DIRECTORATE OF SCHOOL EDUCATION
TAMILNADU

12JPCM15 (2023-24)	JEE PRACTICE QUESTIONS (TEST-15)	Class : XII Time : 1.15 hrs Total Marks : 180

Answer key

12th – MATHS

31. Ans: D) $2e^{x/y^2} = \frac{1}{y} + c$

$2y^2 e^{\frac{x}{y^2}} dx + (y^2 - 4xe^{x/y^2} y)\,dy = 0$

$2e^{x/y^2}(y^2 dx - 2yxdy) = -y^2 dy$

$2e^{x/y^2} \frac{(y^2 dx - 2xy\ dy)}{y^4} = \frac{-1}{y^2} dy$

$\int 2e^{x/y^2} d\left(\frac{x}{y^2}\right) = \int \frac{-1}{y^2} dy$

$2e^{x/y^2} = \frac{1}{y} + c$

32. Ans B) one

$x = e^{\frac{dy}{dx}} \Rightarrow \frac{dy}{dx} = \log x$

degree = 1

33. Ans : A) 1

Put $x^2 = \sin\alpha$, $y^2 = \sin\beta$

The given eqn becomes,

$\cos\alpha + \cos\beta = a(\sin\alpha - \sin\beta)$

$2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = 2a\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

$\cot\left(\frac{\alpha-\beta}{2}\right) = a \Rightarrow \alpha - \beta = 2\cot^{-1}(\alpha)$

$sin^{-1}(x^2) - sin^{-1}(y^2) = 2cot^{-1}(\propto)$

diff w.r to x

$$\frac{1}{\sqrt{1-x^4}}2x - \frac{1}{\sqrt{1-y^4}}2y\frac{dy}{dx} = 0$$

order = 1

34. Ans: C) $3e^{6y} = 2e^{9x+6} + e^9$

$$log\ \frac{dy}{dx} = 9x - 6y + 6$$

$$\frac{dy}{dx} = e^{9x-6y+6} = e^{9x+6-}e^{6y}$$

$$\int e^{6y}dy = \int e^{9x+6}dx$$

$$\frac{e^{6y}}{6} = \frac{e^{9x+6}}{9} + c$$

When x =0, y = 1 $\Rightarrow c = \frac{e^9}{18}$

$$\Rightarrow \frac{e^{6y}}{6} = \frac{e^{9x+6}}{9} + \frac{e^9}{18}$$

$$\Rightarrow 3e^{6y} = 2e^{9x+6} + e^9$$

35. Ans : A) $(y - xy_1)^2 = P^2(1 + y_1^2)$

$$x\cos\alpha + y\sin\alpha = P \qquad (1)$$

diff w.r to x

$$\cos\alpha + \sin\alpha\frac{dy}{dx} = 0$$

$$\tan\alpha = \frac{-1}{y_1} \text{ where } y_1 = \frac{dy}{dx}$$

$$\Rightarrow \sin\alpha = \frac{1}{\sqrt{1+y_1^2}}, \cos\alpha = \frac{-y_1}{\sqrt{1+y_1^2}}$$

$$(1) \Rightarrow x\frac{-y_1}{\sqrt{1+y_1^2}} + y\frac{1}{\sqrt{1+y_1^2}} = P$$

$$(y - xy_1)^2 = P^2(1 + y_1^2)$$

36. Ans: B) $x^3y = 2$

$$xdy + (y + x^3y^2)dx = 0 \Rightarrow xdy + ydx = x^3y^2dx$$

$$\frac{xdy + ydx}{x^2y^2} = -xdx$$

$$\int \frac{d(xy)}{(xy)^2} = -\int xdx$$

$$\frac{-1}{(xy)} = \frac{-x^2}{2} + C \qquad (1)$$

At (1, 2) $\frac{-1}{2} = \frac{-1}{2} + C$

C = 0

(1) $\Rightarrow -\frac{1}{xy} = \frac{-x^2}{2}$

$$xy = \frac{2}{x^2} \Rightarrow x^3y = 2$$

37. Ans: A) $x^y = cy^x$

$$\frac{dy}{dx} = \frac{y(x\log y - y)}{x(y\log x - x)}$$

$$x(y\log x - x)\frac{dy}{dx} = y(x\log y - y)$$

$$(\log x - \frac{x}{y})\frac{dy}{dx} = \log y - \frac{y}{x}$$

$$\frac{y}{x} + \log x\frac{dy}{dx} = \log y + \frac{x}{y}\frac{dy}{dx}$$

$$\frac{d}{dx}(y\log x) = \frac{d}{dx}(x\log y)$$

$y\ log\ gx = x\ log\ y + log\ c$

$log\ x^y = log\ y^x\ c$

$x^y = cy^x$

38. Ans: A) $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$

$y = (\sin^{-1}x)^2 + A\cos^{-1}x + B$

$y = (\sin^{-1}x)^2 - A\sin^{-1} + \frac{\pi}{2}A + B$

diff w.r to x

$$\frac{dy}{dx} = \frac{2\sin^{-1}x}{\sqrt{1-x^2}} - \frac{A}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\frac{dy}{dx} = 2\sin^{-1}x - A$$

$$(1-x^2)\left(\frac{dy}{dx}\right)^2 = 4(\sin^{-1}x)^2 - 4A\sin^{-1}x + A^2$$

$$(1-x^2)\left(\frac{dy}{dx}\right)^2 = 4y - 2\pi A - 4B + A^2$$

differentiating again w.r to x

$$(-2x)\left(\frac{dy}{dx}\right)^2 = 2(1-x^2)\frac{dy}{dx}\ \frac{d^2y}{dx^2} = 4\frac{dy}{dx}$$

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$$

39. Ans: A) $= \frac{3}{4}$

$P(E_1 \cup E_2 \cup E_3)$ = 1-P $(\overline{E_1 \cup E_2 \cup E_3})$

$$= 1 - P(\overline{E_1} \cap \overline{E_2} \cap \overline{E_3})$$

$$= 1 - P(\overline{E_1})P(\overline{E_2})P(\overline{E_3})$$

$$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$$

$$= 1 - \frac{1}{4}$$

$$= \frac{3}{4}$$

40. Ans: $\frac{7}{2^{12}}$

$$p(n) = \frac{1}{n}$$

Favourable cases (16, 16, 16), (32, 8, 8), (8, 32, 8), (8, 8, 32)

Probability $\frac{1}{16} \times \frac{1}{16} \times \frac{1}{16} + 3\frac{1}{8} \times \frac{1}{8} \times \frac{1}{32}$

$$= \frac{1}{2^{12}} + \frac{3}{2^{11}} = \frac{7}{2^{12}}$$

41. Ans: B) $P(A\cap B)\geq 6, (A\cap \bar{B})\leq \frac{1}{3}$

$P(A\cap B)\geq \frac{1}{2}+\frac{2}{3}-1$

$P(A\cap B)\geq \frac{3+4-6}{6}$

$P(A\cap B)\geq \frac{1}{6}$

$P(A\cap \bar{B}) = P(A) - P(A\cap B)$

$\leq \frac{1}{2}-\frac{1}{6}$

$\leq \frac{1}{3}$

42. Ans: B) $\frac{103}{108}$

Required probability = P (sum ≥ 6)

= I – P (sum < 6)

sum = 3, 4, 5

sum 3 = (1, 1, 1)

sum 4 = (1, 1, 2), (1, 2, 1), (2, 1, 1)

sum 5 = (1, 1, 3), (1, 3, 1), (3, 1, 1), (2, 2, 1), (2, 1, 2), (1, 2, 2)

Total case = 10

n(s) = 216

Req. Prob = $1-\frac{10}{216}=\frac{103}{108}$

43. Ans : A) $\frac{3}{8}$

Let A be the event that a six occurs and B the event that the man reports that it is a six.

$P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}$ ---------- (1)

$P(A\cap B)=\frac{1}{6}\times\frac{3}{4}=\frac{3}{24}$

$P(B)= P(A\cap B)+P(\bar{A}\cap B)$

$=\frac{1}{6}\times\frac{3}{4}+\frac{5}{6}\times\frac{1}{4}=\frac{8}{24}$

$$P\left(A/B\right)=\frac{3/24}{8/24}=\frac{3}{8}$$

44. Ans: C) independent but not equally likely

$P(A\cup B)=\frac{5}{6};P(A)=\frac{3}{4}\quad P(A\cap B)=\frac{1}{4}$ (given)

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\frac{5}{6}=\frac{3}{4}+P(B)-\frac{1}{4}$

$P(B)=\frac{5}{6}-\frac{1}{2}=\frac{5-3}{6}=\frac{1}{3}$

$P(A).P(B)=\frac{3}{4}\times\frac{1}{3}=\frac{1}{4}=P(A\cap B)$

independent, but not equally likely

45. Ans: A) $P(C/D)\geq P(C)$

$C\cap D=C\Rightarrow P(C\cap D)=P(C)$

$P\left(\frac{C}{D}\right)=\frac{P(C\cap D)}{P(D)}=\frac{P(C)}{P(D)}\geq P(C)$

DIRECTORATE OF SCHOOL EDUCATION
TAMILNADU

11JPCM15 (2023-24)	JEE PRACTICE QUESTIONS (TEST-15)	Class : XI Time : 1.15 hrs Total Marks : 180

Answer key

11th – MATHS

31. Ans: A) $=\frac{3}{4}$

$P(E_1 \cup E_2 \cup E_3) = 1\text{-}P(\overline{E_1 \cup E_2 \cup E_3})$

$=1-P(\overline{E_1} \cap \overline{E_2} \cap \overline{E_3})$

$=1-P(\overline{E_1})P(\overline{E_2})P(\overline{E_3})$

$=1-\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}$

$=1-\frac{1}{4}$

$=\frac{3}{4}$

32. Ans: $\frac{7}{2^{12}}$

$p(n)=\frac{1}{n}$

Favourable cases (16, 16, 16), (32, 8, 8), (8, 32, 8), (8, 8, 32)

Probability $\frac{1}{16}\times\frac{1}{16}\times\frac{1}{16}+3\frac{1}{8}\times\frac{1}{8}\times\frac{1}{32}$

$=\frac{1}{2^{12}}+\frac{3}{2^{11}}=\frac{7}{2^{12}}$

33. Ans: B) $P(A \cap B) \geq 6$, $P(A \cap \bar{B}) \leq \frac{1}{3}$

$P(A \cap B) \geq \frac{1}{2}+\frac{2}{3}-1$

$P(A \cap B) \geq \frac{3+4-6}{6}$

$P(A \cap B) \geq \frac{1}{6}$

$P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$\leq \frac{1}{2} - \frac{1}{6}$

$\leq \frac{1}{3}$

34. Ans: B) $\frac{103}{108}$

Required probability = P (sum $\geq$ 6)

= I – P (sum < 6)

sum = 3, 4, 5

sum 3 = (1, 1, 1)

sum 4 = (1, 1, 2), (1, 2, 1), (2, 1, 1)

sum 5 = (1, 1, 3), (1, 3, 1), (3, 1, 1), (2, 2, 1), (2, 1, 2), (1, 2, 2)

Total case = 10

n(s) = 216

Req. Prob = $1 - \frac{10}{216} = \frac{103}{108}$

35. Ans : A) $\frac{3}{8}$

Let A be the event that a six occurs and B the event that the man reports that it is a six.

$P\left(A/B\right) = \frac{P(A \cap B)}{P(B)}$ (1)

$P(A \cap B) = \frac{1}{6} \times \frac{3}{4} = \frac{3}{24}$

$P(B) = P(A \cap B) + P(\bar{A} \cap B)$

$= \frac{1}{6} \times \frac{3}{4} = \frac{5}{6} \times \frac{1}{4} = \frac{8}{24}$

$P\left(A/B\right) = \frac{3/24}{8/24} = \frac{3}{8}$

36. Ans: C) independent but not equally likely

$P(A\cup B)=\frac{5}{6}; P(A)=\frac{3}{4}\quad P(A\cap B)=\frac{1}{4}$ (given)

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\frac{5}{6}=\frac{3}{4}+P(B)-\frac{1}{4}$

$P(B)=\frac{5}{6}-\frac{1}{2}=\frac{5-3}{6}=\frac{1}{3}$

$P(A).P(B)=\frac{3}{4}\times\frac{1}{3}=\frac{1}{4}=P(A\cap B)$

independent, but not equally likely

37. Ans: A) $P(C/D)\geq P(C)$

$C\cap D=C\Rightarrow P(C\cap D)=P(C)$

$P\left(\frac{C}{D}\right)=\frac{P(C\cap D)}{P(D)}=\frac{P(C)}{P(D)}\geq P(C)$

38. Ans: A) 0.54

$P(A)=\frac{60}{100}\Rightarrow P(\bar{A})=\frac{40}{100}$

$P(B)=\frac{70}{100}\Rightarrow P(\bar{B})=\frac{30}{100}$

Required prob = $P(A\cap B)+P(\bar{A}\cap\bar{B})$

= $P(A)P(B)+(\bar{A})P(\bar{B})$

= 0.60 × 0.70 + 0.40 × 0.30

= 0.42 + 0.12

= 0.54

39. Ans : D) $\frac{1}{2}$

Required prob = $\frac{^2C_2\times{}^2C_1}{^4C_3}=\frac{1\times 2}{4}=\frac{1}{2}$

40. Ans : B) $\frac{2}{5}$

$P(s)=\frac{2}{6}=\frac{1}{3}; P(F)=\frac{2}{3}$

Probability that the success occurs in even number of times = P(FS) + P(FFFS) + P(FFFFFS) + ……

$= qp + q^3p + q^5p + \ldots\ldots$

$= qp\,(1 + q^2 + q^4 + \ldots\ldots)$

$= qp\,(1 - q^2)^{-1}$

$= \frac{2}{3} \times \frac{1}{3}\left(1 - \frac{4}{9}\right)^{-1}$

$= \frac{2}{9}\left(\frac{5}{9}\right)^{-1} = \frac{2}{9} \times \frac{9}{5} = \frac{2}{5}$

41. Ans: $\frac{7}{8} \le P(A) + P(B) \le \frac{11}{8}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$1 \ge P(A) + P(B) - P(A \cap B) \ge 3/4$

$P(A) + P(B) - \frac{1}{8} \ge 3/4$

$P(A) + P(B) \ge 7/8$

Also $1 \ge P(A) + P(B) - \frac{3}{8}$

$P(A) + P(B) \le \frac{11}{8}$

$\frac{7}{8} \le P(A) + P(B) \le \frac{11}{8}$

42. Ans: D) 4

$n(s) = 6^3 = 216$

$n(A) = 6 \times 5 \times 4$

Prob $= \frac{6 \times 5 \times 4}{216} = \frac{5}{9} = \frac{p}{q}$

So $q - p = 4$

43. Ans: B) $\frac{19}{36}$

Total number of three digit number = 900

Total numbers with at least two odd digit numbers

= OOO + EOO + OEO + OOE

= 5 × 5 × 5 +4 × 5 × 5 + 5 × 5 × 5 + 5 × 5 × 5

= 25 × (5 + 4 + 5 + 5)

= 25 × 19

Req. prob $= \frac{25\times 19}{900} = \frac{19}{36}$

44. Ans B) 33

No of numbers formed by 8 and 1 of 8 digits which is divisible by 3 as well as 7 will contain three 8 and three 1 like given below

8	8	8	1	1	1

1	8	1	8	1	8

etc

and any number of 6 digits using same number will be divisible by 3 and 7 so two cases will arise

like

1	1	1	1	1	1

&

8	8	8	8	8	8

prob = p $= \frac{\frac{\lfloor 6}{\lfloor 3 \lfloor 3}+2}{2^6} = \frac{11}{32}$

$96P = 96\times\frac{11}{32} = 33$

45. Ans D) $P(E_1 \cap \overline{E_2}) = P(E_1)P(E_2)$

$P(E_1 \cap E_2) = \frac{1}{8} = P(E_1)P\left(\frac{E_2}{E_1}\right) = P(E_2)P\left(\frac{E_1}{E_2}\right)$

$P(E_1) = \frac{\frac{1}{8}}{\frac{3}{4}} = \frac{1}{8}\times\frac{4}{3} = \frac{1}{6}$

$P(E_2) = \frac{1}{8}\times\frac{2}{1} = \frac{1}{4}$

(1) $P(E_1)P(E_2)=\frac{1}{24}\neq P(E_1\cap E_2)$

(2) $P(\overline{E}_1)P(\overline{E}_2)=\frac{5}{6}\times\frac{3}{4}=\frac{5}{8}$

$$P(\bar{E}_1\cap\bar{E}_2)=P(\overline{E_1\cup E_2})=1-P(E_1\cup E_2)=1-\left(\frac{1}{6}+\frac{1}{4}-\frac{1}{8}\right)$$

$$=1-\left(\frac{8+12-6}{48}\right)=1-\frac{14}{48}=1-\frac{7}{24}=\frac{17}{24}$$

$P(\overline{E}_1\cap\overline{E}_2)\neq P(E_1)P(E_2)$

(3) $P(\bar{E}_1\cap \mathrm{E}_2)=P(E_2)-P(E_1\cap E_2)=\frac{1}{4}-\frac{1}{8}=\frac{1}{8}\neq P(E_1)P(E_2)$

(4) $P(\bar{E}_1\cap \mathrm{E}_2)=P(E_1)-P(E_1\cap E_2)=\frac{1}{6}-\frac{1}{8}=\frac{8-6}{48}=P(E_1)P(E_2)$