) samples through random selection procedures.
(iii) If the selected first random sample is 5 , then the rest of the samples are automatically selected by incrementing the value of the sampling interval (k = 10) i.e., 5,15,25,35,45,55,65,75,85,95.

Ex: Suppose we have to select 20 items out of 6,000 . The procedure is to number all the 6,000 items from 1 to 6,000 . The sampling interval is calculated as $k = \frac{N}{n} = \frac{6000}{20} = 300$. Thus sampling interval= 300 denotes that for every 300 samples one sample has to be selected. The first sample is selected from the first 300 (sampling interval) samples through random selection procedures. If the selected first random sample is 50 , then the rest of the samples are automatically selected by incrementing the value of the sampling interval (k = 300) ie, 50,350,650,950,1250,1550,1850,2150,2450,2750 , 3050,3350,3650,3950,4250,4550,4850,5150,5450,5750 . Items bearing those numbers will be selected as samples from the population.

Question 19.
A random sample of 60 observations was drawn from a large population and its standard deviation was found to be 2.5 . Calculate the suitable standard error that this sample is taken from a population with standard deviation 3 ?
Solution:
Given sample size n = 60
Sample standard deviation = 2.5
Population standard deviation $\sigma = 3$
S.E. $= \sqrt{\sigma^2/2n} = \sqrt{\frac{9}{120}} = \sqrt{0.075} = 0.2739$

Question 20.

In a sample of 400 population from a village 230 are found to be eaters of vegetarian items and the rest non-vegetarian items. Compute the standard error assuming that both vegetarian and nonvegetarian foods are equally popular in that village?

Solution:

Given sample size 400 and 230 are vegetarian eaters.

So sample proportionp $= \frac{230}{400} = 0.575$

Population proportion

$\boldsymbol{P}$ **= Prob (vegetarian eaters from the village) =** $\frac{1}{2}$

(Since vegetarian and non-vegetarian foods are equally popular)

$Q = 1 - P = 1 - \frac{1}{2} = \frac{1}{2}$

The standard error SE $= \sqrt{\frac{PQ}{N}} = \sqrt{\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}{400}} = \sqrt{\frac{0.25}{400}}$

$= \sqrt{0.000625} = 0.025$

Exercise 8.2

Question 15.

A sample of 400 individuals is found to have a mean height of 67.47 inches. Can it be reasonably regarded as a sample from a large population with a mean height of 67.39 inches and standard deviation of 1.30 inches?

Solution:

Given Sample size n = 400

POPULATION DATA	SAMPLE DATA
Popuation mean $= \mu = 67.39$	Sample mean $= \bar{x} = 67.47$
Population S.D. $= \sigma = 1.3$	-

Null hypothesis $H_0: \mu = 67.39$ inches

Alternative hypothesis $H_1: \mu \neq 67.39$ inches

The level of significance $\alpha = 5\% = 0.05$

The table value $Z_{\alpha/2} = 1.96$

Test statistic

$$Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{67.47-67.39}{\frac{1.3}{\sqrt{400}}} = \frac{0.08}{0.065} = 1.2308$$

$|z| = 1.2308 < 1.96$ (i.e) $Z < Z_{\alpha/2}$.

Since the calculated value is less than the table value at 5% level of significance, the null hypothesis is accepted.

Hence we conclude that the data does not provide us with any evidence against the null hypothesis. Thus, the sample has been drawn from a large population with a mean height of 67.39 inches and S.D 1.3 inches.

Question 16.

The average score on a nationally administered aptitude test was 76 and the corresponding standard deviation was 8 . In order to evaluate a state's education system, the scores of 100 of the state's students were randomly selected. These students had an average score of 72 . Test at a significance level of 0.05 if there is a significant difference between the state scores and the national scores.

Solution: n = 100

POPULATION DATA	SAMPLE DATA
Popuation mean $= \mu = 76$	Sample mean $= \bar{x} = 72$
Population S.D. $= \sigma = 8$	-

Null hypothesis $H_0 : \mu = 76$

Alternative hypothesis $H_1: \mu \neq 76$

(i.e) there is a significant difference between the state scores and the nationals scores of the aptitude test.

level of significance $\alpha = 5\% = 0.05$

The table value $Z_{\alpha/2} = 1.96$

Test statistic $Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{72-76}{\frac{8}{\sqrt{100}}} = \frac{-4}{0.8} = -5$

$|Z| = |-5| = 5$

we find that $|Z| > Z_{\alpha/2}$ (i.e) $5 > 1.96$.

So the null hypothesis is rejected and we accept the alternative hypothesis.

we conclude that at the significance level of 5%, there is a difference between the state scores and the national scores of the nationally administered amplitude test.

Question 17.

The mean breaking strength of cables supplied by a manufacturer is 1,800 with a standard deviation of 100 . By a new technique in the manufacturing process it is claimed that the breaking strength of the cables has increased. In order to test this claim a sample of 50 cables is tested. It is found that the mean breaking strength is 1,850 . Can you support the claim at 0.01 level of significance?

Solution:

n = 50

POPULATION DATA	SAMPLE DATA
Popuation mean $= \mu = 1800$	Sample mean $= \bar{x} = 1850$
Population S.D. $= \sigma = 100$	-

Null hypothesis $H_0: \mu = 1800$

(i.e) the breaking strength of the cables has not increased, after the new technique in the manufacturing process.

Alternative hypothesis $H_1: \mu > 1800$ (i.e) the new technique was successful.

The level of significance $\alpha = 1\% = 0.01$

The table value $Z_{\alpha} = 2.33$

Test statistic: $Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{1850-1800}{\frac{100}{\sqrt{150}}} = \frac{50}{14.144} = 3.536$

$|z| = 3.536$

we find that $Z > Z\alpha$ (i.e.) $3.536 > 2.33$.

Since the calculated value is greater than the table value at 1% level of significance, the null hypothesis is rejected and

we accept the alternative hypothesis. We conclude that by the new technique in the manufacturing process the breaking strength of the cables is increased. So the claim is supported at 0.01 level of significance.

Example 8.11

A machine produces a component of a product with a standard deviation of 1.6 cm in length. A random sample of 64 componentsvwas selected from the output and this sample has a mean length of 90 cm. The customer will reject the part if it is either less than 88 cm or more than 92 cm. Does the 95% confidence interval for the true mean length of all the components produced ensure acceptance by the customer?

Solution:

Here φ is the mean length of the components in the population.

The formula for the confidence interval is

$\bar{x} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

Here $\sigma = 1.6, Z_{\alpha/2} = 1.96, \bar{x} = 90$ and $n = 64$

Then $S.E. = \frac{\sigma}{\sqrt{n}} = \frac{1.6}{\sqrt{64}} = 0.2$

Therefore, $90 - (1.96 \times 0.2) < \varphi < 90 + (1.96 \times 0.2)$

i.e. $(89.61 < \varphi < 90.39)$

population mean length of the components will fall in this interval (89.61,90.39) at 95%.

Hence we concluded that 95% confidence interval ensures acceptance of the component by the consumer.

Example 8.12

A sample of 100 measurements at breaking strength of cotton thread gave a mean of 7.4 and a standard deviation of 1.2 gms. Find 95% confidence limits for the mean breaking strength of cotton thread.

Solution:

Given, sample size $= 100, \bar{x} = 7.4$, since σ is unknown but $s = 1.2$ is known.

In this problem, we consider $\breve{\sigma} = s,\ Z_{\alpha/2} = 1.96$

$S.E. = \frac{\breve{\sigma}}{\sqrt{n}} = \frac{s}{\sqrt{n}} = \frac{1.2}{\sqrt{100}} = 0.12$

Hence 95% confidence limits for the population mean are

$\bar{x} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

$7.4 - (1.96 \times 0.12) < \mu < 7.4 + (1.96 \times 0.12)$

$7.4 - 0.2352 < \mu < 7.4 + 0.2352$

$7.165 < \mu < 7.635$

This implies that the probability that the true value of the population mean breaking strength of the cotton threads will fall in this interval (7.165,7.635) at 95%.

Example 8.13

The mean life time of a sample of 169 light bulbs manufactured by a company is found to be 1350 hours with a standard deviation of 100 hours. Establish 90% confidence limits within which the mean life time of light bulbs is expected to lie.

Solution:

Given: $n = 169, \bar{x} = 1350$ hours, $\sigma = 100$ hours, since the level of significance is (10090)% = 10% thus α is 0.1, hence the significant value at 10% is $Z_{\alpha/2} = 1.645$

$S.E. = \frac{\sigma}{\sqrt{n}} = \frac{100}{\sqrt{169}} = 7.69$

Hence 90% confidence limits for the population mean are

$\bar{x} - Z_{\alpha/2}SE < \mu < \bar{x} + Z_{\alpha/2}SE$

$1350 - (1.645 \times 7.69) < \mu < 1350 + (1.645 \times 7.69)$

$1337.35 < \mu < 1362.65$

Hence the mean life time of light bulbs is expected to lie between the interval (1337.35, 1362.65)

Example 8.14

An auto company decided to introduce a new six cylinder car whose mean petrol consumption is claimed to be lower than that of the existing auto engine. It was found that the mean petrol consumption for the 50 cars was 10 km per litre with a standard deviation of 3.5 km per litre. Test at 5% level of significance, whether the claim of the new car petrol consumption is 9.5 km per litre on the average is acceptable.

Solution:

Sample size $n = 50$ Sample mean $\bar{x} = 10$ km Sample standard deviation $s = 3.5$ km

Population mean $\mu = 9.5$ km

Since population SD is unknown we consider $\sigma = s$

Null Hypothesis $H_0: \mu = 9.5$

Alternative Hypothesis: $H_1: \mu \neq 9.5$ (two tailed test)

The level of significance $\alpha = 5\% = 0.05$

Applying the test statistic $Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$;

$Z = \frac{10-9.5}{\frac{3.5}{\sqrt{50}}} = \frac{0.5}{0.495} = 1.01$

Thus the calculated value 1.01 and the significant value or table value $Z_{\alpha/2} = 1.96$

Comparing the calculated and table value ,

Here $Z < Z_{\alpha/2}$ i.e., $1.01 < 1.96$.

Inference :Since the calculated value is less than table value i.e., $Z < Z_{\alpha}$ at 5% level of sinificance, the null hypothesis H_0 is accepted. Hence we conclude that the company's claim that the new car petrol consumption is 9.5 km per litre is acceptable.

Example 8. 15

A manufacturer of ball pens claims that a certain pen he manufactures has a mean writing life of 400 pages with a standard deviation of 20 pages. A purchasing agent selects a sample of 100 pens and puts them for test. The mean writing life for the sample was 390 pages. Should the purchasing agent reject the manufactures claim at 1% level?

Solution:

$n = 100$,

POPULATION DATA	SAMPLE DATA
Popuation mean $= \mu = 400$	Sample mean $= \bar{x} = 390$
Population S.D. $= \sigma = 20$	-

Null Hypothesis: $H_0: \mu = 400$

Alternative Hypothesis: $H_1: \mu \neq 400$ (two tailed test)

The level of significance $\alpha = 1\% = 0.01$; $\therefore Z_{\alpha/2} = 2.58$

The test statistic $Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$

$Z = \frac{390-400}{\frac{20}{\sqrt{100}}} = \frac{-10}{2} = -5$, $\therefore |Z| = 5$

Thus the calculated value $|Z| = 5$ $\therefore Z_{\alpha/2} = 2.58$

Comparing the calculated and table values,

$Z > Z_\alpha$ i.e., $5 > 2.58$

1% level of significance, the null hypothesis is rejected and Therefore we concluded that $\mu \neq 400$ and the manufacturer's claim is rejected at 1% level of significance.

Example 8.17

The mean weekly sales of soap bars in departmental stores were 146. 3 bars per store. After an advertising campaign the mean weekly sales in 400 stores for a typical week increased to 153. 7 and showed a standard deviation of 17.2. Was the advertising campaign successful at 95% confidence limit?

Solution:

$n = 400$ stores ;

POPULATION DATA	SAMPLE DATA
Popuation mean $= \mu = 146.3$	Sample mean $= \bar{x} = 153.7$
Population S.D. $= \sigma = s = 17.2$	Sample SD $s = 17.2$

Null Hypothesis. i.e, $H_0: \mu = 146.3$

Alternative Hypothesis $H_1: \mu > 143.3$ (Right tail test). The advertising campaign was successful

Level of significance $\alpha = 0.05$

Test statistic $Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$

$Z = \frac{153.7-146.3}{\frac{17.2}{\sqrt{400}}} = \frac{7.4}{0.86} = 8.605$ $\therefore |Z| = 8.605$

Comparing the calculated value $Z = 8.605$ and the significant value or table value $Z_\alpha = 1.645$. we get $8.605 > 1.645$. Inference: Since, the calculated value is much greater than table value i.e., $Z > Z_\alpha$, it is highly significant at 5% level of significance. Hence we reject the null hypothesis H_0 and conclude that the advertising campaign was definitely successful in promoting sales.

Example 8. 16

(i) A sample of 900 members has a mean 3. 4 cm and SD2. 61 cm. Is the sample taken from a large population with mean 3. 25 cm. and SD 2. 62 cm ?

(95% confidence limit)

(ii) If the population is normal and its mean is unknown, find the 95% and 98% confidence limits of true mean.

Solution:

(i) Given:

Sample size $n = 900$,

POPULATION DATA	SAMPLE DATA
Popuation mean $= \mu = 3.25$	Sample mean $= \bar{x} = 3.4$
Population S.D. $= \sigma = s = 2.61$	Sample SD $s = 2.61$ cm. -

Null Hypothesis $H_0: \mu = 3.25$ cm

,Alternative Hypothesis $H_1: \mu \neq 3.25$ cm (two tail) Test statistic: $Z = \frac{3.4-3.25}{\frac{2.61}{\sqrt{900}}} = \frac{0.15}{0.087} = 1.724$

$\therefore Z = 1.724$

$Z_{\alpha/2} = 1.96$

Comparing the calculated and table values, $Z < Z_{\alpha/2}$ i.e., $1.724 < 1.96$

Inference: Since the calculated value is less than table value i.e., $Z < Z_{\alpha/2}$ at 5% level of significance, the null hypothesis is accepted.

Hence we conclude that the data doesn't provide us any evidence against the null hypothesis. Therefore, the sample has been drawn from the population mean $\mu = 3.25$ cm and SD, $\sigma = 2.61$ cm

(ii) Confidence limits 95% confidential limits for the population

mean μ are :

$\bar{x} - Z_{\alpha/2}SE < \mu < \bar{x} + Z_{\alpha/2}SE$

$3.4 - (1.96 \times 0.087) < \mu < 3.4 + (1.96 \times 0.087)$

$3.229 < \mu < 3.571$

34. 98% confidential limits for the population

mean μ are :

$\bar{x} - Z_{\alpha/2}SE < \mu < \bar{x} + Z_{\alpha/2}SE$

$3.4 - (2.33 \times 0.087) < \mu < 3.4 + (2.33 \times 0.087)$

$3.197 < \mu < 3.603$

Therefore, 95% confidential limits is (3.229,3.571) and 98% confidential limits is (3.197,3.603).

Example 8. 18

The wages of the factory workers are assumed to be normally distributed with mean and variance 25. A random sample of 50 workers gives the total wages equal to ₹$2,550$. Test the hypothesis $\mu = 52$, against the alternative hypothesis $\mu = 49$ at 1% level of significance.

Solution:

Sample size $n = 50$ workers

Total wages $\Sigma x = 2550$

Sample mean $\bar{x} = \frac{\text{total wages}}{n} - \frac{\Sigma x}{n} := \frac{2550}{50} = 51$ units

Population mean $\mu = 52$; Population variance $\sigma^2 = 25$

Population SD $\sigma = 5$

Null hypothesis $H_0 : \mu = 52$

alternative hypothesis $H_1 : \mu \neq 52$ (Two tail)

Level of significance $\mu = 0.01$

Test statistic $Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$

$Z = \frac{51-52}{\frac{5}{\sqrt{50}}} = \frac{-1}{0.7071} =- 1.4142$

$|Z| = 1.4142$

Critical value at 1% level of significance is $Z_{\alpha/2} = 2.58$

Inference: Since the calculated value is less than table value i.e., $Z < Z_{\alpha/2}$ at 1% level of significance, the null hypothesis H_0 is accepted.

Therefore, we conclude that there is no significant difference between the sample mean and population mean $\mu = 52$ and SD $\sigma = 5$. Therefore $\mu = 49$ is rejected.

Example 8. 19

An ambulance service claims that it takes on the average 8.9 minutes to reach its destination in emergency calls. To check on this claim, the agency which licenses ambulance services has then timed on 50 emergency calls, getting a mean of 9.3 minutes with a standard deviation of 1.6 minutes. What can they conclude at 5% level of significance.

Solution:

Sample size $n = 50$: Sample mean $\bar{x} = 9.3$ minutes

Sample S.D $s = 1.6$ minutes:

Population mean $\mu = 8.9$ minutes

Null hypothesis $H_0 : \mu = 8.9$

Alternative hypothesis $H_1 : \mu = 8.9$ (Two tail)

Level of significance $\mu = 0.05$

Test statistic $Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$

$Z = \frac{9.3-8.9}{\frac{1.6}{\sqrt{50}}} = \frac{0.4}{0.2263} = 1.7676$

Calculated value $Z = 1.7676$

Critical value at 5% level of significance is $Z_{\alpha/2} = 1.96$

Inference: Since the calculated value is less than table value i.e., $Z < Z_{\alpha}$ at 5% level of significance, the null hypothesis is accepted. Therefore we conclude that an ambulance service claims on the average 8.9 minutes to reach its destination in emergency calls.

CHAPTER 9 - APPLIED STATISTICS
(2, 3 and 5 Marks)

2 Marks : **Exercise 9.1**

Question 1. **Define Time series.**
Answer:
A time series consists of data arranged chronologically when Quantitative data are arrainged in order of their occurances. The resulting series is called the Time series.

Question 2. **What is the need for studying time series?**

Answer:
Time series helps us to study and analyze the time-related data which involves in business fields, economics, industries, etc…

We should study time series for the following reasons.

- It helps in the analysis of past behaviour.
- It helps in forecasting and for future plans.
- It helps in the evaluation of current achievements. It helps in making comparative studies between one time period and others.

Question 3. **State the uses of time series.**

Answer:
1. It helps in the analysis of the past behaviour
2. It helps in forecasting and for future plans
3. It helps in the evaluation of current achievements
4. It helps in making comparatives studies between one time period and other

Question 4. **Mention the components of the time series.**
Answer:
There are four types of components in a time series.
They are 1. Secular Trend 2. Seasonal variations
3. Cyclic variations 4. Irregular variations

Question 5. **Define the secular trend.**

Answer:
It is a general tendency, of time series to increase or decrease or stagnates during a long period of time. An upward tendency is usually observed in the population of a country, production, sales, prices in industries, the income of individuals etc., A downward tendency is observed in deaths, epidemics, prices of electronic gadgets, water sources, mortality rate etc….

Question 7. **Explain cyclic variations.**
Answer:
Cyclic Variations: These variations are not necessarily uniformly periodic in nature. That is, they may or may not follow exactly similar patterns after equal intervals of time. Generally, one cyclic period ranges from 7 to 9 years and there is no hard and fast rule in the fixation of years for a cyclic period. For example, every business cycle has a Start-Boom-Depression- Recover, maintenance during booms and depressions, changes in government monetary policies, changes in interest rates.

Question 8. **Discuss irregular variation.**

Answer:
Irregular Variations: These variations do not have a particular pattern and there is no regular period of time of their occurrences. These are accidental changes which are purely random or unpredictable. Normally they are short – term variations, but its occurrence sometimes has its effect so intense that they may give rise to new cyclic or other movements of variations. For example floods, wars, earthquakes, Tsunami, strikes, lockouts etc…

Question 9. **Define the seasonal index.**

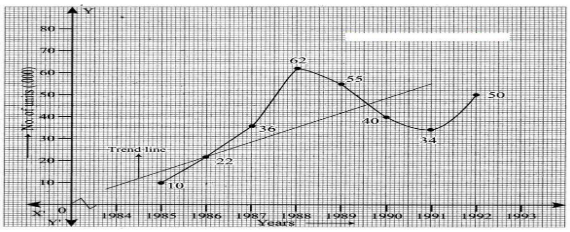
Answer:
Seasonal Index for every season (i.e) months, quarters or year is given by Seasonal Index (S.I) $= \frac{\text{Seasonal Average}}{\text{Grand average}} \times 100$
Where seasonal average is calculated for month, (or) quarter depending on the problem and Grand Average (G) is the average of averages.

Question 11.
State the two normal equations used in fitting a straight line.
Answer:
The normal equations used in fitting a straight line are
$\Sigma Y = na + b\Sigma X$ and $\Sigma XY = a\Sigma X + b\Sigma X^2$
Where n = number of years given in the data,
X = time Y = actual value a, b = constants

Question 16.
The following table gives the number of small - scale units registered with the Directorate of Industries between 1985 and 1991 . Show the growth on a trend line by the freehand method.

Year	1985	1986	1987	1988	1989	1990	1991	1992
No.of units (in'000)	10	22	36	62	55	40	34	50

Solution:



Exercise : 9. 2

Question 1. Define Index Number.
Answer:
"An Index Number is a device which shows by its variations the Changes in a magnitude which is not capable of accurate measurements in itself or of direct valuation in practice". - **Wheldon**

"An Index number is a statistical measure of fluctuations in a variable arranged in the form of a series and using a base period for making comparisons" - **Lawrence J Kalpan**

Question 2. State the uses of Index Number.
Answer:
The uses of Index number are

- It is an important tool for formulating decision and management policies.
- It helps in studying the trends and tendencies.
- It determines the inflation and deflation in an economy

Question 3. Mention the classification of Index Number.
Answer:
Classification of Index Numbers:

Index number can be classified as follows

1. **Price Index Number:** It measures the general changes in the retail or wholesale price level of a particular or group of commodities.
2. **Quantity Index Number:** These are indices to measure the changes in the number of goods manufactured in a factory.
3. **Cost of living Index Number:** These are intended to study the effect of change in the price level on the cost of living of different classes of people.

Question 4. Define Laspeyre's price index number
Answer:
The weighted aggregate index number using base period weights is called Laspeyre's price index number.

$$P_{01}^{L} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

Where p_1 is current year price; p_0 is base year price

q_0 is base year quantity

Question 5.: Explain Paasche's price index number.
Answer:
If both prices and quantities were permitted to change, then it is impossible to isolate the part of movement due to price changes alone. In this case, the current year quantities appear more realistic weights than the base year quantities. The index number based on current year quantities is called Paasche's price index number.

$$P_{01}^{P} = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$$

Where p_1 is the current year price

q_1 is the current year quantity; p_0 is the base year price

Question 6. Write a note on Fisher's price index number.
Answer:
Fisher defined a weighted index number as the geometric mean of Laspeyre's index number and Paasche's Index number

$$P_{01}^{F} = \sqrt{\frac{\sum p_1 q_0 \times \sum p_1 q_1}{\sum p_0 q_0 \times \sum p_0 q_1}} \times 100$$

The Fisher-price index number is also known as the "ideal" price index number. This requires more data than the other two index numbers and as a result, may often be impracticable. But this is a good index number because it satisfies both the time-reversal test and factor reversal test.

(i.e) $P_{01}^{F} \times P_{10}^{F} = 1$ and $P_{01}^{F} \times Q_{01}^{F} = \frac{\sum p_1 q_1}{\sum p_0 q_0}$

Question 7. State the test of the adequacy of the index number.
Answer:
Index numbers are studied to know the- relative changes in price and quantity for any two years compared. There are two tests which are used to test the adequacy for an index number.

The two tests are as follows: Time Reversal Test & Factor Reversal Test

The criterion for a good index number is to satisfy the above two tests.

Question 8. Define Time Reversal Test.
Answer:
It is an important test for testing the consistency of a good index number. This test maintains time consistency by working both forward and backward with respect to time (here time refers to the base year and current year). Symbolically the following relationship should be satisfied, $P_{01} \times P_{10} = 1$

Fisher's index number formula satisfies the above relationship

$$P_{01}^{F} = \sqrt{\frac{\sum p_1 q_0 \times \sum p_1 q_1}{\sum p_0 q_0 \times \sum p_0 q_1}}$$

when the base year and current year are interchanged,

$$P_{10}^{F} = \sqrt{\frac{\sum p_0 q_1 \times \sum p_0 q_0}{\sum p_1 q_1 \times \sum p_1 q_0}} \quad \& \quad P_{01}^{F} \times P_{10}^{F} = 1$$

Question 9. **Explain Factor Reversal Test.**
Answer:
Factor Reversal Test:
This is another test for testing the consistency of a good index number. The product of price index number and quantity index number from the base year to the current year should be equal to the true value ratio. That is the ratio between the total value of the current period and total value pf the base period is known as the true value ratio. Factor Reversal Test is given by, $P_{01} \times Q_{01} = \frac{\Sigma\ p_1 q_1}{\Sigma\ p_0 q_0}$

Where $P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}}$

Now interchanging P by Q, $Q_{01} = \sqrt{\frac{\Sigma\ q_1 p_0 \times \Sigma\ q_1 p_1}{\Sigma\ q_0 p_0 \times \Sigma\ q_0 p_1}}$

where P_{01} is the relative change in price.
Q_{01} is the relative change in quantity.

Question 10. **Define true value ratio.**
Answer:
The ratio between the total value of the current period and the total value of the base period is known as the true value ratio. (i.e) true value ratio $= \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$

Question 11. **Discuss Cost of Living Index Number.**
Answer:
Cost of Living Index Number is constructed to study the effect of changes in the price of goods and services of consumers for a current period as compared with the base period. The change in the cost of living index number between any two periods means the change in income which will be necessary to maintain the same standard of living in both the periods. Therefore the cost of living index number measures the average increase in the cost to maintain the same standard of life.
Further, the consumption habits of people differ widely from class to class (rich, poor, middle class) and even with the region. The changes in the price level affect the different classes of people, consequently, the general price index numbers fail to reflect the effect of changes in their cost of living in different classes of people. Therefore, the cost of living index number measures the general price movement of the commodities consumed by different classes of people.

Question 12. **Define Family Budget Method.**
Solution:
Family Budget Method:
In this method, the weights are calculated by multiplying prices and quantity of the base year. (i.e.) $V = \Sigma p_0 q_0$. The formula is given by,
Cost of Living Index Number $= \frac{\Sigma PV}{\Sigma V}$
where $P = \frac{p_1}{p_0} \times 100$ is the price relative
$V = \Sigma p_0 q_0$ is the value relative

Question 13.
State the uses of the Cost of Living Index Number.
Answer:
Uses of Cost of Living Index Number
- It indicates whether the real wages of workers are rising or falling for a given time.
- It is used by the administrators for regulating dearness allowance or grant of bonus to the workers.

Exercise 9.3

Question 1. **Define Statistical Quality Control.**
Answer:
Statistical quality control (SQC) refers to the use of statistical methods in the monitoring and
maintaining of the quality of products and services. This method is used to determine the tolerance
limits for accepting a production process.

Question 2. **Mention the types of causes for variation in a production process.**
Answer:
There are two causes of variations between items produced under identical conditions in large
production process. They are called assignable causes and non-assignable causes (chance causes).

Question 3. **Define Chance Cause.**
Answer:
:The minor causes which do not affect the quality of the products to an extent are called as chance causes or Random causes. For example rain, floods, power cuts, etc.

Question 4.Define Assignable Cause.
Answer:
The variations in input factors which are the causes for the variations in the output productions are called assignable causes. For example defective raw materials, fault in instruments used, fatigue of workers employed, unskilled technicians, worn out tools etc.

Question 5. **What do you mean by product control?**
Answer:
Product control means controlling the quality of the product by a sampling technique calledacceptance sampling. It aims at a certain quality level to he guaranteed to the customers. It is concerned with classification of raw materials, semi-finished goods or finished goods into acceptable or rejectable products.

Question 6. What do you mean by process control?
Answer:
A production process is said to be under control if the products produced are according to the
specifications; that is the characteristics are within the tolerance limits. This is tested through the
control charts.

Question 7. **Define a control chart.**
Answer: Control charts are statistical tools to test whether a production process is under control. It was introduced by Watter.A.Shewhart. It is a simple technique used for detecting patterns of variations in the data. It consists of three lines namely, centre line (CL), Upper control limit (UCL) and Lower control limit (LCL)

Question 8. Name the control charts for variables.
Answer:

A quality characteristic which can be expressed in terms of a numerical value in the production process is called as a variable. There are two types of control charts for variables. Mean chart ($\bar{X}$ chart) & Range chart (R chart).

Question 9. Define the mean chart.
Answer:

The mean chart ($\bar{X}$ chart) is used to show the quality averages of the samples taken from the given process. The mean of the samples is first calculated. Then the mean of the sample means is found to get the control limits.

$\bar{\bar{X}} = \frac{\Sigma\bar{X}}{\text{number of sample means}}$ where $\Sigma\bar{X} =$ total of all the sample means and $\bar{X}_i = \frac{\Sigma X_i}{n}, i = 1,2,3,4, \ldots$ where $\sum X_i =$ total of ' n ' values included in the sample X_i

Question 10. Define R Chart.
Answer

The R chart is used to show the variability or dispersion of the samples taken from the given process. The average range is given by $\bar{R} = \frac{\Sigma R}{n}$, where $R = x_{\max} - x_{\min}$ for each ' n ' samples. For samples of size less than 20 , the range provides a good estimate of σ . Hence to measure the variance in the variable, range chart is used.

Question 11. What are the uses of statistical quality control?
Answer:

The term Quality means a level or standard of a product which depends on Material, Manpower, Machines, and Management (4M's). Quality Control ensures the quality specifications all along with them from the arrival of raw materials through each of their processing to the final delivery of goods. This technique is used in almost all' production industries such as automobile, textile, electrical equipment, biscuits, bath soaps, chemicals, petroleum products etc.

Question 12.Write the control limits for the mean chart.
Solution:

The calculation of control limits for $\bar{X}$ chart in two different cases are

Case (i) when $\bar{X}$ and SD are given	Case (i) when $\bar{X}$ and SD are not given
$UCL = \bar{\bar{X}} + 3\frac{\sigma}{\sqrt{n}}$	$UCL = \bar{\bar{X}} + A_2\bar{R}$
$CL = \bar{\bar{X}}$	$CL = \bar{\bar{X}}$
$LCL = \bar{\bar{X}} - 3\frac{\sigma}{\sqrt{n}}$	$LCL = \bar{\bar{X}} - A_2\bar{R}$

Question 13. Write the control limits for the R chart.
Solution:

The calculation of control limits for R chart in two different cases are

Case (i) when SD is given	Case (i) when SD is not given
$UCL = \bar{R} + 3\sigma_R$	$UCL = D_4\bar{R}$
$CL = \bar{R}$	$CL = \bar{R}$
$LCL = \bar{R} - 3\sigma_R$	$UCL = D_3\bar{R}$

3 - Marks

Exercise 9.1

Question 6. Write a brief note on seasonal variations.
Answer:

Seasonal Variations: As the name suggests, tendency movements are due to nature which repeats themselves periodically in every season. These variations repeat themselves in less than one year time. It is measured in an interval of time. Seasonal variations may be influenced by natural force, social customs and traditions. These variations are the results of such factors which uniformly and regularly rise and fall in the magnitude. For example, selling of umbrellas' and raincoat in the rainy season, sales of cool drinks in the summer season, crackers in Deepawali season, purchase of dresses in a festival season, sugarcane in Pongal season

Question 10 . Explain the method of fitting a straight line.
Answer:

The method of fitting a straight line is as follows Procedure:
(i) The straight-line trend is represented by the equation

$Y = a + bX$...... (1)

where Y is the actual value, X is time, a, b are constants
(ii) The constants ' a ' and ' b ' are estimated by solving the following two normal Equations

$\Sigma Y = na + b\Sigma X$......(2) $\Sigma XY = a\Sigma X + b\Sigma X^2$...... (3)

Where n = number of years given in the data.
(iii) By taking the mid-point of the time as the origin, we get $\Sigma X = 0$
(iv) When $\Sigma X = 0$, the two normal equations reduces to

$\Sigma Y = na + b(0); a = \frac{\Sigma Y}{n} = \bar{Y}$ $\Sigma XY = a(0) + b\Sigma X^2; b = \frac{\Sigma YY}{\Sigma X^2}$

The constant ' a ' gives the mean of Y and ' 6 ' gives the rate of change (slope).
(v) By substituting the values of ' a ' and ' b ' in the trend equation (1), we get the Line of Best Fit.

Question 12.
State the different methods of measuring trend.
Solution: Measurements of Trends

Following are the methods by which we can measure the trend.

1. Freehand or Graphic Method
2. Method of Semi-Averages
3. Method of Moving Averages
4. Method of Least Squares

Question 14.

The following figures relate to the profits of a commercial concern for 8 years.Find the trend of profits by the method of three year moving averages.

Solution: Computation of three-yearly moving averages

The last column gives the trend of profits.

Year	Profit (₹)	3-yearly moving Total(₹)	3-yearly moving averages(₹)
1986	15420	…	…
1987	15470	46410	15470
1988	15520	52010	17336.667
1989	21020	63040	21013.333
1990	26500	79470	26490
1991	31950	94050	31350
1992	35600	102450	34150
1993	34900	…	…

Question 15 .

Find the trend of production by the method of a five-yearly period of moving average for the following data:

Year	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
Production ('000)	126	123	117	128	125	124	130	114	122	129	118	123

Solution:

Year	Production ('000)	5-yearly moving Total	5-yearly moving averages
1979	**126**	…	
1980	**123**	…	
1981	**117**	**619**	**123.8**
1982	**128**	**617**	**123.4**
1983	**125**	**624**	**124.8**
1984	**124**	**621**	**124.2**
1985	**130**	**615**	**123**
1986	**114**	**619**	**123.8**
1987	**122**	**613**	**122.6**
1988	**129**	**606**	**121.2**
1989	**118**	…	…
1990	**123**	…	

Exercise 9.2

Question 14.:Calculate by a suitable method, the index number of price from the following data:

Commodity	2002		2012	
	Price	Quantity	Price	Quantity
A	10	20	16	10
B	12	34	18	42
C	15	30	20	26

Solution:

Commodity	(Base Year) 2002		(Current Year) 2012		p_1q_0	p_0q_0	p_1q_1	p_0q_1
	Price (p_0)	QTY (q_0)	Price (p_1)	QTY (q_1)				
A	10	20	16	10	320	200	160	100
B	12	34	18	42	612	408	756	504
C	15	30	20	26	600	450	520	390
Total					1532	1058	1436	994

The Laspeyres price index number

$P_{01}^{L} = \frac{\sum p_1q_0}{\sum p_0q_0} \times 100 = \frac{1532}{1058} \times 100 = 144.8$

Paasche's price index number

$P_{01}^{P} = \frac{\sum p_1q_1}{\sum p_0q_1} \times 100 = \frac{1436}{994} \times 100 = 144.4$

Question 20.

The following are the group index numbers and the group weights of an average working-class family's budget. Construct the cost of living index number:

Groups	Food	Fuel and Lighting	Clothing	Rent	Miscellaneous
Index Number	2450	1240	3250	3750	4190
Weight	48	20	12	15	10

Solution:

Group	Weight(W)	Index Number (I)	WI
Food	48	2450	117600
Fuel and lighting	20	1240	24800
Clothing	12	3250	39000
Rent	15	3750	56250
Miscellaneous	10	4190	41900
Total	105		279550

Cost of living index number $= \frac{\sum WI}{\sum W} = \frac{279550}{105} = 2662.38$

Question 21.
Construct the cost of living Index number for 2015 on the basis of 2012 from the following data using the family budget method.

Commodity	Price		Weights
	2012	2015	
Rice	250	280	10
Wheat	70	85	5
Corn	150	170	6
Oil	25	35	4
Dhal	85	90	3

Solution:

Commodity	Price (Rs)		Weights (V)	$\mathbf{P}=\frac{p_1}{p_0}\times 100$	PV
	2012 (p_0)	2015 (p_1)			
Rice	250	280	10	112	1120
Wheat	70	85	5	121.43	607.15
Corn	150	170	6	113.33	679.98
Oil	25	35	4	140	560
Dhal	85	90	3	105.88	317.64
Total			28		3284.77

Cost of living index number $=\frac{\sum PV}{\sum V}=\frac{3284.77}{28}=\mathbf{117.31}$

Question 22 .

Calculate the cost of living index by aggregate expenditure method:

Commodity	Weights	Price (Rs.)	
	2010	2010	2015
P	80	22	25
Q	30	30	45
R	25	42	50
S	40	25	35
T	50	36	52

Solution:

Commodity	Price (Rs)		Weights (V)	$\mathbf{P}=\frac{p_1}{p_0}\times 100$	PV
	2010 (p_0)	2015 (p_1)			
P	22	25	80	113.63	9090.4
Q	30	45	30	150	4500
R	42	50	25	119.05	2976.25
S	25	35	40	140	5600
T	36	52	50	144.44	7222
Total			225		29388.65

Cost of living index number $=\frac{\sum PV}{\sum V}=\frac{29388.65}{225}=130.62$

5 - MARKS

EXERCISE 9.1

Question 13.: Compute the average seasonal movement for the following series.

Year	Quarterly Production			
	I	II	III	IV
2002	3.5	3.8	3.7	3.5
2003	3.6	4.2	3.4	4.1
2004	3.4	3.9	3.7	4.2
2005	4.2	4.5	3.8	4.4
2006	3.9	4.4	4.2	4.6

Solution:

Year	Quarterly Production			
	I	II	III	IV
2002	3.5	3.8	3.7	3.5
2003	3.6	4.2	3.4	4.1
2004	3.4	3.9	3.7	4.2
2005	4.2	4.5	3.8	4.4
2006	3.9	4.4	4.2	4.6
Quartely Total	18.6	20.8	18.8	20.8
Average	3.72	4.16	3.76	4.16

Grand average $=\frac{3.72+4.16+3.76+4.16}{4}=3.95$

Seasonal Index (S.I) for I quarter $=\frac{\text{Average of } I \text{ quarter}}{\text{Grand average}}\times 100$

S.I. for I quarter $=\frac{3.72}{3.95}\times 100=94.1772$

S.I. for II quarter $=\frac{4.16}{3.95}\times 100=105.3165$

S.I. for III quarter $=\frac{3.76}{3.95}\times 100=95.1899$

S.I. for IV quarter $=\frac{4.16}{3.95}\times 100=105.3165$

Thus we obtain the average seasonal movement.

Question 17.

The Annual production of a commodity is given as follows:

Year	1995	1996	1997	1998	1999	2000	2001
Production (in tones)	155	162	171	182	158	180	178

Fit a straight line trend by the method of least squares.

Solution: Computation of trend values by the method of least squares

Year (x)	Production (in tonnes) **(Y)**	X= x-1998	$\mathbf{X^2}$	**XY**	Trend values **(Yt)**
1995	155	-3	9	-465	159.57
1996	162	-2	4	-324	162.86
1997	171	-1	1	-171	166.14
1998	182	0	0	0	169.43
1999	158	1	1	158	172.72
2000	180	2	4	360	176.00
2001	178	3	9	534	179.29
N = 7	$\Sigma\mathbf{Y} = 1186$	$\Sigma X = 0$	$\Sigma X^2 = 28$	$\Sigma XY = 92$	$\Sigma Y_t = 1186.01$

$a = \frac{\Sigma Y}{N} = \frac{1186}{7} = 169.429$ $\qquad b = \frac{\Sigma XY}{\Sigma X^2} = \frac{92}{28} = 3.286$

$Y = a + bX$

(i.e) $Y = 169.429 + 3.286X$ (or)

$Y = 169.429 + 3.286(x - 1998)$

The trends values are obtained by

When $x = 1995, Y_t = 169.429 + 3.286(1995 - 1998)$

$= 169.429 + 3.286(-3) = 169.429 - 9.858 = 159.57$

When $x = 1996, Y_t = 169.429 + 3.286(1996 - 1998)$

$= 169.429 + 3.286(-2) = 169.429 - 6.572 = 162.86$

When $x = 1997, Y_t = 169.429 + 3.286(1997 - 1998)$

$= 169.429 + 3.286(-1) = 169.429 - 3.286 = 166.14$

When $x = 1998, Y_t =- 169.429 + 3.286(1998 - 1998)$

$= 169.429 + 3.286(0) = 169.429 - 0 = 169.43$

When $x = 1999, Y_t = 169.429 + 3.286(1999 - 1998)$

$= 169.429 + 3.286(1) = 169.429 + 3.286 = 172{:}72$

When $x = 2000, Y_t = 169.429 + 3.286(2000 - 1998)$

$= 169.429 + 3.286(2) = 169.429 + 6.572 = 176.00$

When $x = 2001, Y_t = 169.429 + 3.286(2001 - 1998)$

$= 169.429 + 3.286(3) = 169.429 + 9.858 = 179.29$

Question 18. **Determine the equation of a straight line which best fits the following data.**

Compute the trend values for all years from 2000 to 2004 .

Year	2000	2001	2002	2003	2004
Sales (₹'000)	35	36	79	80	40

Solution:

Computation of trend values by the method of least squares. (ODD years)

Year (x)	**Sales** $(\Sigma'000)$ **Y**	**X= X -2002**	$\mathbf{X^2}$	**XY**	**Trend values** $(\hat{Y})$
2000	35	-2	4	-70	43.2
2001	36	-1	1	-36	48.6
2002	79	0	0	0	54
2003	80	1	1	80	59.4
2004	40	2	4	80	64.8
	$\Sigma Y = 270$	$\Sigma X = 0$	$\Sigma X^2 = 10$	$\Sigma XY = 54$	$\Sigma\hat{Y} = 270$

$a = \frac{\Sigma Y}{N} = \frac{270}{5} = 54$ $b = \frac{\Sigma XY}{\Sigma X^2} = \frac{54}{10} = 5.4$

Therefore, the equation of the straight line which best fits the data is given by $Y = a + bX$

(i.e) $Y = 54 + 5.4X$

(or) $Y = 54 + 5.4(x - 2002)$

The trends values are obtained as follows

When $x = 2000,\ y = 54 + 5.4(2000 - 2002) = 54 - 10.8 = 43.2$

When $x = 2001,\ y = 54 + 5.4(2001 - 2002) = 54 - 5.4 = 48.6$

When $x = 2002,\ y = 54 + 5.4(2002 - 2002) = 54$

When $x = 2003,\ y = 54 + 5.4(2003 - 2002) = 54 + 5.4 = 59.4$

When $x = 2004,\ y = 54 + 5.4(2004 - 2002) = 54 + 10.8 = 64.8$

Question 19.

The sales of a commodity in tones varied from January 2010 to December 2010 as follows:

In year 2010	Jan	Feb	Mar	Apr	May	June	July	Aug	Sep	Oct	Nov	Dec
Sales (in tones)	280	240	270	300	280	290	210	200	230	200	230	210

Fit a trend line by the method of semi-average.

Solution:

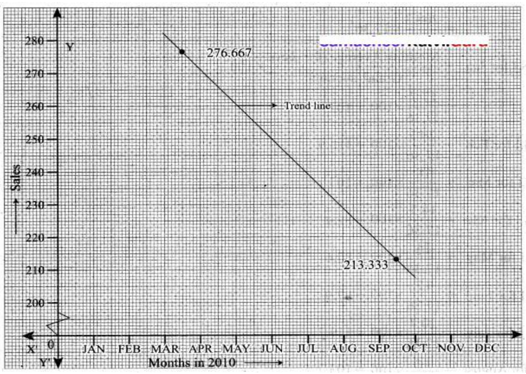
Since the number of months is even (12), we can equally divide the given data in two equal parts and obtain the averages of the first six months and last six months

In Year 2010	Sales in tonnes	Average
JAN	280	$\frac{280+240+270+300+280+290}{6} = 276.667$
FEB	240	
MAR	270	
APR	300	
MAY	280	
JUNE	290	
JULY	210	$\frac{210+200+230+200+230+210}{6} = 213.333$
AUG	200	
SEP	230	
OCT	200	
NOV	230	
DEC	210	

Thus we obtain semi-average $I = 276.667$ and

semi-average $II = 213.333$

To fit a trend line we plot each value at the mid-point (month) of each half, (i.e) we plot 276.667 in the middle of March and April; we plot 213.333 in the middle of September and October. We join the two points by a straight line . This is the required line



Question 20.

Use the method of monthly averages to find the monthly indices for the following data of production of a commodity for the years 2002,2003 and 2004 .

2002	15	18	17	19	16	20	21	18	17	15	14	18
2003	20	18	16	13	12	15	22	16	18	20	17	15
2004	18	25	21	11	14	16	19	20	17	16	18	20

Solution:

Months Years	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2002	15	18	17	19	16	20	21	18	17	15	14	18
2003	20	18	16	13	12	15	22	16	18	20	17	15
2004	18	25	21	11	14	16	19	20	17	16	18	20
Monthly total	53	61	54	43	42	51	62	54	52	51	49	53
Monthly Average	17.7	20.3	18	14.3	14	17	20.7	18	17.3	17	16.3	17.7
Seasonal Indices	102	116.9	103.7	82.4	80.6	97.9	119.2	103.7	99.7	97.9	93.9	102

Grand Average

$= \frac{17.7+20.3+18+14.3+14+17+20.7+18+17.3+17+16.3+17.7}{12}$

$= \frac{208.3}{12} = 17.36$

S.I for Jan $= \frac{\text{Average (for Jan)}}{\text{Grand average}} \times 100 = \frac{17.7}{17.36} \times 100 = 102$

S.I for Feb $= \frac{20.3}{17.36} \times 100 = 116.9$

S.I for Mar $= \frac{18}{17.36} \times 100 = 103.7$

S.I for Apr $= \frac{14.3}{17.36} \times 100 = 82.4$

S.I for May $= \frac{14}{17.36} \times 100 = 80.6$

S.I for June $= \frac{17}{17.36} \times 100 = 97.9$

S.I for July $= \frac{20.7}{17.36} \times 100 = 119.2$

S.I for August $= \frac{18}{17.36} \times 100 = 103.7$

S.I for Sep $= \frac{17.3}{17.36} \times 100 = 99.7$

S.I for Oct $= \frac{17}{17.36} \times 100 = 97.9$

S.I for Nov $= \frac{16.3}{17.36} \times 100 = 93.9$

S.I for Dec $= \frac{17.7}{17.36} \times 100 = 102$

Question 21.

Calculate the seasonal indices from the following data using the average from the following data using the average method:

Year	Quarterly Production			
	I	II	III	IV
2008	72	68	62	76
2009	78	74	78	72
2010	74	70	72	76
2011	76	74	74	72
2012	72	72	76	68

Solution:

Computation of quarterly index by the method of simple averages.

Year	I Quarter	II Quarter	III Quarter	IV Quarter
2008	**72**	**68**	**62**	**76**
2009	**78**	**74**	**78**	**72**
2010	**74**	**70**	**72**	**76**
2011	**76**	**74**	**74**	**72**
2012	**72**	**72**	**76**	**68**
Qly Total	**372**	**358**	**362**	**364**
QLy Avg	$\mathbf{74.4}$	$\mathbf{71.6}$	$\mathbf{72.4}$	$\mathbf{72.8}$
Indices	$\mathbf{102.2}$	$\mathbf{98.35}$	$\mathbf{99.45}$	$\mathbf{100}$

Grand Average $= \frac{74.4+71.6+72.4+72.8}{4} = \frac{291.2}{4} = \mathbf{72.8}$

S.I for I quarter $= \frac{\text{Average of Iquarter}}{\text{Grand average}} \times \mathbf{100} = \frac{74.4}{72.8} \times \mathbf{100} = \mathbf{102.2}$

S. I for II quarter $= \frac{71.6}{72.8} \times \mathbf{100} = \mathbf{98.35}$

S.I for III quarter $= \frac{72.4}{72.8} \times \mathbf{100} = \mathbf{99.45}$

S.I for IV quarter $= \frac{72.8}{72.8} \times \mathbf{100} = \mathbf{100}$

Question 22.

The following table shows the number of salesmen working for a certain concern.

Year	1992	1993	1994	1995	1996
No. of salesmen	46	48	42	56	52

Use the method of least squares to fit a straight line and estimate the number of salesmen in 1997.

Solution:

Year (x)	No. of salesmen (y)	$\mathbf{X = x - 1994}$	$\mathbf{X^2}$	**XY**	Trends Value **Y**
1992	46	-2	4	-92	44.8
1993	48	-1	1	-48	46.8
1994	42	0	0	0	48.8
1995	56	1	1	56	50.8
1996	52	2	4	104	52.8
N = 5	$\sum Y = 244$	$\sum X = 0$	$\sum X^2 = 10$	$\sum XY = 20$	$\sum \hat{Y} = 244$

$a = \frac{\sum Y}{N} = \frac{244}{5} = 48.8$ $b = \frac{\sum XY}{\sum X^2} = \frac{20}{10} = 2$

$Y = a + bX$

$Y = 48.8 + 2X = 48.8 + 2(x - 1994)$

The trend values are obtained as follows:

When $x = 1992, y = 48.8 + 2(1992 - 1994) = 48.8 - 4 = 44.8$

When $x = 1993, y = 48.8 + 2(1993 - 1994) = 48.8 - 2 = 46.8$

When $x = 1994, y = 48.8 + 2(1994 - 1994) = 48.8 - 0 = 48.8$

When $x = 1995, y = 48.8 + 2(1995 - 1994) = 48.8 + 2 = 50.8$

When $x = 1996, y = 48.8 + 2(1996 - 1994) = 48.8 + 4 = 52.8$

In the year 1997,

the estimated number of salesmen is

$\mathbf{Y = 48.8 + 2(1997 - 1994) = 48.8 + 2(3)}$

$\mathbf{= 48.8 + 6 = 54.6 \sim 55}$

Question 15. Calculate price index number for 2005 by

(a) Laspeyre's (b) Paasche's method.

Commodity	1995		2005	
	Quantity	Price	Quantity	Price
A	5	60	15	70
B	4	20	8	35
C	3	15	6	20

Solution:

Commodity	1995(Base Year)		2005(Current Year)		p_1q_0	p_0q_0	p_1q_1	p_0q_1
	Price (p_0)	Qty (q_0)	Price (p_1)	Qty (q_1)				
A	5	60	15	70	900	300	1050	350
B	4	20	8	35	160	80	280	140
C	3	15	6	20	90	45	120	60
Total					1150	425	1450	550

Laspeyre's price index number

$$\mathbf{P_{01}^{L}} = \frac{\sum p_1q_0}{\sum p_0q_0} \times 100 = \frac{1150}{425} \times 100 = \mathbf{270.6}$$

Paasche's price index number

$$\mathbf{P_{01}^{P}} = \frac{\sum p_1q_1}{\sum p_0q_1} \times 100 = \frac{1450}{550} \times 100 = \mathbf{263.63}$$

Question 16.

Compute (i) Laspeyre's (ii) Paasche's (iii) Fisher's Index numbers for 2010 from the following data.

Commodity	Price		Quantity	
	2000	2010	2000	2010
A	12	14	18	16
B	15	16	20	15
C	14	15	24	20
D	12	12	29	23

Solution:

Commodity	2000 (Base Year)		2010 (Current Year)		p_1q_0	p_0q_0	p_1q_1	p_0q_1
	Price (p_0)	Qty (q_0)	Price (p_1)	Qty (q_1)				
A	12	18	14	16	252	216	224	192
B	15	20	16	15	320	300	240	225
C	14	24	15	20	360	336	300	280
D	12	29	12	23	348	348	276	276
					1280	1200	1040	973

Laspeyre's price index number

$$\mathbf{P_{01}^{L}} = \frac{\sum p_1q_0}{\sum p_0q_0} \times 100 = \frac{1280}{1200} \times 100 = \mathbf{106.6}$$

Paasche's price index number

$$\mathbf{P_{01}^{P}} = \frac{\sum p_1q_1}{\sum p_0q_1} \times 100 = \frac{1040}{973} \times 100 = \mathbf{106.8}$$

Fisher's price index number

$$\mathbf{P_{01}^{F}} = \sqrt{\frac{\sum p_1q_0}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_0q_1}} \times 100 = \sqrt{\frac{1280}{1200} \times \frac{1040}{973}} \times 100$$

$$\mathbf{P_{01}^{F}} = \sqrt{\frac{13,31,200}{11,67,600}} \times 100 = \sqrt{1.14} \times 100 = \mathbf{106.7}$$

Question 17. Using the following data, construct Fisher's Ideal index and show how it satisfies Factor Reversal Test and Time Reversal Test?

Commodity	Price in Rupees per unit		Number of units	
	Base year	Current year	Base year	Current year
A	6	10	50	56
B	2	2	100	120
C	4	6	60	60
D	10	12	50	24
E	8	12	40	36

Solution:

Commodity	(Base Year)		Current Year)		p_1q_0	p_0q_0	p_1q_1	p_0q_1
	Price (p_0)	Qty (q_0)	Price (p_1)	Qty (q_1)				
A	6	50	10	56	500	300	560	336
B	2	100	2	120	200	200	240	240
C	4	60	6	60	360	240	360	240
D	10	50	12	24	600	500	288	240
E	8	40	12	36	480	320	432	288
Total					2140	1560	1880	1344

$$\text{Fisher's ideal index} = \sqrt{\frac{\sum p_1q_0}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_0q_1}} \times 100$$

$$= \sqrt{\frac{2140}{1560} \times \frac{1880}{1344}} \times 100 = \sqrt{\frac{40,23,200}{20,96,640}} \times 100 = \sqrt{1.92} \times 100$$

$= 1.385 \times 100 = 138.5$ $\quad \mathbf{P_{01}^{F} = 138.5}$

Time reversal test : To prove $P_{01} \times P_{10} = 1$

$$P_{01} \times P_{10} = \sqrt{\frac{\sum p_1q_0 \times \sum p_1q_1}{\sum p_0q_0 \times \sum p_0q_1}} \times \sqrt{\frac{\sum p_0q_1}{\sum p_1q_1} \times \frac{\sum p_0q_0}{\sum p_1q_0}}$$

$$= \sqrt{\frac{2140}{1560} \times \frac{1880}{1344} \times \frac{1344}{1880} \times \frac{1560}{2140}} \quad \mathbf{P_{01} \times P_{10} = 1}$$

Time reversal test is satisfied.

Factor Reversal Test: To prove $P_{01} \times Q_{01} = \frac{\sum p_1q_1}{\sum p_0q_0}$

$$P_{01} \times P_{10} = \sqrt{\frac{\sum p_1q_0 \times \sum p_1q_1}{\sum p_0q_0 \times \sum p_0q_1}} \times \sqrt{\frac{\sum p_0q_1}{\sum p_1q_1} \times \frac{\sum p_0q_0}{\sum p_1q_0}}$$

$$= \sqrt{\frac{2140}{1560} \times \frac{1880}{1344} \times \frac{1344}{1560} \times \frac{1880}{2140}} = \sqrt{\frac{1880 \times 1880}{1560 \times 1560}} = \frac{1880}{1560} = \frac{\sum p_1q_1}{\sum p_0q_0}$$

Factor Reversal Test is satisfied.

Question 19.

Calculate Fisher's index number to the following data. Also, show that satisfies Time Reversal Test.

Commodity	2016		2017	Qty (Kg)
	Price (Rs.)	Qty (Kg)	Price (Rs.)	
Food	40	12	65	14
Fuel	72	14	78	20
Clothing	36	10	36	15
Wheat	20	6	42	4
Others	46	8	52	6

Solution:

Commodity	2016(Base Year)		2017 CurYear		p_1q_0	p_0q_0	p_1q_1	p_0q_1
	(p_0)	(q_0)	(p_1)	(q_1)				
Food	40	12	65	14	780	480	910	560
Fuel	72	14	78	20	1092	1008	1560	1440
Clothing	36	10	36	15	360	360	540	540
Wheat	20	6	42	4	252	120	168	80
Others	46	8	52	6	416	368	312	276
					2900	2336	3490	2896

Fisher's price index number

$$P_{01}^{F} = \sqrt{\frac{\sum p_1q_0}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_0q_1}} \times 100$$

$$= \sqrt{\frac{2900}{2336} \times \frac{3490}{2896}} \times 100 = \sqrt{\frac{1,01,21,000}{67,65,056}} \times 100$$

$$= \sqrt{1.496 \times 100} = 1.223 \times 100 = 122.3$$

$$P_{01}^{F} = 122.3$$

Time reversal test:

To prove $P_{01} \times P_{10} = 1$

$$P_{01} \times P_{10} = \sqrt{\frac{\sum p_1q_0 \times \sum p_1q_1}{\sum p_0q_0 \times \sum p_0q_1} \times \sqrt{\frac{\sum p_0q_1}{\sum p_1q_1}} \times \frac{\sum p_0q_0}{\sum p_1q_0}}$$

$$= \sqrt{\frac{2900}{2336} \times \frac{3490}{2896} \times \frac{2896}{3490} \times \frac{2336}{2900}} = 1$$

Time reversal test is satisfied.

Question 18.

Using Fisher's Ideal Formula; compute price index number for 1999 with 1996 as the base year, given the following.

Year	Commodity: A		Commodity: B		Commodity: C	
	Price (Rs.)	Qty (Kg)	Price (Rs.)	Qty (Kg)	Price (Rs.)	Qty (Kg)
1996	5	10	8	6	6	3
1999	4	12	7	7	5	4

Solution:

Commodity	1996 (Base Year)		1999 (Cur.Year)		p_1q_0	p_0q_0	p_1q_1	p_0q_1
	Price (p_0)	Qty (q_0)	Price (p_1)	Qty (q_1)				
A	5	10	4	12	40	50	48	60
B	8	6	7	7	42	48	49	56
C	6	3	5	4	15	18	20	24
Total					97	116	117	140

Fisher's index number $P_{01}^{F} = \sqrt{\frac{\sum p_1q_0}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_0q_1}} \times 100$

$$= \sqrt{\frac{97}{116} \times \frac{117}{140}} \times 100 = \sqrt{\frac{1.1349}{16240}} \times 100 = 0.836 \times 100 = 83.6$$

Question 14.

A machine is set to deliver packets of a given weight. Ten samples of size five each were recorded. Below are given relevant data:

Sample number	1	2	3	4	5	6	7	8	9	10
$\bar{X}$	15	17	15	18	17	14	18	15	17	16
R	7	7	4	9	8	7	12	4	11	5

Calculate the control limits for the mean chart and the range chart and then comment on the state of control.
(conversion factors for $n = 5$, $A_2 = 0.58$, $D_3 = 0$ and $D_4 = 2.115$)

Solution:

Sample	1	2	3	4	5	6	7	8	9	10	TOTAL
$\bar{X}$	15	17	15	18	17	14	18	15	17	16	$\sum\bar{X} = 162$
R	7	7	4	9	8	7	12	4	11	5	$\sum R = 74$

$\bar{\bar{X}} = \frac{\sum \bar{X}}{10} = \frac{162}{10} = 16.2$ & $\bar{R} = \frac{\sum R}{n} = \frac{74}{10} = 7.4$

The control limits for $\bar{X}$ chart is

$UCL = \bar{\bar{X}} + A_2\bar{R} = 16.2 + (0.58)(7.4) = 20.49$

$CL = 16.2$

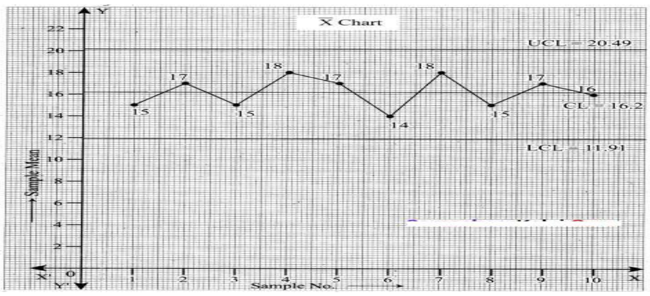
$LCL = \bar{\bar{X}} - A_2\bar{R} = 16.2 - (0.58)(7.4) = 11.91$

The control limits for range chart is

$UCL = D_4\bar{R} = (2.115)(7.4) = 15.65$

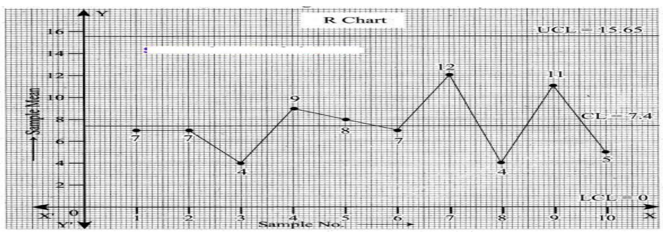
$CL = \bar{R} = 7.4$

$LCL = D_3\bar{R} = 0(7.4) = 0$



The above diagram shows all the three control lines with the data points plotted . We see that all the points of the sample mean are within the control limits .

We now draw the R chart for the given data.



The above diagram shows all the three control lines with the sample range points plotted. We observe that all the points are within the control limits.

Conclusion: From the above two plots of the sample mean $\bar{X}$ and sample range R, we conclude that the process is in control.

Question 15.

Ten samples each of size five are drawn at regular intervals from a manufacturing process. The sample means ($\bar{X}$) and their ranges (R) are given below:

Sample	1	2	3	4	5	6	7	8	9	10
$\bar{X}$	49	45	48	53	39	47	46	39	51	45
R	7	5	7	9	5	8	8	6	7	6

Calculate the control limits in respect of $\bar{X}$ chart.

(Given $A_2 = 0.58, D_3 = 0$ and $D_4 = 2.115$) Comment on the state of control

Solution:

Sample	1	2	3	4	5	6	7	8	9	10	TOTAL
$\bar{X}$	49	45	48	53	39	47	46	39	51	45	462
R	7	5	7	9	5	8	8	6	7	6	68

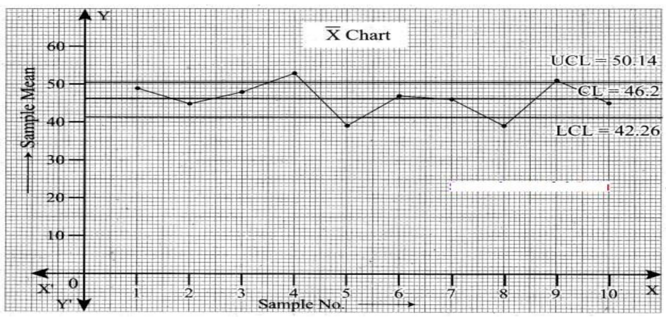
$\bar{\bar{X}} = \frac{\sum \bar{X}}{10} = \frac{462}{10} = 46.2$ & $\bar{R} = \frac{\sum R}{10} = \frac{68}{10} = 6.8$

The control limits for $\bar{X}$ chart is

$UCL = \bar{\bar{X}} + A_2\bar{R} = 46.2 + (0.58)(6.8) = 50.14$

$CL = 46.2$

$LCL = \bar{\bar{X}} - A_2\bar{R} = 46.2 - (0.58)(6.8) = 42.26$

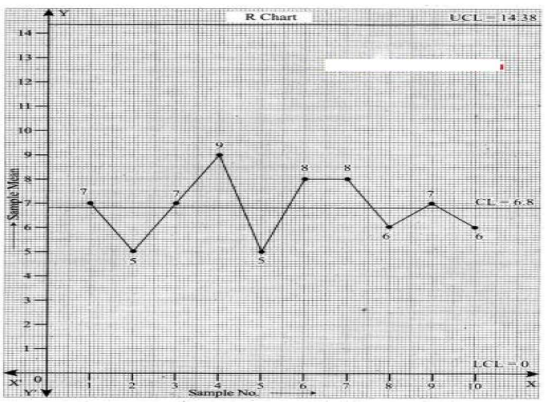


The control limits for range chart is

$UCL = D_4\bar{R} = (2.115)(6.8) = 14.38$

$CL = \bar{R} = 6.8$

$UCL = D_3\bar{R} = 0(6.8) = 0$



From the $\bar{X}$ chart, we see that 4 points are outside the control limit lines. So we say that the process is out of control.

Question 16.

Construct $\bar{X}$ and R charts for the following data:

SAMPLE NUMBER	OBSERVATIONS		
1	32	36	42
2	28	32	40
3	39	52	28
4	50	42	31
5	42	45	34
6	50	29	21
7	44	52	35
8	22	35	44

(Given for $\mathbf{n = 3,\ A_2 = 1.023, D_3 = 0}$ and $\mathbf{D_4 = 2.574}$)

Solution:

We first find the sample mean and range for each of the 8 given samples.

SAMPLE NUMBER	OBSERVATIONS			TOTAL	$\bar{X}$	R (H.V-L.V)
1	32	36	42	110	36.67	10
2	28	32	40	100	33.33	12
3	39	52	28	119	39.67	24
4	50	42	31	123	41	19
5	42	45	34	121	40.33	11
6	50	29	21	100	33.33	29
7	44	52	35	131	43.67	17
8	22	35	44	101	33.67	22
TOTAL					301.67	144

$\bar{\bar{X}} = \frac{\sum \bar{X}}{8} = \frac{301.67}{8} = 37.71$

$\bar{R} = \frac{\sum R}{8} = \frac{144}{8} = 18$

The control limits for $\bar{X}$ chart is

$\text{UCL} = \bar{\bar{X}} + A_2\bar{R} = 37.71 + (1.023)(18) = 56.12$

$\text{CL} = \bar{\bar{X}} = 37.71$

$= \bar{\bar{X}} - A_2\bar{R} = 37.71 - (1.023)(18) = 19.296$

$19.296 < \bar{X} < 56.12$

The process is in control

The control limits for R chart is

$\text{UCL} = D_4\bar{R} = (2.574)(18) = 46.33$

$\text{CL} = \bar{R} = 18$

$\text{LCL} = D_3\bar{R} = 0(18) = 0$

$0 < R < 46.33$

The proces is in control

Question 17.

The following data show the values of the sample mean ($\bar{X}$) and its range (R) for the samples of Size five each. Calculate the values for control limits for mean, range chart and determine whether the process is in control.

Sample	1	2	3	4	5	6	7	8	9	10
Mean	11.2	11.8	10.8	11.6	11.0	9.6	10.4	9.6	10.6	10.0
Range	7	4	8	5	7	4	8	4	7	9

(conversion factors for $\mathbf{n = 5,\ A_2 = 0.58, D_3 = 0}$ and $\mathbf{D_4 = 2.115}$)

Solution:

Sample	1	2	3	4	5	6	7	8	9	10	total
Mean	11.2	11.8	10.8	11.6	11.0	9.6	10.4	9.6	10.6	10.0	106.6
Range	7	4	8	5	7	4	8	4	7	9	63

$\bar{\bar{X}} = \frac{\sum \bar{X}}{10} = \frac{106.6}{10} = \mathbf{10.66}$ $\qquad \bar{R} = \frac{\sum R}{10} = \frac{63}{10} = \mathbf{6.3}$

The control limits for $\bar{X}$ chart is

$\mathbf{UCL = \bar{\bar{X}} + A_2\bar{R} = 10.66 + (0.58)(6.3) = 14.31}$

$\mathbf{CL = \bar{\bar{X}} = 10.66}$

$\mathbf{LCL = \bar{\bar{X}} - A_2\bar{R} = 10.66 - (0.58)(6.3) = 7.006}$

The control limits for R chart is

$\mathbf{UCL = D_4\bar{R} = (2.115)(6.3) = 13.32}$

$\mathbf{CL = \bar{R} = 6.3}$

$\mathbf{LCL = D_3\bar{R} = 0(6.3) = 0}$

From the above control limits values we observe that all the sample means lie between the UCL and LCL (i.e.) $\mathbf{7.006 < \bar{x}_i < 14.31}$ for $\mathbf{i = 1, 2, 3, \ldots\ldots. 10}$. Also all the sample range value lie between the control limits for R (i.e) $\mathbf{0 < R_i < 13.32, i = 1, 2, 3, \ldots, 10}$. Hence we conclude that the process is in control.

Question 18.

A quality control inspector has taken ten samples of size four packets each from a potato chips company. The contents of the sample are given below, Calculate the control limits for the mean and range chart.

Sample number	Observations			
	1	2	3	4
1	12.5	12.3	12.6	12.7
2	12.8	12.4	12.4	12.8
3	12.1	12.6	12.5	12.4
4	12.2	12.6	12.5	12.3
5	12.4	12.5	12.5	12.5
6	12.3	12.4	12.6	12.6
7	12.6	12.7	12.5	12.8
8	12.4	12.3	12.6	12.5
9	12.6	12.5	12.3	12.6
10	12.1	12.7	12.5	12.8

(Given for $\mathbf{n = 4}$, $\mathbf{A_2 = 0.729, D_3 = 0}$ and $\mathbf{D_4 = 2.282}$)

Solution:

Sample number	Observations						
	1	2	3	4	total	$\bar{X}$	R
1	12.5	12.3	12.6	12.7	50.1	12.53	0.4
2	12.8	12.4	12.4	12.8	50.4	12.6	0.4
3	12.1	12.6	12.5	12.4	49.6	12.4	0.5
4	12.2	12.6	12.5	12.3	49.6	12.4	0.4
5	12.4	12.5	12.5	12.5	49.9	12.48	0.1
6	12.3	12.4	12.6	12.6	49.9	12.48	0.3
7	12.6	12.7	12.5	12.8	50.6	12.65	0.3
8	12.4	12.3	12.6	12.5	49.8	12.45	0.3
9	12.6	12.5	12.3	12.6	50	12.5	0.3
10	12.1	12.7	12.5	12.8	50.1	12.53	0.7
Total						125.02	3.7

$\bar{\bar{X}} = \frac{\sum \bar{X}}{10} = \frac{125.02}{10} = 12.5$ & $\bar{R} = \frac{\sum R}{10} = \frac{3.7}{10} = 0.37$

The control limits for mean chart is

$\mathbf{UCL = \bar{\bar{X}} + A_2\bar{R} = 12.5 + (0.729)(0.37) = 12.77}$

$\mathbf{CL = \bar{\bar{X}} = 12.5}$

$\mathbf{LCL = \bar{\bar{X}} - A_2\bar{R} = 12.5 - (0.729)(0.37) = 12.23}$

The control limits for R chart is

$\mathbf{UCL = D_4\bar{R} = (2.282)(0.37) = 0.84}$

$\mathbf{CL = \bar{R} = 0.37}$

$\mathbf{LCL = D_3\bar{R} = (0)(0.37) = 0}$

$12.23 < \bar{X} < 12.77$

The process is in control

Question 19.

The following data show the values of sample means and the ranges for ten samples of size 4 each. Construct the control chart for mean and range chart and determine whether the process is in control

Sample	1	2	3	4	5	6	7	8	9	10
$\bar{X}$	29	26	37	34	14	45	39	20	34	23
R	39	10	39	17	12	20	05	21	23	15

Solution:

Sample	1	2	3	4	5	6	7	8	9	10	total
Mean	29	26	37	34	14	45	39	20	34	23	301
Range	39	10	39	17	12	20	05	21	23	15	201

$\bar{\bar{X}} = \frac{\sum \bar{X}}{10} = \frac{301}{10} = 30.1$

$\bar{R} = \frac{\sum R}{10} = \frac{201}{10} = 20.1$

The control limits for $\bar{X}$ chart is

$\mathbf{UCL = \bar{\bar{X}} + A_2\bar{R} = 30.1 + (0.729)(20.1) = 44.75}$

$\mathbf{CL = \bar{\bar{X}} = 30.1}$

$\mathbf{LCL = \bar{\bar{X}} - A_2\bar{R} = 30.1 - (0.729)(20.1) = 15.45}$

The control limits for R chart is

$\mathbf{UCL = D_4\bar{R} = (2.282)(20.1) = 45.87}$

$\mathbf{CL = \bar{R} = 20.1}$

$\mathbf{LCL = D_3\bar{R} = (0)(20.1) = 0}$

From the values of the control limits for $\bar{X}$, we observe that one sample $\bar{X}$ value (45) is above the UCL and one sample $\bar{X}$ value (14) is below the LCL. Hence we conclude that the process is out of control.

Question 20.

In a production process, eight samples of size 4 are collected and their means and ranges are given below. Construct a mean chart and range chart with control limits.

Sample number	1	2	3	4	5	6	7	8
$\bar{X}$	12	13	11	12	14	13	16	15
R	2	5	4	2	3	2	4	3

Solution:

Sample number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	Total
$\bar{X}$	12	13	11	12	14	13	16	15	106
R	2	5	4	2	3	2	4	3	25

$\bar{X} = \frac{\sum \bar{X}}{8} = \frac{106}{8} = 13.25$

$\bar{R} = \frac{\sum R}{8} = \frac{25}{8} = 3.13$

The control limits for $\bar{X}$ chart is

$UCL = \bar{\bar{X}} + A_2\bar{R} = 13.25 + (0.729)(3.13) = 15.53$

$CL = \bar{\bar{X}} = 13.25$

$LCL = \bar{\bar{X}} - A_2\bar{R} = 13.25 - (0.729)(3.13) = 10.97$

The control limits for R chart is

$UCL = D_4\bar{R} = (2.282)(3.13) = 7.14$

$CL = \bar{R} = 3.13$

$LCL = D_3\bar{R} = (0)(3.13) = 0$

From the values of the control limits for $\bar{X}$, we observe that sample $\bar{X}$ value 16 is above the UCL. Hence we conclude that the process is out of control.

Question 21.

In a certain bottling industry, the quality control inspector recorded the weight of each of the 5 bottles selected at random during each hour of four hours in the morning.

Time	Weights in ml					
8.00AM	43	41	42	43	41	
9.00AM	40	39	40	39	44	
10.00AM	42	42	43	38	40	
11.00AM	39	43	40	39	42	

Solution:

Time	**Weights in ml**					$\bar{X}$	**R**
8.00AM	**43**	**41**	**42**	**43**	**41**	**42**	**2**
9.00AM	**40**	**39**	**40**	**39**	**44**	**40.4**	**5**
10.00AM	**42**	**42**	**43**	**38**	**40**	**41**	**5**
11.00AM	**39**	**43**	**40**	**39**	**42**	**40.6**	**4**
Total						**164**	**16**

$\bar{X} = \frac{\sum \bar{X}}{4} = \frac{164}{4} = 41$

$\bar{R} = \frac{\sum R}{4} = \frac{16}{4} = 4$

The control limits for $\bar{X}$ chart is

$UCL = \bar{\bar{X}} + A_2\bar{R} = 41 + (0.58)(4) = 43.32$

$CL = \bar{\bar{X}} = 41$

$LCL = \bar{\bar{X}} - A_2\bar{R} = 41 - (0.58)(4) = 38.68$

The control limits for R chart is

$UCL = D_4\bar{R} = (2.115)(4) = 8.46$

$CL = \bar{R} = 4$

$LCL = D_3\bar{R} = (0)4 = 0$

From the above control limit values. We observe that all the sample $\bar{X}$ values are within UCL and LCL values.

Also, all the R values are also within UCL and LCL of R chart. Hence we conclude that the process is within Control.

CHAPTER 10 - OPERATIONS RESEARCH

(2, 3 AND 5 MARKS)

Exercise 10.1

2 - Marks

Question 1. What is the transportation problem?

Answer:

The transportation problem is to identify the quantity of homogeneous items **to be transported** from each **origin (source)** to each **destination** with the objective of **minimising the total transportation cost**.

Example : Managing water supply from water distribution points to various places in a city, so as to minimise the transportation cost.

Question 2.

Write the mathematical form of transportation problem.

Answer:

Objective function: Minimize $Z=\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}x_{ij}$

Subject to the constraints

$\sum_{j=1}^{n} x_{ij}=a_i, i=1,2,\ldots\ldots m$ (supply constraints)

$\sum_{i=1}^{m} x_{ij}=b_j, j=1,2,\ldots\ldots n$ (demand constraints)

$x_{ij}\geq 0$ for all i, j. (non-negative restrictions)

Question 3.

What are a feasible solution and non-degenerate solution in the transportation problem?

Solution:

Feasible Solution: A feasible solution to a transportation problem is a set of non-negative values $x_{ij}(i=1,2,\ldots,m,$ $j=1,2,\ldots n)$ that satisfies the constraints.

Non-degenerate basic feasible Solution: If a basic feasible solution to a transportation problem contains exactly $m+n-1$ allocation in independent positions, it is called a Non-degenerate basic feasible solution. Here m is the number of rows and n is the number of columns in a transportation problem.

Question 4.

What do you mean by balanced transportation problem?

Answer:

In a transportation problem, if the total supply is **equal to** the total demand, it is said to be balanced transportation problem.

(i. e) $\sum_{i=1}^{m} a_i=\sum_{j=1}^{n} b_i$

A feasible solution can be obtained to these problems by Northwest comer method, minimum cost method (or) Vogel's approximation method.

Exercise 10.2

Question 1.

What is the Assignment problem?

Answer:

For 'm' jobs to be performed on 'n' machines (one job per machine).

The assignment of different jobs to the different machines to **minimize** the **overall cost** is known as Assignment problem.

Question 2.

Give the mathematical form of the assignment problem.

Answer:

The mathematical form of assignment problem is

Minimize $Z=\sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}x_{ij}$ Subject to the constraints

$\sum_{i=1}^{n} x_{ij}=1$, and $\sum_{j=1}^{n} x_{ij}=1; x_{ij}=0$ (or) 1

for all i = 1,2, ……. n and j = 1,2, ……n where C_{ij} is the cost of assigning ith job to j th machine and x_{ij} represents the assignment of ith job to jth machine.

Question 3.

What is the difference between Assignment Problem and Transportation Problem?

Answer:

The assignment problem is a special case of the transportation problem. The differences are given below.

Transportation Problem	Assignment Problem
1. This is about reducing cost of transportation merchandise	1. This is about assigning finite sources to finite destinations where only one destination is allotted for one source with minimum cost
2. Number of sources and number of demand need not be equal	2. Number of sources and the number of destinations must be equal
3. If total demand and total supply are not equal then the problem is said to be unbalanced.	3. If the number of rows are not equal to the number of columns then problems are unbalanced.
4. It requires 2 stages to solve: Getting initial basic feasible solution, by NWC, LCM, VAM and optimal solution by MODI method	4. It has only one stage. Hungarian method is sufficient for obtaining an optimal solution

3 - MARKS

EXERCISE 10.1

Question 5 .

Find an initial basic feasible solution of the following problem using north-west corner rule.

	D_1	D_2	D_3	D_4	Supply
O_1	5	3	6	2	19
O_2	4	7	9	1	37
O_3	3	4	7	5	34
Demand	16	18	31	25	

Solution:

Given the transportation table is

	D_1	D_2	D_3	D_4	Supply a_i
O_1	5	3	6	2	19
O_2	4	7	9	1	37
O_3	3	4	7	5	34
Demand (b_j)	16	18	31	25	90

Total supply = Total Demand = 90.

First allocation:

	D_1	D_2	D_3	D_4	a_i
O_1	(16) 5	3	6	2	~~19~~ 3
O_2	4	7	9	1	37
O_3	3	4	7	5	34
(b_j)	~~16~~ 0	18	31	25	90

Second allocation:

	D_1	D_2	D_3	D_4	a_i
O_1	(16) 5	(3) 3	6	2	~~19/3~~/0
O_2	4 -	7	9	1	37
O_3	3 -	4	7	5	34
(b_j)	~~16~~ 0	~~18~~/15	31	25	90

Third allocation:

	D_1	D_2	D_3	D_4	a_i
O_1	(16) 5	(3) 3	6	2	19/3/0
O_2	4	(15) 7	9	1	37/22
O_3	3	4 -	7	5	34
(b_j)	16/0	18/15/0	31	25	90

Fourth allocation:

	D_1	D_2	D_3	D_4	a_i
O_1	(16) 5	(3) 3	6	2	19/3/0
O_2	4	(15) 7	(22) 9	1	37/22/0
O_3	3	4	7	5	34
(b_j)	16/0	18/15/0	31/9	25	90

Fifth allocation :

	D_1	D_2	D_3	D_4	a_i
O_1	(16) 5	(3) 3	6	2	19/3/0
O_2	4	(15) 7	(22) 9	1	37/22/0
O_3	3	4	(9) 7	5	34/25
(b_j)	16/0	18/15/0	31/9/0	25	35

Final allocation :

	D_1	D_2	D_3	D_4	a_i
O_1	(16) 5	(3) 3	6	2	19/3/0
O_2	4	(15) 7	(22) 9	1	37/22/0
O_3	3	4	(9) 7	(25) 5	34/25/0
(b_j)	16/0	18/15/0	31/9/0	25/0	35

Transportation schedule:

$O_1 \to D_1, O_1 \to D_2, O_2 \to D_2, O_2 \to D_3, O_3 \to D_3, O_3 \to D_4$

$x_{11} = 16, x_{12} = 3, x_{22} = 15, x_{23} = 22, x_{33} = 9, x_{34} = 25.$

Total transportation cost

$= (16 \times 5) + (3 \times 3) + (15 \times 7) + (22 \times 9) + (9 \times 7) + (25 \times 5) = 80 + 9 + 105 + 198 + 63 + 125$

$= 580$

Question 6.

Determine an initial basic feasible solution of the following transportation problem by north-west corner method.

	Bangalore	Nasik	Bhopal	Delhi	Capacity
Chennai	6	8	8	5	30
Madurai	5	11	9	7	40
Trichy	8	9	7	13	50
Demand (Units/day)	35	28	32	25	

Solution:

Total capacity = Total Demand = 120.

First allocation:

	B	N	Bh	D	(a_i)
C	(30) 6	8	8	5	30/0
M	5	11	9	7	40
T	8	9	7	13	50
(b_j)	35/5	28	32	25	120

Second allocation:

	B	N	Bh	D	(a_i)
C	(30) 6	8	8	5	30/0
M	(5) 5	11	9	7	40/35
T	8	9	7	13	50
(b_j)	35/5/0	28	32	25	120

Third allocation :

	B	N	Bh	D	(a_i)
C	(30) 6	8	8	5	30/0
M	(5) 5	(28) 11	9	7	40/35/7
T	8	9	7	13	50
(b_j)	35/5/0	28/0	32	25	120

Fourth allocation :

	B	N	Bh	D	(a_i)
C	(30) 6	8	8	5	30/0
M	(5) 5	(28) 11	(7) 9	7	40/35/7
T	8	9	7	13	50
(b_j)	35/5/0	28/0	32/25	25	120

Fifth allocation :

	B	N	Bh	D	(a_i)
C	(30) 6	8	8	5	30/0
M	(5) 5	(28) 11	(7) 9	7	40/35/7
T	8	9	(25) 7	13	50/25
(b_j)	35/5/0	28/0	32/25/0	25	120

Final allocation :

	B	N	Bh	D	(a_i)
C	(30) 6	8	8	5	30/0
M	(5) 5	(28) 11	(7) 9	7	40/35/7
T	8	9	(25) 7	(25) 13	50/25/0
(b_j)	35/5/0	28/0	32/25/0	25/0	120

Transportation schedule:

(i.e) $x_{11} = 30, x_{21} = 5, x_{22} = 28, x_{23} = 7, x_{33} = 25, x_{34} = 25$

The total transportation cost

$= (30 \times 6) + (5 \times 5) + (28 \times 11) + (7 \times 9) + (25 \times 7) + (25 \times 13)$

$= 180 + 25 + 308 + 63 + 175 + 325 = 1076$

Thus the minimum cost is Rs. 1076 by the north west comer method.

Question 7.

Obtain an initial basic feasible solution to the following transportation problem by using the least-cost method.

	D_1	D_2	D_3	Supply
O_1	9	8	5	25
O_2	6	8	4	35
O_3	7	6	9	40
Demand	30	25	45	

Solution:

Total supply = Total demand = 100

First allocation:

	D_1	D_2	D_3	a_i
O_1	9	8	5	25
O_2	6	8	(35) 4	35/0
O_3	7	6	9	40
(b_j)	30	25	45/10	

The least-cost 4 corresponds to cell (O_2, D_3). So first we allocate to this cell.

Second allocation:

	D_1	D_2	D_3	a_i
O_1	9	8	(10) 5	25/15
O_2	6	8	(35) 4	35/0
O_3	7	6	9	40
(b_j)	30	25	45/10/0	

The least-cost 5 corresponds to cell (O_1, D_3). So we have allocated min(10,25) to this cell.

Third allocation:

	D_1	D_2	D_3	a_i
O_1	9	8	(10) 5	25/15
O_2	6	8	(35) 4	35/0
O_3	7	(25) 6	9	40
(b_j)	30	25/0	45/10/0	

The least-cost 6 corresponds to cell (O_3, D_2). So we have allocated min(25,40) to this cell.

Fourth allocation:

	D_1	D_2	D_3	a_i
O_1	9	8	(10) 5	25/15
O_2	6	8	(35) 4	35/0
O_3	(15) 7	(25) 6	9	40/15/0
(b_j)	30/15	25/0	45/10/0	

The least-cost 7 corresponds to cell (O_3, D_1). So we have allocated min(30,15) to this cell.

Final allocation:

Although the next least cost is 8 , we cannot allocate to cells (O_1, D_2) and (O_2, D_2) because we have exhausted the demand 25 for this column. So we allocate 15 to cell (O_1, D_1)

	D_1	D_2	D_3	a_i
O_1	(15) 9	8	(10) 5	25/15/0
O_2	6	8	(35) 4	35/0
O_3	(15) 7	(25) 6	9	40/15/0
(b_j)	30/15/0	25/0	45/10/0	

Transportation schedule: $O_1 \to D_1, O_1 \to D_3, O_2 \to D_3, O_3 \to D_1, O_3 \to D_2$

(i.e) $x_{11} = 15, x_{13} = 10, x_{23} = 35, x_{31} = 15, x_{32} = 25$

Total cost is $=(15\times9)+(10\times5)+(35\times4)+(15\times7)+(25\times6)$

$= 135 + 50 + 140 + 105 + 150$

$= 580$

Thus by least cost method (LCM) the cost is Rs. 580 .

Question 10.

Determine the basic feasible solution to the following transportation problem using North West Corner rule.

		Sinks					Supply
		A	B	C	D	E	
	P	2	11	10	3	7	4
Origins	Q	1	4	7	2	1	8
	R	3	9	4	8	12	9
Demand		3	3	4	5	6	

Solution: Total demand = Total supply = 21

First allocation:

	A	B	C	D	E	a_i
P	(3) 2	11	10	3	7	4
Q	1	4	7	2	1	8
R	3	9	4	8	12	9
b_j	3	3	4	5	6	

Second allocation:

	B	C	D	E	a_i
P	(1) 11	10	3	7	1/0
Q	4	7	2	1	8
R	9	4	8	12	9
b_j	3/2	4	5	6	

Third allocation :

	B	C	D	E	a_i
Q	(2) 4	7	2	1	8/6
R	9	4	8	12	9
b_j	2/0	4	5	6	

Fourth allocation :

	C	D	E	a_i
Q	(4) 7	2	1	6/2
R	4	8	12	9
b_j	4/0	5	6	

Fifth allocation :

	D	E	a_i
Q	(2) 2	1	2/0
R	8	12	9
b_j	5/3	6	

Final allocation :

	D	E	a_i
R	(3) 8	(6) 12	9/6/0
b_j	3/0	6/0	

	A	B	C	D	E	a_i
P	(3) 2	(1) 11	10	3	7	4
Q	1	(2) 4	(4) 7	(2) 2	1	8
R	3	9	4	(3) 8	(6) 12	9
b_j	3	3	4	5	6	

Transportation schedule:

P → A, P → B, Q → B, Q → C, Q → D, R → D, R → E

(i.e)$x_{11} = 3, x_{12} = 1, x_{22} = 2, x_{23} = 4, x_{24} = 2, x_{34} = 3, x_{35} = 6$

Total cost =(3×2)+(1×11)+(2×4)+(4×7)+(2×2)+(3×8)+(6×12)
= 6 + 11 + 8 + 28 + 4 + 24 + 72 = 153

Question 12.

Obtain an initial basic feasible solution to the following transportation problem by north-west corner method.

	D	E	F	G	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Required	200	225	275	250	

Solution:

Total availability = Total requirement = 950

First:

	D	E	F	G	a_i
A	(200) 11	13	17	14	250/5
B	16	18	14	10	300
C	21	24	13	10	400
b_j	200/0	225	275	250	

Second :

	E	F	G	a_i
A	(50) 13	17	14	50/0
B	18	14	10	300
C	24	13	10	400
b_j	225/175	275	250	

Third allocation :

	E	F	G	a_i
B	(175) 18	14	10	300/125
C	24	13	10	400
b_j	175/0	275	250	

Fourth allocation:

	F	G	a_i
B	(125) 14	10	125/0
C	13	10	400
b_j	275/150	250	

Fifth allocation :

	F	G	a_i
C	(150) 13	(250) 10	400/250/0
b_j	150/0	250/0	

	D	E	F	G	Available
A	(200) 11	(50) 13	17	14	250
B	16	(175) 18	(125) 14	10	300
C	21	24	(150) 13	(250) 10	400
Required	200	225	275	250	

Transportation schedule: A → D, A → E, B → E, B → F, C → F, C → G

(i.e) $\mathbf{x_{11} = 200, x_{12} = 50, x_{22} = 175, x_{23} = 125, x_{33} = 150, x_{34} = 250}$

Total cost =(200×11)+(50×13)+(175×18)+(125×14)+(150×13)+(250×10)
= 2200 + 650 + 3150 + 1750 + 1950 + 2500

= Rs. 12, 200

EXERCISE 10.2

Question 4.

Three jobs A, B and C one to be assigned to three machines U, V and W. The processing cost for each job machine combination is shown in the matrix given below. Determine the allocation that minimizes the overall processing cost.

Machine

Job		U	V	W
	A	17	25	31
	B	10	25	16
	C	12	14	11

(cost is in ₹ per unit)

Solution: Here the number of rows and columns are equal.

Step 1: We select the smallest element from each row and subtract from other elements in its row.

Machine

Job		U	V	W
	A	0	8	14
	B	0	15	6
	C	1	3	0

Column V has no zero. Go to step 2.

Step 2: Select the smallest element from each column and subtract from other elements in its column.

Machine

Job		U	V	W
	A	0	5	14
	B	0	12	6
	C	1	0	0

Since each row and column contains at least one zero, assignments can be made.

Step 3: (Assignment) : Row A contains exactly one zero. We mark it by □ and other zeros in its column by x.

Machine

Job		U	V	W
	A	0	5	14
	B	[0]	12	6
	C	X	0	0

Now proceed column wise. Column V has exactly one zero. Mark by □ and other zeros in its row by X.

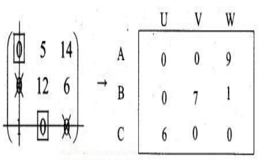
Step 4:

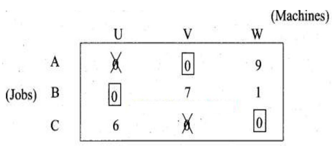
Machine

Job		U	V	W
	A	0	5	14
	B	[0]	12	6
	C	X	[0]	X

Now there is no zero in row B to assign the job. So proceed as follows. Draw a minimum number of lines to cover all the zeros in the reduced matrix. Subtract 5 from all the uncovered elements and add to the element at the intersection of 2 lines as shown below.

Step 5:





Now start the whole procedure once again for assignment to get the following matrix .

Thus all the 3 assignments have been made. The optimal assignment schedule and the total cost is Job Machine Cost

Job	Machine	Cost
A	V	25
B	U	10
C	W	11
TOTAL		46

Exercise 10.3

Question 1.

Given the following pay-off matrix (in rupees) for three strategies and two states of nature.

Strategy	States-of-nature	
	E_1	E_2
S_1	40	60
S_2	10	-20
S_3	-40	150

Select a strategy using each of the following rule

(i) Maximin (ii) Minimax

Solution:

Strategy	States - of - nature		MINIMUM PAY OFF	MAXIMUM PAY OFF
	E_1	E_2		
S_1	40	60	40	60
S_2	10	-20	-20	10
S_3	-40	150	-40	150

(I) Max min Principle :

Max $(40, -20, -40) = 40$. Since the maximum pay-off is Rs. 40, the best strategy is S_1 according to maximin rule.

(ii) Minimax principle:

Min $(60,10,150) = 10$. Since the minimum pay- off is Rs. 10 , the best strategy is S_2 according to minimax rule.

Question 2.

A farmer wants to decide which of the three crops he should plant on his 100 -acre farm. The profit from each is dependent on the rainfall during the growing season. The farmer has categorized the amount of rainfall as high, medium and low. His estimated profit for each is shown in the table.

Rainfall	Estimated Conditional Profit (Rs.)		
	Crop A	Crop B	Crop C
High	**8000**	3500	5000
Medium	4500	4500	5000
Low	2000	5000	4000

If the farmer wishes to plant the only crop, decide which should be his best crop using (i) Maximin (ii) Minimax

Solution:

Estimated conditional profit (Rs.)	**Rainfall**			**Minimum PAY OFF**	**Maximum payoff**
	High	**Medium**	**Low**		
Crop A	**8000**	**4500**	**2000**	**2000**	**8000**
Crop B	**3500**	**4500**	**5000**	**5000**	**5000**
Crop C	**5000**	**5000**	**4000**	**4000**	**5000**

(i) Maxmn principle:

Max $(2000,3500,4000) = 4000$. Since the maximum profit is Rs. 4000 , he must choose crop C as the best crop.

(ii) Minimax principle:

Min $(8000,5000,5000) = 5000$. Since the minimum cost is Rs. 5000 , he can choose crop B and crop C as the best crop.

Question 3 .

The research department of Hindustan Ltd. has recommended paying the marketing department to launch a shampoo of three different types. The marketing types of shampoo to be launched under the following estimated pay-offs for various level of sales.

Types of shampoo	Estimated sales		
	15000	10000	5000
Egg shampoo	**30**	**10**	**10**
Clinic shampoo	**40**	**15**	**5**
Deluxe shampoo	**55**	**20**	**3**

What will be the marketing manager's decision if (i) Maximin and (ii) Minimax principle applied?

Solution:

Types of shampoo	Estimated sales			Min Pay off	Max pay off
	15000	10000	5000		
Egg shampoo	30	10	10	10	30
Clinic shampoo	40	15	5	5	40
Deluxe shampoo	55	20	3	3	55

<u>(i) Maximin principle</u>

Max $(10,5,3) = 10$. Since the maximum pay-off is 10 units, the marketing manager has to choose Egg shampoo by Maximin rule.

<u>(ii) Minimax principle</u>

Min $(30,40,55) = 30$. Since the minimum pay-off is 30 units, the marketing manager has to choose Egg shampoo by minimax rule.

Question 4.

Following pay-off matrix, which is the optimal decision under each of the following rule (i) Maximin (ii) Minimax

Act	States of nature			
	S_1	S_2	S_3	S_4
A_1	14	9	10	5
A_2	11	10	8	7
A_3	9	10	10	11
A_4	8	10	11	13

Solution:

Act	States of nature				Min pay-off	Max pay off
	S_1	S_2	S_3	S_4		
A_1	14	9	10	5	5	14
A_2	11	10	8	7	7	11
A_3	9	10	10	11	9	11
A_4	8	10	11	13	8	13

(i) Maximin principle

Max $(5,7,9,8) = 9$. Since the maximum pay-off is 9 , the optimal decision is A_3 according to maximin rule.

(ii) Minimax principle

Min $(14,11,11,13) = 11$. Since the minimum pay-off is 11 , the optimal decision A_2 and A_3 according to minimax rule.

5 MARKS

EXERCISE 10.1

Question 8.

Explain Vogel's approximation method by obtaining an initial feasible solution of the following transportation problem

	D_1	D_2	D_3	D_4	Supply
O_1	2	3	11	7	6
O_2	1	0	6	1	1
O_3	5	8	15	9	10
Demand	7	5	3	2	

Solution: Total supply = Total demand = 17

First allocation :

	D_1	D_2	D_3	D_4	a_i	Penalty
O_1	2	3	11	7	6	(1)
O_2	1	0	6	(1) 1	1/0	(1)
O_3	5	8	15	9	10	(3)
b_j	7	5	3	2/1		
Penalty	(1)	(3)	(5)	(6)		

The largest difference is 6 corresponding to column D_4 .
In this column least cost is (O_2, D_4). Allocate min(2,1) to this cell.

Second allocation:

	D_1	D_2	D_3	D_4	a_i	Penalty
O_1	2	(5) 3	11	7	6/1	(1)
O_3	5	8	15	9	10	(3)
b_j	7	5/0	3	1		
Penalty	(3)	(5)	(4)	(2)		

The largest difference is 5 in column D_2. Here the least cost is (O_1, D_2). So allocate min (5,6) to this cell.

Third allocation:

	D_1	D_3	D_4	a_i	Penalty
O_1	(1) 2	11	7	1/0	(5)
O_3	5	15	9	10	(4)
b_j	7/6	3	1		
Penalty	(3)	(4)	(2)		

The largest penalty is 5 in row O_1. The least cost is in (O_1, D_1). So allocate min(7,1) here.

Fourth allocation:

	D_1	D_3	D_4	a_i	Penalty
O_3	(6) 5	15	9	10/4	(4)
b_j	6/0	3	1		
Penalty	-	-	-		

Fifth allocation:

	D_3	D_4	a_i	Penalty
O_3	15	9	4/3/0	(6)
b_j	3/0	1/0		
Penalty	-	-		

We allocate min(1,4) to (O_3, D_4) cell since it has the least cost. Finally the balance we allot to cell (O_3, D_3).Thus we have the following allocations:

	D_1	D_2	D_3	D_4	a_i
O_1	(1) 2	(5) 3	11	7	6
O_2	1	0	6	(1) 1	1
O_3	(6) 5	8	(3) 15	(1) 9	10
b_j	7	5	3	2	

Transportation schedule:

$O_1 \to D_1, O_1 \to D_2, O_2 \to D_4, O_3 \to D_1, O_3 \to D_3, O_3 \to D_4$

(i.e) $x_{11} = 12, x_{12} = 5, x_{24} = 1, x_{31} = 6, x_{33} = 3, x_{34} = 1$

Total cost $= (1 \times 2) + (5 \times 3) + (1 \times 1) + (6 \times 5) + (3 \times 15) + (1 \times 9)$

$= 2 + 15 + 1 + 30 + 45 + 9 = 102$

Question 9.

Consider the following transportation problem.

	D_1	D_2	D_3	D_4	Availability
O_1	5	8	3	6	30
O_2	4	5	7	4	50
O_3	6	2	4	6	20
Requirement	30	40	20	10	

Determine initial basic feasible solution by VAM
Solution:
Total availability = Total requirement = 100
First allocation:

	D_1	D_2	D_3	D_4	a_i	Penalty
O_1	5	8	3	6	30	(2)
O_2	4	5	7	4	50	(1)
O_3	6	(20) 2	4	6	20/0	(2)
b_j	30	40/20	20	10		
Penalty	(1)	(3)	(1)	(2)		

Largest penalty = 3. allocate min(40,20) to (O_3, D_2)

Second allocation:

	D_1	D_2	D_3	D_4	a_i	Penalty
O_1	5	8	(20) 3	6	30/10	(2)
O_2	4	5	7	4	50	(1)
b_j	30	20	20/0	10		
Penalty	(1)	(3)	(4)	(2)		

The largest penalty is 4 . Allocate min(20,30) to (O_1, D_3)

Third allocation :

	D_1	D_2	D_4	a_i	Penalty
O_1	5	8	6	10	(1)
O_2	4	(20) 5	4	50/30	(1)
b_j	30	20/0	10		
Penalty	(1)	(3)	(2)		

The largest penalty is 3 . Allocate min(20,50) to (O_2, D_2)

Fourth allocation:

	D_1	D_4	a_i	Penalty
O_1	5	6	10	(1)
O_2	4	(10) 4	30/20	(0)
b_j	30	10/0		
Penalty	(1)	(2)		

The largest penalty is 2 , Allocate min(10,30) to (O_2, D_4)

Fifth allocation:

	D_1	a_i	Penalty
O_1	(10) 5	10/0	-
O_2	(20) 4	20/0	-
b_j	30/10/0		
Penalty	(1)		

The largest penalty is 1 . Allocate min(30,20) to (O_2, D_1)

Balance 10 units we allot to (O_1, D_1).

Thus we have the following allocations:

	D_1	D_2	D_3	D_4	a_i
O_1	(10) 5	8	(20) 3	6	30
O_2	(20) 4	(20) 5	7	(10) 4	50
O_3	6	(20) 2	4	6	20
b_j	30	40	20	10	

Transportation schedule:

$O_1 \to D_1, O_1 \to D_3, O_2 \to D_1, O_2 \to D_2, O_2 \to D_4, O_3 \to P_2$
(i.e) $x_{11} = 10, x_{13} = 20, x_{21} = 20, x_{22} = 20, x_{24} = 10, x_{32} = 20$

Total cost $= (10 \times 5) + (20 \times 3) + (20 \times 4)$

$+(20 \times 5) + (10 \times 4) + (20 \times 2)$

$= 50 + 60 + 80 + 100 + 40 + 40 = 370$

Thus the least cost by YAM is Rs. 370 .

Question 11.

Find the initial basic feasible solution of the following transportation problem:

	I	II	III	Demand
A	**1**	**2**	**6**	**7**
B	**0**	**4**	**2**	**12**
C	**3**	**1**	**5**	**11**
Supply	**10**	**10**	**10**	

Using (i) North West Corner rule (ii) Least Cost method

(iii) Vogel's approximation method

Solution: Total demand = total supply = 30.

(i) North West Corner rule

First allocation :

	I	II	III	a_i
A	(7) 1	2	6	7/0
B	0	4	2	12
C	3	1	5	11
b_j	10/3	10	10	

Second allocation :

	I	II	III	a_i
B	(3) 0	4	2	12/9
C	3	1	5	11
b_j	3/0	10	10	

Third allocation :

	II	III	a_i
B	(9) 4	2	9/0
C	1	5	11
b_j	10/1	10	

Fourth allocation :

	II	III	a_i
C	(1) 1	(10) 5	11/10/0
b_j	1/0	10/0	

We first allot 1 unit to (C, II) cell and then the balance 10 units to (C, III) cell.

Thus we have the following allocations:

	I	II	III	Demand
A	(7) 1	2	6	7
B	(3) 0	(9) 4	2	12
C	3	(1) 1	(10) 5	11
Supply	10	10	10	

Transportation schedule: A → I, B → I, B → II, C → II, C → III

(i.e) $x_{11} = 7, x_{21} = 3, x_{22} = 9, x_{32} = 1, x_{33} = 10$

Total cost $= (7 \times 1) + (3 \times 0) + (9 \times 4) + (1 \times 1) + (10 \times 5)$

$= 7 + 0 + 36 + 1 + 50 =$ Rs. 94

(ii) Least Cost method: Total demand = total supply = 30.

First allocation:

	I	II	III	a_i
A	1	2	6	7
B	(10) 0	4	2	12/2
C	3	1	5	11
b_j	10/0	10	10	

Second allocation :

	II	III	a_i
A	2	6	7
B	4	2	2
C	(10) 1	5	11/1
b_j	10/0	10	

Third allocation :

	III	a_i
A	6	7
B	(2) 2	2/0
C	5	1
b_j	10/8	

Fourth allocation :

	III	a_i
A	(7) 6	7/0
C	(1) 5	1/0
b_j	8/7/0	

We first allot 1 unit to cell (C, III) and the balance 7 units to cell (A, III).

Thus we have the following allocations:

	I	II	III	Demand
A	1	2	(7) 6	7
B	(10) 0	4	(2) 2	12
C	3	(10) 1	(1) 5	11
Supply	10	10	10	

Transportation schedule:

A → III, B → I, B → III, C → II, C → III

(i.e) $x_{13} = 7, x_{21} = 10, x_{23} = 2, x_{32} = 10, x_{33} = 1$

Total cost = $(7 \times 6) + (10 \times 0) + (2 \times 2) + (10 \times 1) + (1 \times 5)$

$= 42 + 0 + 4 + 10 + 5 =$ Rs. 61

(iii) Vogel's approximation method (VAM)

Total demand = total supply = 30.

First allocation:

	I	II	III	a_i	Penalty
A	1	2	6	7	(1)
B	0	4	(10) 2	12/2	(2)
C	3	1	5	11	(2)
b_j	10	10	10/0		
Penalty	(1)	(1)	(3)		

Largest penalty = 3. Allocate min (10, 12) to (B, III)

Second allocation:

	I	II	III	a_i	Penalty
A	1	2	6	7	(1)
B	0	4	(10) 2	12/2	(2)
C	3	1	5	11	(2)
b_j	10	10	10/0		
Penalty	(1)	(1)	(3)		

Largest penalty = 4. Allocate min (10, 2) to cell (B, I)

Third allocation:

	I	II	a_i	Penalty
A	1	2	7	(1)
B	(2) 0	4	2/0	(4)
C	3	1	11	(2)
b_j	10/8	10		
Penalty	(1)	(1)		

The largest penalty is 2. We can choose I column or C row.
Allocate min (8, 7) to cell (A, I)

Fourth allocation:

	I	II	a_i	Penalty
A	(7) 1	2	7/0	(1)
C	3	1	11	(2)
b_j	8/1	10		
Penalty	(2)	(1)		

First, we allocate 10 units to cell (C, II). Then balance 1 unit we allot to cell (C, I); Thus we have the following allocations:

	I	II	a_i	Penalty
C	(1) 3	(10) 1	11/1/0	(2)
b_j	1/0	10/0		
Penalty	-	-		

Final allocation

	I	II	III	Demand
A	(7) 1	2	6	7
B	(2) 0	4	(10) 2	12
C	(1) 3	(10) 1	5	11
Supply	10	10	10	

A → I, B → I, B → III, C → I, C → II

(i.e) $x_{11} = 7, x_{21} = 2, x_{23} = 10, x_{31} = 1, x_{32} = 10$

Total cost $= (7 \times 1) + (2 \times 0) + (10 \times 2) + (1 \times 3) + (10 \times 1) = 7 + 0 + 20 + 3 + 10 =$ Rs. 40

EXERCISE 10.2

Question 5.

A computer centre has got three expert programmers. The centre needs three application programmes to be developed. The head of the computer centre, after studying carefully the programmes to be developed, estimates the computer time in minutes required by the experts to the application programme as follows. Now all the subordinates have been assigned tasks. The optimal assignment schedule and the total cost is

		Programmes		
		P	Q	R
Programmers	1	120	100	80
	2	80	90	110
	3	110	140	120

Assign the programmers to the programme in such a way that the total computer time is least.

Solution: Here the number of rows equals the number of columns.

Step 1:

		Programmes		
		P	**Q**	**R**
Programmer	**1**	40	20	0
	2	0	10	30
	3	0	30	10

Step 2 :

		Programmes		
		P	**Q**	**R**
Programmer	**1**	40	10	0
	2	0	0	30
	3	0	20	10

Step 3 : (Assignment)

		Programmes		
		P	**Q**	**R**
Programmer	**1**	40	10	[0]
	2	~~0~~	[0]	30
	3	[0]	20	10

Now all the 3 programmes have been assigned to the programmers.
The optimal assignment schedule and the total cost is
The optimal assignment (minimum) cost is ₹ 280.

Programmer	Programme	Cost
1	R	80
2	Q	90
3	P	110
TOTAL COST		280

Question 6. A departmental head has four subordinates and four tasks to be performed. The subordinates differ inefficiency and the tasks differ in their intrinsic difficulty. His estimates of the time each man would take to perform each task is given below
How should the tasks be allocated to subordinates so as to minimize the total man-hours?

		Tasks			
		1	2	3	4
Subordinates	P	8	26	17	11
	Q	13	28	4	26
	R	38	19	18	15
	S	9	26	24	10

Solution: A number of tasks equal the number of subordinates.
Step 1: Subtract minimum hours of each row from other elements of that row.

		Tasks			
		1	2	3	4
Subordinates	P	0	18	9	3
	Q	9	24	0	22
	R	23	4	3	0
	S	0	17	15	1

Since column 2 has no zero , proceed further

Step 2 :

		Tasks			
		1	2	3	4
Subordinates	P	0	14	9	3
	Q	9	20	0	22
	R	23	0	3	0
	S	0	13	15	1

We can proceed with the assignment since all the rows and columns have zeros .
Step 3 : (Assignment)

	1	2	3	4
P	[0]	14	9	3
Q	9	20	[0]	22
R	23	[0]	3	~~0~~
S	~~0~~	13	15	1

Now there is no zero in row S . So we proceed as below

Step 4:

	1	2	3	4
P	[0]	14	9	3
Q	9	20	[0]	22
R	23	[0]	3	~~0~~
S	~~0~~	13	15	1

We have drawn the minimum number of lines to cover all the zeros in the reduced matrix obtained. The smallest element from all the uncovered elements is 1. We subtract this from all the uncovered elements and add them to the elements which lie at the intersection of two lines. Thus we obtain another reduced problem for fresh assignment.

	1	2	3	4
P	[0]	13	8	3
Q	10	20	[0]	22
R	24	[0]	3	~~0~~
S	~~0~~	12	14	[0]

Now all the subordinates have been assignment tasks .

Subordinates	P	Q	R	S
Task	1	3	2	4
No. Of hours	8	4	19	10

Total = 41

Question 7.

Find the optimal solution for the assignment problem with the following cost matrix.

Area

		1	2	3	4
	P	11	17	8	16
Salesman	Q	9	7	12	6
	R	13	16	15	12
	S	14	10	12	11

Solution: Number of rows = Number of columns

Step 1:

Area

		1	2	3	4
	P	3	9	0	8
Salesman	Q	3	1	6	0
	R	1	4	3	0
	S	4	0	2	1

Step 2 :

Area

		1	2	3	4
	P	2	9	0	8
Salesman	Q	2	1	6	0
	R	0	4	3	0
	S	3	0	2	1

Step 3 : (Assignment)

Area

		1	2	3	4
	P	2	9	[0]	8
Salesman	Q	2	1	6	[0]
	R	[0]	4	3	~~0~~
	S	3	[0]	2	1

Now all the salesmen have been assigned areas. The optimal assignment schedule and the total cost is

Salesman	Area	Cost
P	3	8
Q	4	6
R	1	13
S	2	10
Total hours		37

Thus the optimal cost is Rs 37.

Question 8.

Assign four trucks 1, 2, 3 and 4 to vacant spaces A, B, C, D, E and F so that distance travelled is minimized. The matrix below shows the distance.

	1	2	3	4
A	4	7	3	7
B	8	2	5	5
C	4	9	6	9
D	7	5	4	8
E	6	3	5	4
F	6	8	7	3

Solution:

Here the number of trucks is 4 and vacant spaces are 6 . So the given assignment problem is the unbalanced problem. So we introduce two dummy columns with all the entries zero to make is balanced. So the problem is

(Trucks)

		1	2	3	4	d	d
(spaces)	A	4	7	3	7	0	0
	B	8	2	5	5	0	0
	C	4	9	6	9	0	0
	D	7	5	4	8	0	0
	E	6	3	5	4	0	0
	F	6	8	7	3	0	0

Here only 4 vacant spaces can be assigned to four trucks

Step 1 : Not necessary since all rows have zeros

Step 2 : (Trucks)

		1	2	3	4	d	d
(spaces)	A	0	5	0	4	0	0
	B	4	0	2	2	0	0
	C	0	7	3	6	0	0
	D	3	3	1	5	0	0
	E	2	1	2	1	0	0
	F	2	6	4	0	0	0

Step 3 : (Assignment)

(Trucks)

		1	2	3	4	d	d
	A	~~0~~	5	[0]	4	~~0~~	~~0~~
	B	4	[0]	2	2	~~0~~	~~0~~
(Spaces)	C	[0]	7	3	6	~~0~~	~~0~~
	D	3	3	1	5	0	0
	E	2	1	2	1	0	0
	F	2	6	4	[0]	~~0~~	~~0~~

The optimal assignment schedule and total distance travelled is

Vacant space	Truck	Distance
A	3	3
B	2	2
C	1	4
D	d	0
E	d	0
F	4	3
Total		12

Thus the minimum distance travelled is 12km

CHAPTER – I

1. If A=(1 2 3), then the rank of AA^T is **Ans : 1**

2. The rank of m × n matrix whose elements are unity is **Ans : 1**

3. If $T = \begin{matrix} A \\ B \end{matrix}\begin{pmatrix} 0.4 & 0.6 \\ 0.2 & 0.8 \end{pmatrix}$ is a transition probability matrix, then at equilibrium A is equal to **Ans :** ¼

4. If $A = \begin{pmatrix} 2 & 0 \\ 0 & 8 \end{pmatrix}$, then ρ(A) is **Ans : 2**

5. The rank of the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$ is **Ans : 3**

6. The rank of the unit matrix of order n is **Ans : n**

7. If ρ (A) = r then which of the following is correct? **Ans :** A has at least one minor of order r which does not vanish

8. If $A = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ then the rank of AA^T is **Ans : 1**

9. If the rank of the matrix $\begin{pmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -1 & 0 & \lambda \end{pmatrix}$ is 2. Then λ is **Ans : 1**

10. The rank of the diagonal matrix $\begin{pmatrix} \mathbf{1} & & & & & \\ & \mathbf{2} & & & & \\ & & \mathbf{-3} & & & \\ & & & \mathbf{0} & & \\ & & & & \mathbf{0} & \\ & & & & & \mathbf{0} \end{pmatrix}$ **Ans : 3**

11. If $T = \begin{matrix} A \\ B \end{matrix}\begin{pmatrix} 0.7 & 0.3 \\ 0.6 & x \end{pmatrix}$ is a transition probability matrix, then the value of x is **Ans : 0.4**

12. Which of the following is not an elementary transformation? **Ans :** $R_i \to 2R_i + 2C_j$

13. If ρ(A) = ρ(A, B) then the system is **Ans : Consistent**

14. If ρ(A) = ρ(A, B)= the number of unknowns, then the system is **Ans :** Consistent and has a unique solution

15. If ρ (A) ≠ ρ (A, B) , then the system is **Ans :** inconsistent

16. In a transition probability matrix, all the entries are greater than or equal to **Ans : 0**

17. If the number of variables in a non- homogeneous system AX = B is n, then the system possesses a unique solution only when

Ans : ρ (A) = ρ (A, B) = n

18. The system of equations 4x + 6 y = 5 , 6x + 9 y = 7 has **Ans :** no solution

19. For the system of equations x + 2 y + 3z = 1 , 2x + y + 3z = 2 , 5x + 5y + 9z = 4 **Ans :** there is only one solution

20. If |A| ≠ 0, then A is **Ans :** non- singular matrix

21. The system of linear equations x + y + z = 2 , 2x + y − z = 3 , 3x + 2 y + k = 4 has unique solution, if k is not equal to **Ans : 0**

22. Cramer's rule is applicable only to get an unique solution when **Ans :** Δ ≠ 0

23. If $\frac{a_1}{x} + \frac{b_1}{y} = c_1$, $\frac{a_2}{x} + \frac{b_2}{y} = c_2$, $\Delta_1 = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$; $\Delta_2 = \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$; $\Delta_3 = \begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}$ then (x, y) is **Ans :** $\left(\frac{-\Delta_1}{\Delta_2}, \frac{-\Delta_1}{\Delta_3}\right)$

24. $|A_{n\times n}| = 3$ |adjA| = 243 then the value n is **Ans : 6**

25. Rank of a null matrix is **Ans : 0**

CHAPTER - II

1. $\int \frac{1}{x^3} dx$ is **Ans :** $\frac{-1}{-2x^2} + c$

2. $\int 2^x dx$ is **Ans :** $\frac{2^x}{\log 2} + c$

3. $\int \frac{\sin 2x}{2\sin x} dx$ is **Ans :** $\sin x + c$

4. $\int \frac{\sin 5x - \sin x}{\cos 3x} dx$ is **Ans :** $-\cos 2x + c$

5. $\int \frac{\log x}{x} dx, x > 0$ is **Ans :** $\frac{1}{2}(\log x)^2 + c$

6. $\int \frac{e^x}{\sqrt{1+e^x}} dx$ is **Ans :** $2\sqrt{1+e^x} + c$

7. $\int \sqrt{e^x} dx$ is **Ans :** $2\sqrt{e^x} + c$

8. $\int e^{2x}[2x^2 + 2x]dx$ **Ans :** $e^{2x}x^2 + c$

9. $\int \frac{e^x}{e^x+1} dx$ is **Ans :** $\log|e^x + 1| + c$

10. $\int \left[\frac{9}{x-3} - \frac{1}{x+1}\right] dx$ is **Ans :** $9\log|x-3| - \log|x+1| + c$

11. $\int \frac{2x^3}{4+x^4} dx$ is **Ans :** $\frac{1}{2}\log|4+x^4| + c$

12. $\int \frac{dx}{\sqrt{x^2-36}}$ is **Ans :** $\log|x + \sqrt{x^2-36}| + c$

13. $\int \frac{2x+3}{\sqrt{x^2+3x+2}} dx$ is **Ans :** $2\sqrt{x^2+3x+2} + c$

14. $\int_0^1 (2x+1)\, dx$ is **Ans : 2**

15. $\int_2^4 \frac{dx}{x}$ is **Ans : log 2**

16. $\int_0^\infty e^{-2x} dx$ is **Ans :** ½

17. $\int_{-1}^1 x^3 e^{x^4} dx$ is **Ans : 0**

18. If f(x) is a continuous function and a < c < b , then $\int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ is **Ans :** $\int_a^b f(x)\,dx$

19. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\, dx$ is **Ans : 2**

20. $\int_0^1 \sqrt{x^4(1-x)^2} dx$ is **Ans :** 1 /12

21. If $\int_0^1 f(x)\,dx = 1$, $\int_0^1 x f(x) = a$ and $\int_0^1 x^2 f(x)dx = a^2$, $\int_0^1 (a-x)^2 f(x)dx$ is **Ans : 0**

22. The value of $\int_2^3 f(5-x)dx - \int_2^3 f(x)\,dx$ is **Ans : 0**

23. $\int_0^4 \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$ is **Ans :** 28/3

24. $\int_0^{\frac{\pi}{3}} \tan x\, dx$ is **Ans : log 2**

25. Using the factorial representation of the gamma function, which of the following is the solution for the gamma function Γ(n) when n = 8 **Ans : 5040**

26. Γ(n) is **Ans :** (n – 1)!

27. Γ(1) is **Ans : 1**

28. If n > 0, then Γ(n) is **Ans :** $\int_0^\infty e^{-x}x^{n-1}\,dx$

29. $\Gamma\left(\frac{3}{2}\right)$ **Ans :** $\frac{\sqrt{\pi}}{2}$

30. $\int_0^\infty x^4 e^{-x} dx$ is **Ans :** 4!

CHAPTER - III

1. Area bounded by the curve $y = x(4 - x)$ between the limits 0 and 4 with x – axis is **Ans :** 32/3 sq.units

2. Area bounded by the curve $y = e^{-2x}$ between the limits $0 \le x \le \infty$ is **Ans :** ½ sq.unit

3. Area bounded by the curve $y = \frac{1}{x}$ between the limits 1 and 2 is **Ans :** log2 sq.units

4. If the marginal revenue function of a firm is $MR = e^{\frac{-x}{10}}$, then revenue is **Ans :** $10\left(1 - e^{\frac{-x}{10}}\right)$

5. If MR and MC denotes the marginal revenue and marginal cost functions, then the profit functions is **Ans :** $P = \int (MR - MC)\,dx + k$

6. The demand and supply functions are given by $D(x) = 16 - x^2$ and $S(x) = 2x^2 + 4$ are under perfect competition, then the equilibrium price x is **Ans : 2**

7. The marginal revenue and marginal cost functions of a company are $MR = 30 - 6x$ and $MC = -24 + 3x$ where x is the product, then the profit function is **Ans :** $54x - \frac{9x^2}{2} + k$

8. The given demand and supply function are given by $D(x) = 20 - 5x$ and $S(x) = 4x + 8$ if they are under perfect competition then the equilibrium demand is **Ans :** 40/3

9. If the marginal revenue $MR = 35 + 7x - 3x^2$, then the average revenue AR is **Ans :** $35 + \frac{7x}{2} - x^2$

10. The profit of a function p(x) is maximum when **Ans :** $MC - MR = 0$

11. For the demand function p(x), the elasticity of demand with respect to price is unity then **Ans :** revenue is constant

12. The demand function for the marginal function $MR = 100 - 9x^2$ is **Ans :** $100 - 3x^2$

13. When $x_0 = 5$ and $p_0 = 3$ the consumer's surplus for the demand function $p_d = 28 - x^2$ is **Ans :** 250/3 units

14. When $x_0 = 2$ and $p_0 = 12$ the producer's surplus for the supply function $p_s = 2x^2 + 4$ is **Ans :** 32/3 units

15. Area bounded by $y = x$ between the lines $y = 1$, $y = 2$ with y = axis is **Ans :** 3/2 sq.units

16. The producer's surplus when the supply function for a commodity is $P = 3 + x$ and $x_0 = 3$ is **Ans :** 9/2

17. The marginal cost function is $MC = 100\sqrt{x}$. find AC given that TC =0 when the out put is zero is **Ans :** $\frac{200}{3}x^{\frac{1}{2}}$

18. The demand and supply function of a commodity are $P(x) = (x - 5)^2$ and $S(x) = x^2 + x + 3$ then the equilibrium quantity x_0 is **Ans : 2**

19. The demand and supply function of a commodity are $D(x) = 25 - 2x$ and $S(x) = \frac{10+x}{4}$ then the equilibrium price P0 is **Ans : 5**

20. If MR and MC denote the marginal revenue and marginal cost and $MR - MC = 36x - 3x^2 - 81$, then the maximum profit at x is equal to **Ans : 9**

21. If the marginal revenue of a firm is constant, then the demand function is **Ans : MR**

22. For a demand function p, if $\int \frac{dp}{p} = k \int \frac{dx}{x}$ then k is equal to **Ans :** $\frac{-1}{\eta_d}$

23. Area bounded by $y = e^x$ between the limits 0 to 1 is **Ans :** $(e - 1)$ sq.units

24. The area bounded by the parabola $y^2 = 4x$ bounded by its latus rectum is **Ans :** 8/3 sq.units

25. Area bounded by $y = |x|$ between the limits 0 and 2 is **Ans :** 2 sq.units

CHAPTER – IV

1. The degree of the differential equation $\frac{d^4y}{dx^4} - \left(\frac{d^2y}{dx^2}\right)^4 + \frac{dy}{dx} = 3$ **Ans : 1**

2. The order and degree of the differential equation $\sqrt{\frac{d^2y}{dx^2}} = \sqrt{\frac{dy}{dx} + 5}$ are respectively **Ans :** 2 and 1

3. The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}} - \sqrt{\left(\frac{dy}{dx}\right)} - 4 = 0$ are respectively. **Ans :** 2 and 6

4. The differential equation $\left(\frac{dx}{dy}\right)^3 + 2y^{\frac{1}{2}} = x$ is **Ans :** of order 1 and degree 3

5. The differential equation formed by eliminating a and b from $y=ae^x + be^{-x}$ is **Ans :** $\frac{d^2y}{dx^2} - y = 0$

6. If $y = cx + c - c^3$ then its differential equation is **Ans :** $y = x\frac{dy}{dx} + \frac{dy}{dx} - \left(\frac{dy}{dx}\right)^3$

7. The integrating factor of the differential equation $\frac{dx}{dy} + Px = Q$ is **Ans :** $e^{\int Pdy}$

8. The complementary function of $(D^2 + 4)\, y = e^{2x}$ is **Ans :** A cos 2x + B sin 2x

9. The differential equation of y = mx + c is (m and c are arbitrary constants) **Ans :** $\frac{d^2y}{dx^2} = 0$

10. The particular integral of the differential equation is $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 16y = 2e^{4x}$ **Ans :** x^2e^{4x}

11. Solution of $\frac{dx}{dy} + Px = 0$ **Ans :** $x = ce^{-py}$

12. If $\sec^2 x$ is an integrating factor of the differential equation $\frac{dy}{dx} + Py = Q$ then P = **Ans :** 2 tan x

13. The integrating factor of $x\frac{dy}{dx} - y = x^2$ is **Ans :** $\frac{1}{x}$

14. The solution of the differential equation $\frac{dy}{dx} + Py = Q$ where P and Q are the function of x is **Ans :** $ye^{\int Pdx} = \int Q\, e^{\int Pdx}dx + c$

15. The differential equation formed by eliminating A and B from $y = e^{-2x}(A\cos x + B\sin x)$ is **Ans :** $y_2 + 4y_1 + 5 = 0$

16. The particular integral of the differential equation $f(D)y = e^{ax}$ where $f(D) = (D - a)^2$ **Ans :** $\frac{x^2}{2}e^{ax}$

17. The differential equation of $x^2 + y^2 = a^2$ **Ans :** xdx+ydy=0

18. The complementary function of $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$ is **Ans :** $A + Be^x$

19. The P.I of $(3D^2 + D - 14)y = 13e^{2x}$ is **Ans :** xe^{2x}

20. The general solution of the differential equation $\frac{dy}{dx} = \cos x$ is **Ans :** y = sin x + c, c is an arbitrary constant

21. A homogeneous differential equation of the form $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$ can be solved by making substitution, **Ans :** y = v x

22. A homogeneous differential equation of the form $\frac{dx}{dy} = f\left(\frac{x}{y}\right)$ can be solved by making substitution, **Ans :** x = v y

23. The variable separable form of $\frac{dy}{dx} = \frac{y(x-y)}{x(x+y)}$ by taking y = vx and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ is **Ans :** $\frac{1+v}{2v^2}dv = -\frac{dx}{x}$

24. Which of the following is the homogeneous differential equation? **Ans :** $y^2dx + (x^2 - xy - y^2)\, dy = 0$

25. The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{f\left(\frac{y}{x}\right)}{f'\left(\frac{y}{x}\right)}$ is **Ans :** $f\left(\frac{y}{x}\right) = kx$

CHAPTER - V

1. $\Delta^2 y_0$ = **Ans :** $y_2 - 2y_1 + y_0$

2. Δ f (x) = **Ans :** f (x + h) – f (x)

3. E ≡ **Ans :** 1 + Δ

4. If h=1, then $\Delta (x^2)$ = **Ans :** 2x + 1

5. If c is a constant then Δ c = **Ans : 0**

6. If m and n are positive integers then $\Delta^m \Delta^n f(x)$ = **Ans :** $\Delta^{m+n} f(x)$

7. If 'n' is a positive integer $\Delta^n [\Delta^{-n} f(x)]$ **Ans :** f (x)

8. E f (x) = **Ans :** f (x + h)

9. ∇ ≡ **Ans :** $1 - E^{-1}$

10. ∇ f(a) = **Ans :** f(a) – f(a – h)

11. For the given points (x_0 , y_0) and (x_1 , y_1) the Lagrange's formula is **Ans :** $y(x) = \frac{x-x_1}{x_0-x_1} y_0 + \frac{x-x_0}{x_1-x_0} y_1$

12. Lagrange's interpolation formula can be used for **Ans :** both equal and unequal intervals

13. If f(x) = x2 + 2x + 2 and h=1 the interval of differencing is unity then Δ f(x) **Ans :** 2x + 3

14. For the given data find the value of $\Delta^3 y_0$ is **Ans : 0**

x	5	6	9	11
y	12	13	15	18

CHAPTER VI

1. Value which is obtained by multiplying possible values of random variable with probability of occurrence and is equal to weighted average is called **Ans :** Expected value

2. Demand of products per day for three days are 21, 19, 22 units and their respective probabilities are 0.29, 0.40, 0.35. Pofit per unit is 0.50 paisa then expected profits for three days are **Ans :** 3.045, 3.8, 3.85

3. Probability which explains x is equal to or less than particular value is classified as **Ans :** cumulative probability

4. Given E(X) = 5 and E(Y) = –2, then E(X – Y) is **Ans : 7**

5. A variable that can assume any possible value between two points is called **Ans :** continuous random variable

6. A formula or equation used to represent the probability distribution of a continuous random variable is called **Ans :** probability density function

7. If X is a discrete random variable and p(x) is the probability of X , then the expected value of this random variable is equal to **Ans :** $\sum xp(x)$

8. Which of the following is not possible in probability distribution? **Ans :** p(x) = – 0.5

9. If c is a constant, then E(c) is **Ans : c**

10. A discrete probability distribution may be represented by **Ans :** table , graph , mathematical equation

11. A probability density function may be represented by: **Ans :** graph , mathematical equation

12. If c is a constant in a continuous probability distribution, then p(x = c) is always equal to **Ans :** zero

13. E [X – E (X)] is equal to **Ans : 0**

14. E $[X - E(X)]^2$ is **Ans :** V (X)

15. If the random variable takes negative values, then the negative values will have **Ans :** positive probabilities

16. If we have f(x) = 2 x , 0 ≤ x ≤1 , then f (x) is a **Ans :** probability density function

17. $\int_{-\infty}^{\infty} f(x)dx$ is always equal to **Ans :** one

18. A listing of all the outcomes of an experiment and the probability associated with each outcome is called **Ans :** probability distribution

19. Which one is not an example of random experiment? **Ans :** All medical insurance claims received by a company in a given year.

20. A set of numerical values assigned to a sample space is called **Ans :** random variable

21. A variable which can assume finite or countably infinite number of values is known as **Ans :** discrete

22. The probability function of a random variable is defined as

X=x	-1	-2	0	1	2
P(x)	k	2k	3k	4k	5k

Then k is equal to **Ans :** 1/15

23. If $p(x) = \frac{1}{10}$, x = 10, then E(X) is **Ans : 1**

24. A discrete probability function p(x) is always **Ans :** non-negative

25. In a discrete probability distribution the sum of all the probabilities is always equal to **Ans :** one

26. An expected value of a random variable is equal to it's **Ans :** mean

27. A discrete probability function p(x) is always non-negative and always lies between **Ans :** 0 and 1

28. The probability density function p(x) cannot exceed **Ans :** one

29. The height of persons in a country is a random variable of the type **Ans :** continuous random variable

30. The distribution function F(x) is equal to **Ans :** P (X ≤ x)

CHAPTER - VII

1. Normal distribution was invented by **Ans :** De-Moivre
2. If X ~N(9,81) the standard normal variate Z will be **Ans :** $Z = \frac{X-9}{9}$
3. If Z is a standard normal variate, the proportion of items lying between Z = –0.5 and Z = –3.0 is **Ans :** 0.3072
4. If X ~N(μ, σ2), the maximum probability at the point of inflexion of normal distribution is **Ans :** $\left(\frac{1}{\sigma\sqrt{2\pi}}\right) e^{-\frac{1}{2}}$
5. In a parametric distribution the mean is equal to variance is : **Ans :** poisson
6. In turning out certain toys in a manufacturing company, the average number of defectives is 1%. The probability that the sample of 100 toys there will be 3 defectives is **Ans :** 0.0613
7. The parameters of the normal distribution $f(x) = \left(\frac{1}{\sqrt{72\pi}}\right)\frac{e^{-(x-10)^2}}{72}$ $-\infty < x < \infty$ **Ans :** (10,36)
8. A manufacturer produces switches and experiences that 2 per cent switches are defective. The probability that in a box of 50 switches, there are atmost two defective is : **Ans :** $2.5\ e^{-1}$
9. An experiment succeeds twice as often as it fails. The chance that in the next six trials, there shall be at least four successes is **Ans :** 496/729
10. If for a binomial distribution b(n,p) mean = 4 and variance = 4/3, the probability, P(X ≥ 5) is equal to : **Ans :** 4(2/3)6
11. The average percentage of failure in a certain examination is 40. The probability that out of a group of 6 candidates atleast 4 passed in the examination are : **Ans :** 0.5443
12. Forty percent of the passengers who fly on a certain route do not check in any luggage. The planes on this route seat 15 passengers. For a full flight, what is the mean of the number of passengers who do not check in any luggage? **Ans :** 6.00
13. Which of the following statements is/are true regarding the normal distribution curve?

    Ans :

    (a) it is symmetrical and bell shaped curve
    (b) it is asymptotic in that each end approaches the horizontal axis but never reaches it
    (c) its mean, median and mode are located at the same point
14. Which of the following cannot generate a Poisson distribution?

    Ans : The number of customers arriving at a petrol station
15. The random variable X is normally distributed with a mean of 70 and a standard deviation of 10. What is the probability that X is between 72 and 84? **Ans :** 0.340
16. The starting annual salaries of newly qualified chartered accountants (CA's) in South Africa follow a normal distribution with a mean of ₹ 180,000 and a standard deviation of ₹ 10,000. What is the probability that a randomly selected newly qualified CA will earn between ₹ 165,000 and ₹ 175,000 per annum? **Ans :** 0.242
17. In a large statistics class the heights of the students are normally distributed with a mean of 172cm and a variance of 25cm. What proportion of students are between 165cm and 181cm in height? **Ans :** 0.883
18. A statistical analysis of long-distance telephone calls indicates that the length of these calls is normally distributed with a mean of 240 seconds and a standard deviation of 40 seconds. What proportion of calls lasts less than 180 seconds? **Ans :** 0.067
19. Cape town is estimated to have 21% of homes whose owners subscribe to the satelite service, DSTV. If a random sample of your home in taken, what is the probability that all four home subscribe to DSTV? **Ans :** 0.0019
20. Using the standard normal table, the sum of the probabilities to the right of z = 2.18 and to the left of z = -1.75 is: **Ans :** 0.0547
21. The time until first failure of a brand of inkjet printers is normally distributed with a mean of 1,500 hours and a standard deviation of 200 hours. What proportion of printers fails before 1000 hours? **Ans :** 0.0062
22. The weights of newborn human babies are normally distributed with a mean of 3.2kg and a standard deviation of 1.1kg. What is the probability that a randomly selected newborn baby weighs less than 2.0kg? **Ans :** 0.138
23. Monthly expenditure on their credit cards, by credit card holders from a certain bank, follows a normal distribution with a mean of ₹ 1,295.00 and a standard deviation of ₹ 750.00. What proportion of credit card holders spend more than ₹ 1,500.00 on their credit cards per month? **Ans :** 0.392
24. Let z be a standard normal variable. If the area to the right of z is 0.8413, then the value of z must be: **Ans :** –1.00
25. If the area to the left of a value of z (z has a standard normal distribution) is 0.0793, what is the value of z? **Ans :** -1.41
26. If P(Z > z) = 0.8508 what is the value of z (z has a standard normal distribution)? **Ans :** –1.04
27. If P(Z > z) = 0.5832 what is the value of z (z has a standard normal distribution)? **Ans :** –0.21
28. In a binomial distribution, the probability of success is twice as that of failure. Then out of 4 trials, the probability of no success is **Ans :** 1/81

CHAPTER – VIII

1. A ………… may be finite or infinite according as the number of observations or items in it is finite or infinite. **Ans :** Population
2. A ……… of statistical individuals in a population is called a sample. **Ans :** finite subset
3. A finite subset of statistical individuals in a population is called ……….. **Ans :** a sample
4. Any statistical measure computed from sample data is known as ………….. **Ans :** statistic
5. A…………..is one where each item in the universe has an equal chance of known opportunity of being selected. **Ans :** random sample
6. A random sample is a sample selected in such a way that every item in the population has an equal chance of being included **Ans :** Harper
7. Which one of the following is probability sampling **Ans :** simple random sampling
8. In simple random sampling from a population of units, the probability of drawing any unit at the first draw is **Ans :** 1/N units
9. In …………….. the heterogeneous groups are divided into homogeneous groups. **Ans :** a stratified random sample
10. Errors in sampling are of **Ans :** Two types
11. The method of obtaining the most likely value of the population parameter using statistic is called …… **Ans :** estimation
12. An estimator is a sample statistic used to estimate a **Ans :** population parameter
13. …………..is a relative property, which states that one estimator is efficient relative to another. **Ans :** efficiency
14. If probability $P[|\hat{\theta}-\theta|<\varepsilon] \to 1$ as $n \to \infty'$, for any positive ε then $\hat{\theta}$ is said to …………..estimator of θ . **Ans :** consistent
15. An estimator is said to be ………… if it contains all the information in the data about the parameter it estimates. **Ans :** sufficient
16. An estimate of a population parameter given by two numbers between which the parameter would be expected to lie is called an…………….interval estimate of the parameter. **Ans :** interval estimation
17. A _______ is a statement or an assertion about the population parameter. **Ans :** hypothesis
18. Type I error is **Ans :** Reject H0 when it is true
19. Type II error is **Ans :** Accept H0 when it is wrong
20. The standard error of sample mean is **Ans :** $\sigma/\sqrt{n}$

CHAPTER – IX

1. A time series is a set of data recorded **Ans :** Periodically , Weekly , successive points of time
2. A time series consists of **Ans :** Four components
3. The components of a time series which is attached to short term fluctuation is **Ans :** Irregular variation
4. Factors responsible for seasonal variations are **Ans :** Weather, Festivals, Social customs
5. The additive model of the time series with the components T, S, C and I is **Ans :** y=T+S+C+I
6. Least square method of fitting a trend is **Ans :** Most exact
7. The value of 'b' in the trend line y = a+bx is **Ans :** Either positive or negative
8. The component of a time series attached to long term variation is trended as **Ans :** Secular variations
9. The seasonal variation means the variations occurring with in **Ans :** within a year
10. Another name of consumer's price index number is: **Ans :** Cost of living index
11. Cost of living at two different cities can be compared with the help of **Ans :** Consumer price index
12. Laspeyre's index = 110, Paasche's index = 108, then Fisher's Ideal index is equal to: **Ans : 109**
13. Most commonly used index number is: **Ans :** Price index number
14. Consumer price index are obtained by: **Ans :** Family budget method formula
15. Which of the following Index number satisfy the time reversal test? **Ans :** Fisher Index number
16. While computing a weighted index, the current period quantities are used in the: **Ans :** Paasche's method
17. The quantities that can be numerically measured can be plotted on a **Ans :** x bar chart
18. How many causes of variation will affect the quality of a product? **Ans : 2**
19. Variations due to natural disorder is known as **Ans :** random cause
20. The assignable causes can occur due to **Ans :** poor raw materials , unskilled labour , faulty machines
21. A typical control charts consists of **Ans :** CL, LCL, UCL
22. $\overline{X}$ chart is a **Ans :** variable control chart
23. R is calculated using **Ans :** $x_{max} - x_{min}$
24. The upper control limit for $\overline{X}$ chart is given by **Ans :** $\overline{\overline{X}} + A_2\overline{R}$
25. The LCL for R chart is given by **Ans :** $D_3\overline{R}$

CHAPTER – X

1. The transportation problem is said to be unbalanced if ________ **Ans :** Total supply ≠ Total demand

2. In a non – degenerate solution number of allocations is **Ans :** Equal to m+n–1

3. In a degenerate solution number of allocations is **Ans :** less than m+n–1

4. The Penalty in VAM represents difference between the first _______ **Ans :** Smallest two costs

5. Number of basic allocation in any row or column in an assignment problem can be **Ans :** Exactly one

6. North-West Corner refers to _______ **Ans :** top left corner

7. Solution for transportation problem using _________method is nearer to an optimal solution. **Ans :** VAM

8. In an assignment problem the value of decision variable xij is ________. **Ans :** 1 or 0

9. If number of sources is not equal to number of destinations, the assignment problem is called_____________ **Ans :** unbalanced

10. The purpose of a dummy row or column in an assignment problem is to

Ans : balance between total activities and total resources

11. The solution for an assignment problem is optimal if **Ans :** each row and each column has exactly one assignment

12. In an assignment problem involving four workers and three jobs, total number of assignments possible are **Ans : 3**

13. Decision theory is concerned with **Ans :** analysis of information that is available , decision making under certainty, selecting optimal decisions in sequential problem

14. A type of decision –making environment is **Ans :** certainity , uncertainity , risk