DIRECTORATE OF SCHOOL EDUCATION TAMILNADU

12JPCM14 (2023-24)	JEE PRACTICE QUESTIONS (TEST-14)	Class : XII Time : 1.15 hrs Total Marks : 180

Answer key

12th – MATHS

31. Ans : B)

Required area

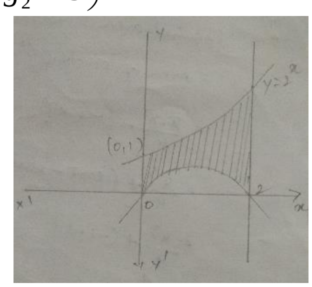
$$=\int_0^2 [2^x-(2x-x^2)]dx$$

$$=\int_0^2 (2^x-2x+x^2)dx$$

$$=\left[\frac{2x}{10g_2}-x^2+\frac{x^3}{3}\right]_0^2$$

$$=\frac{4}{10g_2}-4+\frac{8}{3}-\frac{1}{10g_2}$$

$$=\left(\frac{3}{10g_2}-\frac{4}{3}\right)\text{sq. unit}$$



32. Ans : C)

The curve $y(x^2+4a^2) = 8a^3$ is symmetrical about y – axis and cut it at A $(0, 2a)$

This curve meet S $x^2 = 4ay$

By solving this

$x^4 + 4a^2x^2 - 32a^4 = 0$

$(x^2 - 4a^2)\ (x^2+8a^2) = 0$

$\therefore x = \pm 2a$

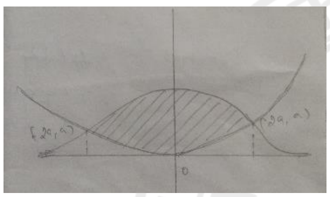
Required area

$$= \; = 2\left[\int_0^{2a} \frac{8a^3}{x^2+4a^2}dx - \int_\theta^{2a} \frac{x^2}{4a}dx\right]$$

$$= 16a^3.\frac{1}{2a}\left[\tan^{-1}\frac{x}{2a}\right]_0^{2a} - \frac{1}{2a}\left[\frac{x^3}{3}\right]_0^{2a}$$

$$= 8a^2.\,\frac{\pi}{4} - \frac{1}{6a}8a^3 \quad = 2\pi a^2 = \frac{4}{3}a^2$$

$$= \;\frac{a^2}{3}\,(6\pi - 4)$$



33. Ans : A)

Required area

= Area of circle (1st quad vent) $- \int_\theta^\pi \sin x dx$

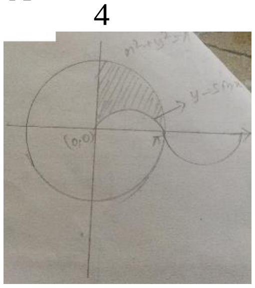
$$= \frac{\pi.\pi^2}{4} - [-\cos x]_0^\pi$$

$$= \frac{\pi^3}{4} + [\cos\pi - \cos 0]$$

$$= \frac{\pi^3}{4} + (-1-1)$$

$$= \frac{\pi^3}{4} - 2$$

$$A = \frac{\pi^3 - 8}{4}$$



34. Ans : B)

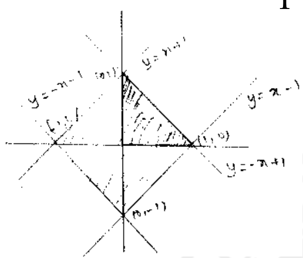
$$y = |x| - 1 = \begin{cases} x-1 & x \geq 0 \\ -x-1 & x < 0 \end{cases}$$

$$y = -|x| + 1 = \begin{cases} -x+1 & x \geq 0 \\ x+1 & x < 0 \end{cases}$$

Required area = $4(\frac{1}{2}bh)$

$= 4(\frac{1}{2} \times 1 \times 1)$

= 2 sq. units



35. Ans : D)

Area $A_1 = \int_0^{\pi/4} \sin x \, dx$

$= -[\cos x]_0^{\pi/4}$

$= 1 - \frac{1}{\sqrt{2}}$

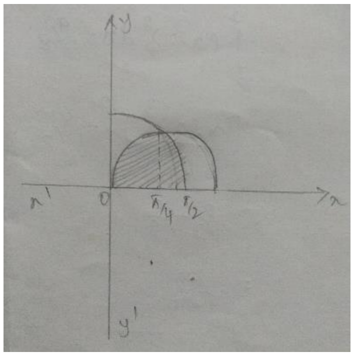
$= \frac{\sqrt{2}-1}{\sqrt{2}}$

Area $A_2 = \int_{\pi/4}^{\pi/2} \cos x \, dx = [\sin x]_{\pi/4}^{\pi/2}$

$= 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}}$

$\therefore A_1 : A_2 = \frac{\sqrt{2}-1}{\sqrt{2}} : \frac{\sqrt{2}-1}{\sqrt{2}}$

$A_1 : A_2 = 1 : 1$



36. Ans : B)

Slope $m = \frac{dy}{dx} = 2x$

$m = 2(2) = 4$

Equation of the tangent

$y - y_1 = m(x - x1)$

$y - 5 = 4(x - 2)$

$4x - y = 3$

Required Area $\int_0^2 [(x^2+1) - (4x-3)]dx - 9/8$

$[Area\ of\ triangle = 1/2 \times 3/4 \times 3 = \frac{9}{8}$

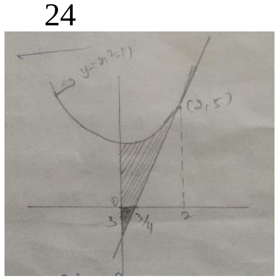
$\int_0^2 [(x - 4x + 4)\, dx - 9/8$

$= \int_0^2 (x-2)^2 dx - 9/8$

$= \left[\frac{(x-2)^3}{3}\right]_0^2 - 9/8$

$= 8/3 - \frac{9}{8}$

$= \frac{37}{24}$



37. Ans : B)

Required Area = $\int_0^4 y\,dx$

$$\int_0^4 2/3\, x^{3/2} + x \Big]_0^4$$

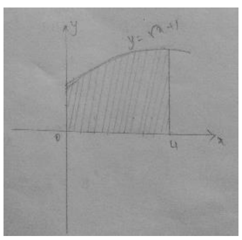
$$= 2/3 (4)^{3/2} + 4$$

$$= 2/3 (4\sqrt{4}) + 4$$

$$A = 28/3$$

$$\therefore\ 3A = 3\left(28/3\right)$$

3A = 28



38. Ans : A)

Let $t = 3 + 2\cos x$

$dt = -2\sin x\, dx$

$$= \frac{1}{2}\int \frac{dt}{\sqrt{t}}$$

$$= \frac{-1}{2}\left[\frac{\sqrt{t}}{1/2}\right] + c$$

$$= -\sqrt{t} + c$$

$$= \sqrt{3 + 2\cos x + c}$$

39. Ans : C)

$$I = \int \frac{\cos^2 x . \cos x}{\sin x(1+\sin x)} dx$$

$$= \int \frac{(1-\sin^2 x)\cos x}{\sin x(1+\sin x)} dx \qquad \text{put } \sin x = t$$

$$= \int \frac{(1+\sin x)(1-\sin x)\cos x}{\sin x(1+\sin x)} dx \qquad \cos x\, dx = dt$$

$$= \int \frac{(1-\sin x)\cos x}{\sin x} dx$$

$$= \int \frac{1-t}{t} dt$$

$$= \log|t| - t + c$$

$$= \log \sin x - \sin x + c$$

40. Ans : C)

$$I = \int \frac{dx}{x(x^n+1)} = \int \frac{dx}{x^{n+1}\left(1+\frac{1}{x^n}\right)}$$

Put $1 + \frac{1}{x_n} = t$

$$\frac{-n}{x^{n+1}} dx = dt$$

$$\frac{dx}{x^{n+1}} = \frac{dt}{-n}$$

$$I = \frac{-1}{n}\int \frac{1}{t} dt = -\frac{1}{n}\log t + c$$

$$= \frac{-1}{n}\log(1+\frac{1}{x^n}) + c$$

$$= \frac{-1}{n}\log(\frac{x^n+1}{x^n}) + c$$

$$\therefore A = \frac{-1}{n}$$

41. Ans: C)

$$I = \int \log x\left(\frac{1}{x^3}\right)dx = \log x\left(\frac{-1}{2x^2}\right) - \int\left(\frac{1}{x}\right)\left(\frac{-1}{2x^2}\right)dx$$

Using integration by parts

$$= \frac{-\log x}{2x^2} - \frac{1}{4x^2} + C$$

$$= \frac{1}{4x^2}(-2\log x - 1) + C$$

Given then $\int \frac{\log x}{x^3} dx = \frac{f(x)}{4x^2} + C$

$$\therefore f(x) = -2\log x - 1$$

$$f(e^2) = -2\log(e^2) - 1$$

$$= -4 \log e - 1$$

$= -4 -1$

$f(e^2) = -5$

42. Ans: B)

$$I = \int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} dx$$

$$= \int \frac{(\sin^4 x - con^4 x)(\sin^4 x + \cos^4 x)}{1 - 2\sin^2 x \cos^2 x} dx$$

$$= \int \frac{(\sin^2 x - con^2 x)(\sin^2 x + \cos^2 x)(\sin 2x + cons^2 x)^2 - 2\sin^2 x \cos^2 x}{1 - 2\sin^2 x \cos^2 x} dx$$

$$= \int \frac{(-2cosx)(1 - 2\sin^2 x\, cos^2 x)}{1 - 2\sin^2 x \cos^2 x} dx$$

$$= \int (-\cos 2x) dx = -\frac{1}{2} \sin 2x + c$$

43. Ans: A)

$$I = \int e^x \left(\frac{x^2 - 3}{(x-1)^2} \right) dx$$

$$= \int e^x \left(\frac{x^2 - 1 - 2}{(x-1)^2} \right) dx$$

$$= \int e^x \left[\left(\frac{x+1}{x-1} \right) - \frac{2}{(x-1)^2} \right] dx$$

$\therefore \int e^x (f(x) + f^1(x) dx) = e^x f(x) + C$ using formala

$$= e^x \left(\frac{x+1}{x-1} \right) + C$$

44. Ans: B)

We have $I_n = \int \tan^n x) dx$

Given $I_4 + I_6 = \int (\tan^4 x + \tan^6 x) dx$

$$= \int \tan^4 (1 + \tan^2 x)\, dx \qquad put\ tan\ x = t$$

$$= \int \tan^4 \sec^2 x\, dx \qquad sec^2 xd\ x = dt$$

$$= \int t^4 dt = \frac{t^5}{5} + C$$

$$= \frac{\tan^5 x}{5} + C$$

$$I_4 + I_6 = a\tan^5 x + bx^5 + c = \frac{1}{5}\left(\tan^5 x\right) + C$$

The ordered pairs $\left(1/5, 0\right)$

45. Ans: C)

$$f(x) = \int e^x (x-1)(x-2)dx$$

For decreasing function $f^1(x) < 0$

$$\Rightarrow e^x (x-1)(x-2) < 0$$

$$\Rightarrow (x-1)(x-2) < 0$$

$$\Rightarrow 1 < x < 2$$

$$\therefore e^x > 0 \neq x \in R$$

The decreasing interval is (1, 2)

DIRECTORATE OF SCHOOL EDUCATION TAMILNADU

11JPCM14 (2023-24)	JEE PRACTICE QUESTIONS (TEST-14)	Class : XI Time : 1.15 hrs Total Marks : 180

Answer key

11^{th} – MATHS

31. Ans : A)

Let $t = 3 + 2\cos x$

$dt = -2\sin x\, dx$

$$= \frac{1}{2}\int \frac{dt}{\sqrt{t}}$$

$$= \frac{-1}{2}\left[\frac{\sqrt{t}}{1/2}\right] + c$$

$$= -\sqrt{t} + c$$

$$= \sqrt{3 + 2\cos x + c}$$

32. Ans : C)

$$I = \int \frac{\cos^2 x.\cos x}{\sin x(1+\sin x)}dx$$

$$= \int \frac{(1-\sin^2 x)\cos x}{\sin x(1+\sin x)}dx$$

put sin x = t

$$= \int \frac{(1+\sin x)(1-\sin x)\cos x}{\sin x(1+\sin x)}dx$$

cos x dx = dt

$$= \int \frac{(1-\sin x)\cos x}{\sin x}dx$$

$$= \int \frac{1-t}{t}dt$$

$$= \log|t| - t + c$$

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33. Ans : C)

$$I = \int \frac{dx}{x(x^n+1)} = \int \frac{dx}{x^{n+1}\left(1+\frac{1}{x^n}\right)}$$

Put $1 + \frac{1}{x_n} = t$

$$\frac{-n}{x^{n+1}}dx = dt$$

$$\frac{dx}{x^{n+1}} = \frac{dt}{-n}$$

$$I = \frac{-1}{n}\int \frac{1}{t}dt = -\frac{1}{n}\log t + c$$

$$= \frac{-1}{n}\log(1+\frac{1}{x^n}) + c$$

$$= \frac{-1}{n}\log(\frac{x^n+1}{x^n}) + c$$

$$\therefore A = \frac{-1}{n}$$

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Using integration by parts

$$= \frac{-\log x}{2x^2} - \frac{1}{4x^2} + C$$

$$= \frac{1}{4x^2}(-2\log x - 1) + C$$

Given then $\int \frac{\log x}{x^3}dx = \frac{f(x)}{4x^2} + C$

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$$f(e^2) = -2\log(e^2) - 1$$

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$$= -4 - 1$$

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$$I = \int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x}dx$$

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$$= \int \frac{(\sin^2 x - con^2 x)(\sin^2 x + \cos^2 x)(\sin 2x + cons^2 x)^2 - 2\sin^2 x \cos^2 x}{1 - 2\sin^2 x \cos^2 x} dx$$

$$= \int \frac{(-2\cos x)(1 - 2\sin^2 x\, cos^2 x)}{1 - 2\sin^2 x \cos^2 x} dx$$

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$$= \int e^x \left[\left(\frac{x+1}{x-1} \right) - \frac{2}{(x-1)^2} \right] dx$$

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$$= e^x \left(\frac{x+1}{x-1} \right) + C$$

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$= \int \tan^4 (1 + \tan^2 x) dx$ *put tan x = t*

$= \int \tan^4 \sec^2 x\, dx$ *$sec^2 xd\, x = dt$*

$$= \int t^4 dt = \frac{t^5}{5} + C$$

$$= \frac{\tan^5 x}{5} + C$$

$$I_4 + I_6 = a\, tan^5 x + bx^5 + c = \frac{1}{5}\left(\tan^5 x\right) + C$$

The ordered pairs $\left(1/5, 0 \right)$

38. Ans: C)

$$f(x) = \int e^x (x-1)(x-2) dx$$

For decreasing function $f^1(x) < 0$

$$\Rightarrow e^{x}(x-1)(x-2)<0$$
$$\Rightarrow (x-1)(x-2)<0$$
$$\Rightarrow 1<x<2$$
$$\therefore e^{x}>0 \neq x\in R$$

The decreasing interval is (1, 2)

39. Ans: B)

$$I=\int \frac{x^{2}-1}{x^{3}\sqrt{x^{4}-2x^{2}+1}}dx$$
$$I=\int \frac{x^{2}-1}{x^{3}\sqrt{(x^{2}-1)^{2}}}dx$$
$$=\int \frac{(x^{2}-1)}{x^{3}(x^{2}-1)^{2}}dx$$
$$=\int \frac{1}{x^{3}}dx$$
$$I=-\frac{1}{2x^{2}}+C$$

40. Ans: B)

Let $I=\int (x^{x})^{x}\left(2x\log_{e}^{x}+x\right)dx$

Put $y=(x^{x})^{x} \Rightarrow y=x^{x^{2}}$

$$\log y=x^{2}\log x$$
$$\frac{1}{y}\frac{dy}{dx}=x^{2}\frac{1}{x}+\log x.2x$$
$$\frac{1}{y}\frac{dy}{dx}=x+2x\log_{e}x$$
$$\frac{dy}{y}=(x+2x)\log_{e}x)\mathrm{dx}$$
$$\therefore I=\int y.\left(\frac{1}{y}\right)dy$$
$$I=(x^{x})^{x}+C$$

41. Ans: B)

$$I=\int x^{2}e^{x/2}dx$$

Using Bernoulli's theorem

u = x^2 $\qquad dV=e^{x/2}$

$u_1^1 = 2x \qquad V = 2e^{x/2}$

$u^{11} = 2 \qquad V_1 = 4e^{x/2}$

$V_2 = 8e^{x/2}$

$$\therefore I = \int x^2 e^{x/2} dx = 2x^2 e^{x/2} - 2x(4e^{x/2}) + 2(8e^{x/2}) + C$$

$$= 2x^2 e^{x/2} - 8xe^{x/2} + 16e^{x/2} + C$$

$\therefore\ \alpha = 2 \qquad \beta = -8 \qquad \gamma = 16$

$\therefore\ \alpha + \beta + \gamma = 10$

42. Ans: A)

$$I = \int \frac{dx}{\cos x + \sqrt{3}\sin x} dx$$

$$= \frac{1}{2}\int \frac{dx}{\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x}$$

$$= \frac{1}{2} \int \frac{dx}{cos\frac{\pi}{3}\, cos\ x + sin\frac{\pi}{3}.sin\ x}$$

$$= \frac{1}{2}\int \frac{dx}{\cos(x - \pi/3)}$$

$$= \frac{1}{2}\int \sec(x - \pi/3) dx$$

$$= \frac{1}{2}\text{logtan}\left(\frac{x}{2} - \frac{\pi}{6} + \frac{\pi}{4}\right) + C$$

$$= \frac{1}{2}\log\ \tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + C$$

43. Ans: A)

$$I = \int \frac{e^x}{(2+e^x)(e^x+1)} d^x$$

Put $e^x = t \quad \Rightarrow \quad e^x dx = dt$

$$= \int \frac{1}{(2+t)(1+t)} dt$$

$$= \int \left[\left(\frac{1}{1+t}\right) - \left(\frac{1}{2+t}\right)\right] dt$$

$$= \log(1+t) - \log(2+t) + C$$

$$= \log\left(\frac{1+t}{2+t}\right) + C$$

$$= \log\left(\frac{1+e^x}{2+e^x}\right) + C$$

44. Ans: A)

$$I = \int \sin(101^x).\sin 99^x dx$$

$$= \int \sin(100x + x).\sin^{99} x dx$$

$$= \int \sin(100x)\cos x.\sin^{99} xdx \,|+ \int \cos/100x).\sin 100^x d^x$$

$$I = I_1 + I_2$$

$$I_1 = \int \sin(100x).\cos x \sin^{99} xdx$$

Using by parts

$$= \sin(100x).\frac{(\sin x)^{100}}{100} - \int \frac{(\sin x)^{100}}{100}.\cos(100x).100dx$$

$$I = \frac{\sin(100x).(\sin x)^{100}}{100} - \int (\sin x)^{100}.(\cos 100x)dx + \int cas(100x)\sin^{100} xdx$$

[$since\ u = sin(100x)$ du = cos(100x).100]

$$\int dV = \int cos\ x\ sin^{99} x\ dx$$

Put sin x = t

cos x dx = dt

$$\int t^{99}\, dt = \frac{t^{100}}{100} + C$$

$$V = \frac{(sin\ x)^{100}}{100} + C$$

$$= \frac{\sin(100x).(\sin x)^{100}}{100} + C$$

45. Ans : A)

Let

$$\left(\frac{x}{e}\right)^{2x} = t$$

$$\log\left(\frac{x}{e}\right)^{2x} = \log t$$

$$2x\log\left(\frac{x}{2}\right) = \log t$$

$$2x[10gx-1] = \log t$$

$$2(10gx-1)+x\left(\frac{1}{x}\right) = \frac{1}{t}dt$$

$$10gx\,dx = \frac{1}{2t}dt$$

$$I = \int(t+\frac{1}{t})\frac{1}{2t}dt$$

$$= \frac{1}{2}\int(1+\frac{1}{t^2})dt$$

$$= \frac{1}{2}(t-\frac{1}{t})+C$$

$$= \frac{1}{2}\left[\left(\frac{x}{e}\right)^{2x} - \left(e/x\right)^{2x}\right]+C$$

$$\therefore \alpha = \beta = \gamma = \delta = 2$$

$$\therefore \alpha+2\beta+3\gamma-4\delta = 2+4+6=-8$$

$$= 4$$